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Analysis of Single Server Unreliable and Removable Server Queueing model with Partial Breakdown and Balking Behavior

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Abstract: In systems with an unreliable and removable server, this study investigates an M/M/1 queueing model that accounts for consumer behavior and setup. During maintenance, working vacations, and setup, the server experiences partial failures. Customers arrive at a Poisson distribution with an arrival rate of λ. The server goes into a working vacation mode at a rate of ψ while the system is empty. The service rate drops from from to due to unexpected partial server breakdowns during the working vacation. If there are still customers waiting at the end of a vacation, the server resumes normal operations at a rate of . At a rate of α, the server enters a shutdown state when there are no customers. If customers arrive when the system shuts down, the server has to be restarted. With chance q, the server will fail during this restart procedure; with probability p, it will return to its typical busy state. The study develops closed-form equations for the steady-state probabilities and assesses a number of performance measures, such as expected queue length, system waiting time, and probabilities of the system's state during various server operating phases. Numerous system parameter's numerical effects on various performance metrics are displayed in tables and visualizations.

Keywords: Partial breakdown, removable server, repair, setup time, working vacation, unreliable server, Impatient Customer

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INTRODUCTION

One way to improve customer service and resource distribution is to use queuing theory as a scientific basis. This is very important in many fields, like healthcare, industry, telecommunications, and finance, because it's main goal is to find a balance between operating efficiency and cost. Queuing models can help organizations figure out why there is congestion and how to make systems that provide reliable and quick service while cutting costs and wait times? In order to work around real issues, we have greatly expanded the M/M/1 queueing model, which is a basic tool in operations management. It's a big step forward in this field that working vacation models let servers keep running at a lower capacity during downtime instead of stopping all service. Any business, company, or group in the real world doesn't want to lose customers because it can hurt their brand and revenue. Problems with service often cause customers to leave a business. An important reason for these kinds of problems is that the service system breaks down, making customers wait longer. Problems with hardware, software, technical breakdowns, repair needs, or unplanned downtime are some of the reasons why things can go wrong. ISPs, airlines, public transit, retail checkout systems, hospital emergency rooms, banks, and industrial machinery problems are all real-life examples of situations where services go down because of system failures.

When a server has a partial breakdown, it can still provide some level of service even though it isn't fully working. When a server has a full breakdown, on the other hand, it stops working totally until it is fixed or replaced. The repair time is the amount of time needed to fix a computer that has broken down and can't serve customers for a short time. In accordance with the queueing theory, the amount of time required to get a server ready to serve customers once more is referred to as the setup time. The process may include a number of steps, including the configuration of the server, the addition of any data or software that is required, the modification of the settings, and the verification that the system is prepared to process requests.

According to the queueing theory, an unreliable server is one that has the potential to malfunction, experience hardware difficulties, or experience program issues, and thus stop functioning or being available at any given moment. A server that is unreliable or unstable can have a significant influence on the operation of queues as well as on performance metrics for the system, such as throughput, waiting times, and line length measurement. A removable server is a server that may be temporarily switched off for reasons such as maintenance, user weariness, energy-saving measures, or hardware difficulties. This type of server is included in the concept of queues. The activation or deactivation of these servers is determined by the requirements of the system. When a server is restarted, it requires some time to be set up before it can begin functioning normally again. During this stage, tasks like as initializing the system, adding any data or software that it requires, modifying settings, and ensuring that it is prepared to handle requests are carried out. The conduct of customers who are very impatient is a significant issue for commercial organizations, whether they are public or private. People tend to get restless and impatient when they are required to wait in queue for services. When we are waiting, a lot of us experience feelings of anxiety and irritability. When it comes to the study of queues, making assumptions about duration of wait time makes the model more practical and adaptive.

The purpose of this section is to provide a comprehensive review of major research on the M/M/1 model, which is equipped with an unstable server and a working vacation, as well as with impatient customers and repair services. The concept of working vacation was originally presented by Servi and Finn (2002) within the context of a single-server queueing architecture. In order to ascertain the steady-state distributions of system size and sojourn duration, they approached the problem using the PGF method. It was investigated how these findings may be used to the evaluation of the performance of gateway routers in fiber communication networks. Liu et al. (2007) conducted an investigation on the stochastic decomposition of waiting time and queue length. They did this by utilizing the queueing model developed by Servi and Finn (2). It was the matrix geometric approach that they utilized in order to investigate the proposed model.

