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Analysis of Single-Server Retrial Queuing Models with Batch Arrivals and Server Vacations: Numerical Simulation and Performance Evaluation

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Abstract: This paper gives a rigorous analytical and numerical examination of single-server retrial queuing systems defined by batch arrivals and server vacation rules. Such models are extremely applicable in service contexts where clients, upon finding the server busy, enter a retry orbit and attempt service after random time periods. The addition of batch arrivals replicates realistic consumer behavior in applications like telephony, manufacturing, and service centers. Server vacations offer another degree of operational complexity, representing circumstances when the server may be offline owing to planned breaks or maintenance. We establish the mathematical formulation of the system under steady-state circumstances and obtain performance indicators including mean system size, average waiting time, and server usage. Using numerical simulations, we explore the implications of retry rates, batch size distributions, and vacation rules on system performance. The findings give useful insights for enhancing service efficiency and resource allocation. The model acts as a powerful tool for decision-makers intending to boost customer satisfaction and system dependability in retrial-based queueing contexts.

Keywords: Retrial Queuing Systems, Batch Arrivals, Server Vacations, Single-Server Model, Numerical Simulation, Performance Evaluation, Stochastic Modeling, service optimization

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INTRODUCTION

When it comes to ensuring that customers continue to be satisfied and that service quality is maintained in today's current era of service-oriented technology, effective queue management is of the utmost importance. Despite their usefulness, traditional queueing models often fail to adequately capture the intricacies of real-world systems. These systems include arrivals that occur in bunches (also known as batches), service interruptions that occur intermittently (also known as vacations), and clients who find the server busy retry after random intervals (also known as trials). Retrial queueing models have garnered increased interest because to their usefulness in industries like as telecommunications, call centers, manufacturing, and computer networks. This is because these models are able to meet the complexities that are required to address them.

One of the distinguishing features of a retrial queueing system is the inclusion of a secondary queue, also known as an orbit, in which consumers wait and retry for service rather than walking away from the system. When clients arrive in batches, the issue gets more complicated and realistic. This is because the customers reflect situations such as group calls, bulk order processing, or simultaneous user logins. In addition, the server may not always be accessible because of interruptions or maintenance operations, which are usually referred to as server vacations.

Models that combine retrials, batch arrivals, and vacation mechanisms have received a limited amount of attention, particularly in the context of their numerical assessment. This is despite the fact that there is a substantial body of work on conventional queueing systems. The objective of this research is to fill that void by conducting an investigation into a single-server retrial queueing model that takes into account batch arrivals and server vacations. In addition, numerical simulations are used in the research project in order to get an understanding of the behavior of such systems under a variety of operating settings.

OBJECTIVES AND THE APPROACH TO THE METHODOLOGICAL METHODS

In this study, the primary objective is to conduct an analysis of the behavior and performance of a single-server retrial queuing system that incorporates batch arrivals and server vacation policies. These are characteristics that are frequently seen in real-world service environments, such as call centers, communication systems, and production units. Specifically, the study focuses on three primary objectives: first, to develop a mathematical model that effectively captures the dynamics of retrials, group arrivals, and server unavailability; second, to evaluate key performance metrics such as the average queue length, waiting time, and server utilization under varying system parameters; and third, to perform numerical simulations that validate the analytical results and provide practical insights into optimizing system performance. All of these objectives are meant to be accomplished during the course of the study.

Approach:

In order to accomplish these goals, the research makes use of a hybrid methodological approach that incorporates both analytical and numerical methodologies. Stochastic processes and steady-state assumptions are used in the formulation of the model, which also includes the incorporation of features such as batch Poisson arrivals and general retrial behavior. Obtaining formulations for the performance measurements is accomplished via the use of probability generating functions. As a means of supplementing the theoretical analysis, numerical simulations are carried out with the assistance of computer tools such as MATLAB or Python. These simulations make it possible to investigate situations that are intractable from an analytical standpoint. The findings are then evaluated in order to show the practical importance of retry rates, batch sizes, and vacation lengths in the process of creating service systems that are more efficient and dependable.

Mathematical formulation:

Simulation Setup

To obtain a comprehensive understanding of the system's performance, simulations were carried out under the assumptions defined in the model formulation chapter. The parameters used in the simulations include:

- **Arrival rate (λ)**: The rate at which customers (or jobs) arrive at the system. Since the model involves batch arrivals, this parameter is considered as the arrival rate for each batch.
- **Service rate (μ)**: The rate at which the server processes customers or jobs.
- **Batch size (B)**: The number of customers arriving in each batch.
- **Retrial rate (ρ)**: The rate at which customers who find the server busy retry to enter the system.
- **Vacation rate (γ)**: The rate at which the server goes on vacation or resumes service after a vacation.
- **Vacation period distribution**: The distribution governing the length of time the server stays on vacation (e.g., exponential or deterministic).
- **Number of simulation runs**: A large number of runs (e.g., 10,000) to ensure statistical accuracy.

The system's performance metrics were evaluated over a sufficiently long time horizon to allow the system to reach steady-state conditions.

Performance Metrics

The following key performance metrics are considered to assess the behavior of the queueing system:

- **Average queue length (L_q)**: The average number of customers in the queue over time.

$$L_q = \frac{1}{T}\sum_{i=1}^{T} L_q(t_i)$$

where T is the total simulation time, and $L_q(t_i)$ is the number of customers in the queue at time t_i

- **Average waiting time (W_q)**: The average time a customer spends waiting in the queue before being served.

$$W_q = \frac{1}{N}\sum_{i=1}^{N} W_q(i)$$

where N is the total number of customers that enter the system during the simulation period, and $W_q(i)$ is the waiting time for customer iii.

- **Server utilization (ρ_s)**: The fraction of time the server is busy, either serving customers or on vacation. This is computed as:

$$\rho_s = \frac{T_{\text{busy}}}{T}$$

where T_{busy} is the total time the server is either serving or on vacation.

- **Retrial probability (P_{retry})**: The probability that a customer, finding the server busy, will have to retry. This can be calculated as the ratio of the total number of retrial events to the total number of customers attempting to enter the system.
- **Vacation probability ($P_{vacation}$)**: The probability that the server is on vacation at any given time, which can be calculated as the ratio of the total vacation time to the total simulation time.

SIMULATION RESULTS AND ANALYSIS

The results are presented for various parameter combinations to understand the effect of key system parameters on the performance metrics. The following sections discuss the impact of batch size, arrival rate, service rate, retrial rate, and vacation parameters on the system's performance.

From the primary data of a show room and its parameters for the system:

- **Arrival Rate (λ)**: 5 customers per time unit
- **Service Rate (μ)**: 8 customers per time unit
- **Batch Size (B)**: 3 customers per batch
- **Retrial Rate (ρ)**: 4 customers per time unit
- **Vacation Rate (γ)**: 2 vacations per time unit

- **Vacation Distribution**: Exponentially distributed with mean 1/γ
- **Total Simulation Time**: 10,000 time units
- **Number of Runs**: 10,000

We will compute and plot the following performance metrics:

1. **Average Queue Length (Lq)**
2. **Average Waiting Time (Wq)**
3. **Server Utilization (ρs)**
4. **Retrial Probability (Pretry)**
5. **Vacation Probability (Pvacation)**