Yadin and Naor (1963) improved the cost structure of a single-server Markovian queue with partially shut down and setup states by analyzing the system features. An M/M/1/∞ queue with a single unreliable server that could malfunction while in use was studied by Liou (2013), taking into account consumer impatience during outages. In a single-server queueing system that was difficult to manage and unreliable, Singh and Jain (2013) examined how several performance measures evolved over time using Laplace transforms. The idea of a waiting server, as well as malfunctions and repairs during peak hours, was first presented by Gupta and Kumar (2021). To examine the behavior of an unreliable and removable server, Wang and Hsieh (1995) created an economic cost model. A study by Economou et al. (2011) looked at how customers might balk in a single-server line with widely dispersed service and vacation periods. Table 1 presents an overview of the literature on the evaluation of a model with working vacation, an anxious client, an unreliable server, and repair. Scholars employ many methodologies to identify both transient and steady-state solutions for their models.

Table 1 Literature on unreliable server with working vacation and customer's impatience

Authors	Key features	Technique
Jain and Preeti (2014)	MRP, Standby, MWV, Server breakdown	Matrix recursive Method
Yang and Wu (2015)	MWV, Unreliable server, N-policy,	MGM
He et al. (2019)	SWV,MRP	MAM
Laxmi et al. (2021)	VWV, Customer impatience, Server Breakdowns	PGF
Thilaka et al. (2019)	SWV, Catastrophe, Customer impatience	PGF, LST
Pikkala and Edadasari (2021)	VWV, Unreliable server, Second optional service, Reneging	PGF
Seenivasan et al. (2022)	Server breakdown, Feedback, SWV, State Reliant customers	MGM
Majid (2022)	Customers' impatience, Vacation interruption, , MWV	PGF
Manoharan and Jeeva (2020)	WV, Impatient customers, set up times, Vacation interruption	PGF
Azhagappan and Deepa (2020)	Variant impatience, Balking, MWV	Continued fractions, PGF
Ganie and Manoharan (2018)	Impatient customers, SWV, MWV	PGF

The article is organized as follows: section 2 gives a full description of the model, section 3 discusses mathematical formulation, section 4 discusses steady state probabilty, section 5 provides graphical illustrations, section 6 concludes the chapter.

MODEL DESCRIPTION

This article examines the notion of setup with repair, as well as an M/M/1 queueing model with an unreliable and removable server that fails intermittently while on vacation. An exponential distribution with rate μ_a reflects the rate of service during busy periods and a rate μ_w, where $(\mu_w < \mu_a)$, during working vacations, whereas a Poisson distribution with rate λ explains the arrival of customers. When a customer comes, they have the option of joining the queue with probability b or leaving the system with probability 1-b. During idle time, the server takes a single working vacation using the option ψ. Customers receive service at a lesser rate than during a typical busy period when the server encounters a partial breakdown, also known as a soft failure, while on vacation because no physical repair is required. During vacation, the server closes down at an exponential rate (α). Because the server is removable, it is shut off at this point. When the server is not operating, customers must wait till it is turned on. Restarting the server requires a specific period of time, known as setup time, with rates m that follows an exponential distribution. Because the server is considered unreliable, it may fail during setup activation with a probability of q=p-1; otherwise, it will return to the busy phase with a probability of p. The failed or broken server is sent to be fixed at a rate δ with exponential distributed

MATHEMATICAL FORMULATION

Let P_{in} denotes Probability of n customers during i state of server, where i= 0,1,2,3 and n= 0, 1, 2, 3…… and server states at time t by S (t) where

$$S(t) = \begin{cases} 0 & \text{server in working vacation state} \\ 1 & \text{server in busy period state} \\ 2 & \text{server in closed down state} \\ 3 & \text{server in repair state} \end{cases}$$

Here are the balance equations for each server state:

1. **For busy state**

$$P_{11}(\lambda b+\mu_w)=\mu_w P_{12}+\psi P_{01}+mpP_{21}+\delta P_{31}, \qquad n=1 \qquad (1)$$

$$P_{1n}(\lambda b+\mu_w)=\mu_w P_{1n+1}+\psi P_{0n}+mpP_{2n}+\delta P_{3n}+\lambda b P_{1n-1}, \qquad n\geq 2 \qquad (2)$$

2. **For working vacation state**

$$P_{00}(\lambda b+\alpha)=\mu_a P_{01}+\mu_w P_{11}, \qquad n=0 \qquad (3)$$

$$P_{0n}(\lambda b+\mu_a+\psi)=\mu_a P_{0n+1}+\lambda b P_{0n-1}, \qquad n\geq 1 \qquad (4)$$

3. **For closed down state**

$\alpha P_{00}=\lambda b P_{20}$, n=0 (5)

$P_{2n}(\lambda b+mq+mp)=\lambda b P_{2n-1}$, n≥1 (6)

4. **For repair state**

$P_{31}(\lambda b+\delta)=mqP_{21}$, n=1 (7)

$P_{3n}(\lambda b+\delta)=mqP_{2n}+\lambda b P_{3n-1}$, n≥2 (8)

4. Steady State Probabilities and some System Performances (Transient Analysis)

Consider probability generating function (PGF)

$F_0(z)=\sum_{n=0}^{\infty} P_{0n}\, z^n$ (9)

$F_1(z)=\sum_{n=1}^{\infty} P_{1n}\, z^n$ (10)

$F_2(z)=\sum_{n=0}^{\infty} P_{2n}\, z^n$ (11)

$F_3(z)=\sum_{n=1}^{\infty} P_{3n}\, z^n$ (12)

Such that $F_0(1)+F_1(1)+F_2(1)+F_3(1)=1$ (13)

Multiply equation (2) by z^n and taking summation over n, we get,

$(\lambda b+\mu_w)\sum_{n=2}^{\infty} P_{1n}\, z^n =$
$\mu_w \sum_{n=2}^{\infty} P_{1n+1}\, z^n+\psi\sum_{n=2}^{\infty} P_{0n}\, z^n+mp\sum_{n=2}^{\infty} P_{2n}\, z^n+\delta\sum_{n=2}^{\infty} P_{3n}\, z^n+\lambda b$
$\sum_{n=2}^{\infty} P_{1n-1}\, z^n$

On simplifying above equation, and using equation (9), (10), (11) and (12), we get

$F_1(z)\,(\lambda bz-\mu_w)=z[\psi F_0(z)-\psi P_{00}+mpF_2(z)-mpP_{20}+\delta F_3(z)-2\mu_w P_{11}]$
(14)

Taking limit $z\rightarrow 1$ in (14) we get,

$F_1(1) = \frac{1}{(\lambda b - \mu_w)}[\psi F_0(1) - \psi P_{00} + mpF_2(1) - mpP_{20} + \delta F_3(1) - 2\mu_w P_{11}] = P(B)$ (15)

Differentiate equation (14) both side w.r.t. z and taking limit $z \to 1$, we get,

$F_1'(1) = \frac{1}{(\lambda b - \mu_w)}[mpF_2(1) + mpF_2'(1) - mpP_{02} + \psi F_0(1) + \psi F_0'(1) - \psi P_{00} + \delta F_3(1)$

$+ \delta F_3'(1) \quad - 2\mu_w P_{11} - \lambda b F_1(1)] = E(L_1)$ (16)

Multiply equation (4) by z^n and taking summation over n, we get,

$(\lambda b + \mu_a + \psi)\sum_{n=1}^{\infty} P_{0n} z^n = \mu_a \sum_{n=1}^{\infty} P_{0n+1} z^n + \lambda b \sum_{n=1}^{\infty} P_{0n-1} z^n$

When we simplify the equation above and apply the equation (9), we obtain

$F_0(z) = \frac{(\lambda bz + \mu_a z + \psi z - \mu_a)P_{00} - \mu_a z P_{01}}{(\lambda bz + \mu_a z + \psi z - \mu_a - \lambda b z^2)}$ (17)

When we take the limit of $z \to 1$ in (17), we obtain the following:

$F_0(1) = \frac{(\lambda b + \psi)P_{00} - \mu_a P_{01}}{\psi} = P(WV)$ (18)

Equation (17) is differentiated on both sides with respect to z, and by considering the limit $z \to 1$, we obtain

$F_0'(1) = \frac{1}{\psi}[(\lambda b + \mu_a + \psi)P_{00} - \mu_a P_{01} - F_0(1)\,(\mu_a + \psi - \lambda b)] = E(L_0)$ (19)

Multiply equation (6) by z^n and taking summation over n, we get,

$(\lambda b+mp+mq)\sum_{n=1}^{\infty} P_{2n}\, z^n=\lambda b\sum_{n=1}^{\infty} P_{2n-1}\, z^n$

On simplifying above equation, we get

$$F_2(z) = \frac{(\lambda b+mp+mq)P_{20}}{(\lambda b+mp+mq-\lambda bz)} \qquad (20)$$

Taking limit $z\rightarrow 1$ in (20) we get,

$$F_2(1)= \frac{(\lambda b+mp+mq)}{(mp+mq)} P_{20}= P\,(S) \qquad (21)$$

Differentiate equation (20) both side w.r.t. z and taking limit $z\rightarrow 1$, we get,

$$F_2'(1)=\frac{\lambda F_2(1)}{(mp+mq)} = E\,(L_2) \qquad (22)$$

Multiply equation (8) by z^n and taking summation over n, we get,

$(\lambda b + \delta)\sum_{n=2}^{\infty} P_{3n}\, z^n= mq\sum_{n=2}^{\infty} P_{2n}\, z^n+\lambda b\sum_{n=2}^{\infty} P_{3n-1}\, z^n$

On simplifying above equation, we get

$$F_3(z) = \frac{mqF_2(z)-mqP_{20}-mqzP_{21}+(\lambda b+\delta)zP_{31}}{(\lambda b-\lambda bz+\delta)} \qquad (23)$$

Taking limit $z\rightarrow 1$ in (23) we get,

$$F_3(1) = \frac{mqF_2(1)-mqP_{20}-mqP_{21}+ (\lambda b+\delta)P_{31}}{\delta} = P\,(R) \qquad (24)$$

Differentiate equation (23) both side with respect to z, taking limit $z\rightarrow 1$ we get,

$$F_3'(1) = \frac{\lambda bF_3(1)+mqF_2'(1)-mqP_{21}+(\lambda b+\delta)P_{31}}{\delta} = E\,(L_3) \qquad (25)$$

Using the recurrence relation (1), (2), (3), (4), (5), (6), (7), and (8), we are able to obtain the following:

$$P_{00}=\frac{\lambda b}{\alpha}P_{20}=R_1P_{20}$$

$$P_{21}=\left(\frac{\lambda b}{\lambda b+mp+mq}\right)P_{20}=R_2P_{20}$$

$$P_{31}=\frac{\lambda bmq}{(\lambda b+\delta)(\lambda b+mp+mq)}P_{20}=R_3P_{20}$$

$$P_{01}=\frac{(\lambda b)^2}{(\mu_a+\psi)\alpha}P_{20}=R_4P_{20}$$

$$P_{11}=\frac{\lambda b}{\mu_w\alpha}\left[(\lambda b+\alpha)-\frac{\mu_a\lambda b}{(\mu_a+\psi)}\right]P_{20}=R_5P_{20}$$

Where $R_1=\frac{\lambda b}{\alpha}$, $R_2=\frac{\lambda b}{(\lambda b+mp+mq)}$, $R_3=\frac{\lambda bmq}{(\lambda b+\delta)(\lambda b+mp+mq)}$, $R_4=\frac{(\lambda b)^2}{(\mu_a+\psi)\alpha}$, $R_5=\frac{\lambda b}{\mu_w\alpha}\left[(\lambda b+\alpha)-\frac{\mu_a\lambda b}{(\mu_a+\psi)}\right]$