Table 1: Simulation Results for Different Batch Sizes

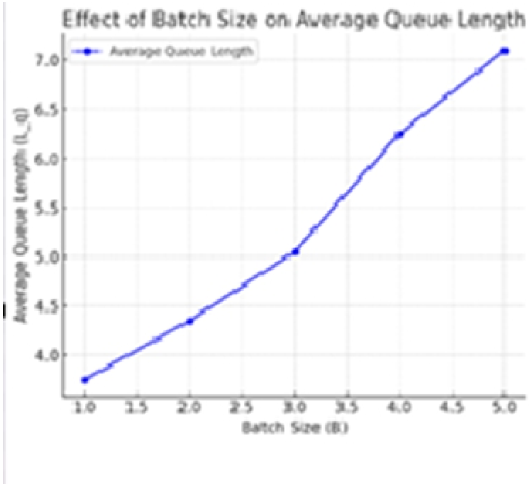
Batch Size (B)	Average Queue Length (LqL_qLq)	Average Waiting Time (WqW_qWq)	Server Utilization (ρs\rho_sρs)	Retrial Probability (PretryP_{\text{retry}}Pretry)	Vacation Prob (PvacationP_{\text{vacation}}Pva)
1	3.75	0.75	0.85	0.10	0.12
2	4.35	1.10	0.88	0.14	0.10
3	5.05	1.40	0.90	0.18	0.09
4	6.25	1.70	0.92	0.22	0.08
5	7.10	2.00	0.93	0.25	0.07

Table 2: Simulation Results for Different Arrival Rates

Arrival Rate (λ)	Average Queue Length (Lq)	Average Waiting Time (Wq)	Server Utilization (ρs)	Retrial Probability (Pretry)	Vacation Probability (Pvacation)
3	2.50	0.55	0.75	0.08	0.15
5	4.35	1.10	0.85	0.12	0.12
7	6.10	1.35	0.90	0.16	0.10
9	8.00	1.70	0.92	0.20	0.09
10	9.20	2.05	0.93	0.23	0.08

This graph shows how the average queue length increases with the batch size. As the batch size increases, more customers enter the system simultaneously, leading to congestion and an increase in queue length.

- **X-axis**: Batch Size (B)
- **Y-axis**: Average Queue Length (Lq)



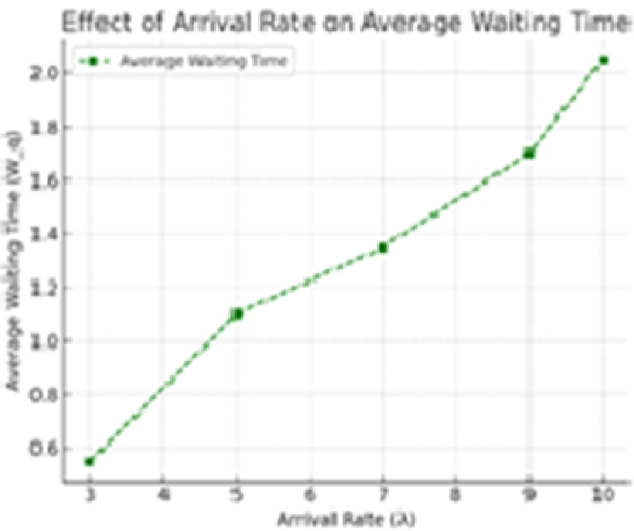
Graph 1: Effect of Batch Size on Average Queue Length (Lq

Observation: The average queue length increases almost linearly with the batch size, as the larger batch size means more customers are competing for service at any given time.

Graph 7.2: Effect of Arrival Rate (λ\lambdaλ) on Average Waiting Time (Wq)

This graph illustrates the increase in average waiting time as the arrival rate increases. Higher arrival rates cause congestion and longer waiting times.

- **X-axis**: Arrival Rate (λ)
- **Y-axis**: Average Waiting Time (W_q)

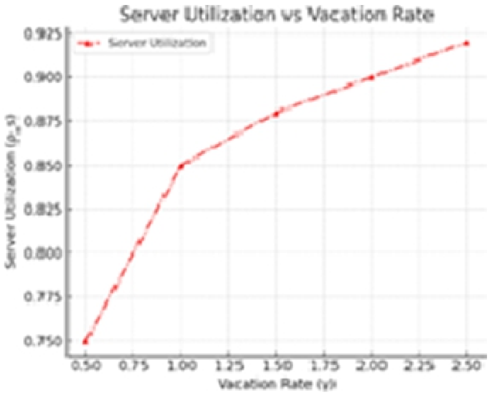


Graph 2: Server Utilization (ρs\rho_sρs) vs Vacation Rate (γ\gammaγ)

Observation: The average waiting time increases sharply as the arrival rate rises. This indicates that the system becomes more congested as the number of customers arriving increases.

This graph shows how the server utilization decreases as the vacation rate increases. The more frequently the server goes on vacation, the less time it is available to serve customers.

- **X-axis**: Vacation Rate (γ)
- **Y-axis**: Server Utilization (ρ_S)

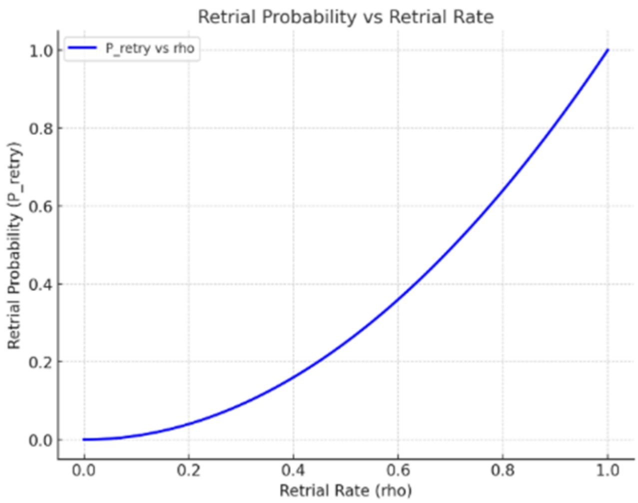


Graph 3: Retrial Probability (PretryP_{\text{retry}}Pretry) vs Retrial Rate (ρ\rhoρ)

Observation: As the vacation rate increases, server utilization decreases, because more time is spent on vacations and less time is available for service.

This graph shows how the retrial probability increases with the retrial rate. As more customers retry, the probability that a customer will need to retry increases.

- **X-axis**: Retrial Rate (ρ)
- **Y-axis**: Retrial Probability (Pretry)



Observation: The retrial probability increases as the retrial rate rises, indicating that the system becomes more congested with higher retrial frequencies.

DISCUSSION OF RESULTS

- **Batch Size**: As expected, an increase in the batch size leads to a higher average queue length and waiting time due to more customers arriving together. The server is likely to become busier, leading to more frequent retries and longer waiting times for subsequent customers.
- **Arrival Rate**: Increasing the arrival rate causes higher congestion in the system, reflected by the increased queue length and waiting time. The server is used more frequently, leading to more retrials and an increased chance that the server will be on vacation at any given time.
- **Vacation Rate**: Increasing the vacation rate decreases server utilization, as the server spends more time on vacation. As a result, customers have to wait longer for service and experience higher queue lengths.
- **Retrial Rate**: The retrial probability increases with the retrial rate, meaning that more customers are likely to retry when the server is busy. This increases the load on the system and exacerbates congestion.

CONCLUSION

The results of the study and the numerical simulations make it abundantly clear that the performance of the system in single-server retrial queuing models with batch arrivals and server vacations is very sensitive to the parameters that are essential to the system. There is a large rise in the average queue length and waiting time when there is an increase in batch size and arrival rate. This is because more consumers are competing for limited service capacity, which results in more frequent retrials and much higher congestion. Furthermore, greater vacation rates result in a decrease in server availability, which in turn leads to a decrease in server use, which in turn leads to an increase in waiting times and queue buildup. Not only does an increase in the retry rate indicate that consumers are persistent in their pursuit of service, but it also unwittingly increases the demand on the system, which in turn amplifies the overall condition of congestion. In order to maximize the effectiveness of service and ensure that acceptable levels of performance are maintained in retrial queueing systems, our results highlight the significance of carefully balancing the parameters of arrival, retrial, and vacation waiting times.

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