By using above P_{in}'s, we can rewrite $F_0(1), F_1(1), F_2(1), F_3(1)$ in terms of P_{20} as follows,

$$F_0(1)=I_0\,P_{20},\quad I_0=\left\{\frac{(\lambda b+\psi)\,R_1-\mu_a\,R_4)}{\psi}\right\}P_{20}$$

$$F_1(1)=I_1\,P_{20},\quad I_1=\frac{1}{(\lambda b-\mu_w)}\left[\psi\,I_0-\psi\,R_1+mp\,I_2-mp+\delta\,I_3-2\mu_w\,R_5\right]$$

$$F_2(1)=I_2\,P_{20},\quad I_2=\frac{(\lambda b+mp+mq)}{(mq+mp)}$$

$$F_3(1)=I_3\,P_{20},\quad I_3=\left\{\frac{mq\,I_2-mq-mq\,R_2+(\lambda b+\delta)\,R_3}{\psi}\right\}$$

Since $F_0(1), F_1(1), F_2(1), F_3(1)$ and all P_{in}'s are expressed in terms of P_{20}, therefore we need to calculate P_{20} which can be determined by using Normalization condition,

$$F_0(1)+F_1(1)+F_2(1)+F_3(1)=1.$$

$$I_0\,P_{20}+I_1\,P_{20}+I_2\,P_{20}+I_3\,P_{20}=1$$

$$\therefore P_{20}=[I_0+I_1+I_2+I_3]^{-1}$$

NUMERICAL ANALYSIS

Table 1 represents Impact of repair rate r on different system Performances E (L_0) , E (L_1) , E (L_2) , E (L_3) , E (L) , W. As the value of r increases E (L_0), E (L_2) remain constant while E (L_1), E (L_3), E (L), W decrease continuously.

Table 1: Impact of repair rate r on various system performances

r	E (Lo)	E (L1)	E (L2)	E (L3)	E(L)	W
0.1	0.2	0.58	0.5	9.899	12	2.98
0.3	0.2	0.356	0.5	2.98	4.124	0.999
0.4	0.2	0.234	0.5	0.99	1.999	0.512
0.6	0.2	0.198	0.5	0.512	1.412	0.412
0.9	0.2	0.088	0.5	0.299	1.178	0.315

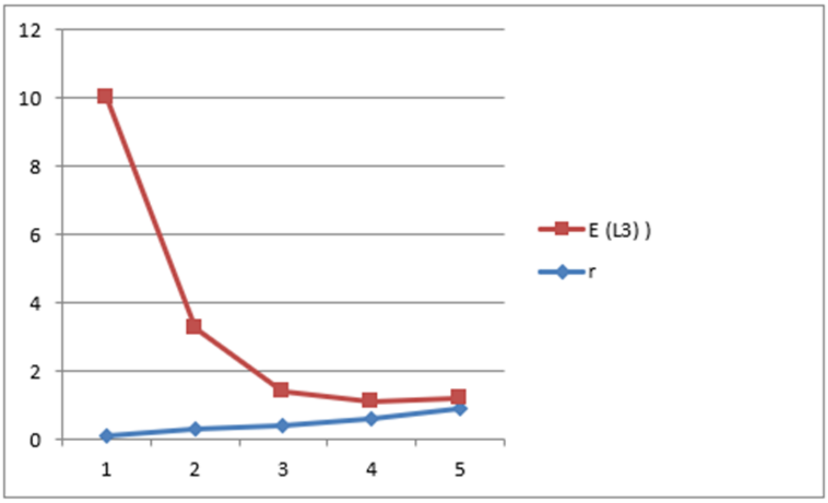

Figure 1:Shows graphical illustration of repair rate r on Expected queue length E(L3) in the when server in repair state

Figure 1 clearly displays the influence of the repair rate r on the expected queue length E (L3) while the server is in repair mode. It has been noticed that when r increases, the value of E (L) decreases steadily.

CONCLUSION

This article examines an M/M/1 queueing system with a removable and unreliable server, partial breakdowns during work vacations, and the setup, repair, and balking operations. Finally, it has been determined that breakdown, slow setup, balking, and repair time inject uncertainty and unpredictability into the queueing system by directly influencing system performance, customer satisfaction, and operating expenses. Effective manipulation of these factors can result in smoother functioning and greater overall efficiency. The model may be expanded by incorporating the idea of bulk arrival and Threshold Policy for Service.

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