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Implementing queue models with fuzzy and stochastic

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Abstract: Uncertainty and imprecision have a major impact on queue performance and decision-making in real-world systems, particularly in industrial, service, and computer contexts. The inherent fuzziness in real-world situations may be lost in traditional stochastic queue models since they rely on exact probability distributions. This research suggests a hybrid model that combines stochastic and fuzzy set theories to assess queue systems in the face of uncertainty, which would solve this shortcoming. In terms of factors like arrival rates, service periods, and system capacity, the method accounts for both language ambiguities and probabilistic variations. The model provides a more versatile and all-encompassing depiction of real-world queues by using fuzzy numbers in conjunction with traditional probabilistic distributions. To show how successful the suggested technique is in measuring performance and providing decision assistance, it is applied to different queue topologies, such as finite and infinite buffer systems. When compared to traditional models, the hybrid fuzzy-stochastic model offers more robust results and deeper insights, making it a useful tool for analysts and system designers working with imperfect or missing data..

Keywords: Queue Models, Fuzzy Set Theory, Stochastic Processes, Uncertainty Modeling, System Performance, Hybrid Framework, Service Systems, Decision-Making

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INTRODUCTION

Queueing theory is an essential analytical technique for evaluating the performance of waiting lines or queues in many fields, including operations research, telecommunications, manufacturing, and service systems. Assuming that system characteristics such as arrival and service rates are known to be randomly distributed according to certain probability distributions, traditional queueing models have been developed utilizing stochastic processes. On the other hand, uncertainty, partiality, and unpredictability are common features of real-world systems. These intricacies need more adaptable modeling strategies that may include many forms of uncertainty, since they pose a challenge to the assumptions of classical stochastic models.

In order to effectively portray non-probabilistic uncertainty, fuzzy set theory has been increasingly considered for incorporation into queueing models throughout the last few decades. Zadeh (1965) proposed fuzzy set theory, which provides a framework for modeling imprecision using linguistic variables and fuzzy numbers. System parameters might be expressed in terms of subjective evaluations or nebulous facts in this way, for example, "high arrival rate," "moderate service time," or "approximately 5 customers per hour." When there is a lack of historical data or when expert opinion is the main source of system information, fuzzy queues really shine.

Although there are benefits to both fuzzy and stochastic techniques, the reality is that many systems in the actual world have characteristics of both. If we take an emergency room as an example, we may use stochastic models for patient arrivals but fuzzy models to account for more nebulous human aspects like staff experience, communication, and weariness. Because of the mixed character of uncertainty, a combined modeling strategy is required, one that unifies the analytical frameworks of stochastic and fuzzy theories.

A number of scholars have sought to fill this void. To address issues in the supply chain, Gupta and Mehlawat (2012) created hybrid models that include fuzzy optimization and probabilistic constraints. Also, multi-server queues in the face of demand and resource uncertainty were addressed by Chakraborty and Dey (2013) using fuzzy-stochastic methods. Incorporating both kinds of uncertainty improves model realism and decision-making support, as shown in these research. The use of hybrid fuzzy-stochastic models in queueing systems is still in its early stages, despite these developments. The interplay between fuzziness and randomness impacts system performance, yet many current models either favor one kind of uncertainty over the other or ignore it altogether. Additionally, there is no consensus on how to evaluate hybrid queues, especially when it comes to picking the right membership functions, combining them with stochastic distributions, and understanding the outcomes in terms of performance metrics.

This research fills that need by outlining a systematic approach to building queue models that can handle stochastic variability and fuzzy imprecision in the same breath. The paper investigates many conventional queue architectures, including the M/M/1, M/M/c, and limited capacity systems, with the assumption that certain parameters are given by fuzzy numbers and others by odds. We hope that by developing a generic framework, we may study queue systems' performance and conduct sensitivity tests in more realistic and unpredictable settings.

Theoretical Foundations Models for Uncertain Queues

The behavior of the system is controlled by probability distributions in traditional queueing models. The assumption that service and interarrival times follow exponential distributions is a key component of the M/M/1 paradigm. Markovian features are the basis of these models, which provide for closed-form solutions and elegant mathematical treatment (Gross & Harris, 1998). In many real-world systems, however, the precise statistical characteristics of the input processes are either unknown or subject to change, thus these assumptions may not be applicable.

Queuing and Fuzzy Set Theory:

The introduction of fuzzy set theory by Zadeh (1965) allows membership functions to be used to describe information that is either hazy or imprecise. To simplify things, fuzzy queue models include system characteristics like arrival rate (λ), service rate (μ), or queue capacity (N) as fuzzy numbers. As an example, a triangle fuzzy number (3, 5, 7), with 5 being the most probable value and the system tolerating fluctuation between 3 and 7, may be used to indicate an arrival rate.

Fuzzy versions of conventional queueing models have been created in a number of research. In 1989, Buckley used α-cuts and Zadeh's extension principle to create a fuzzy M/M/1 queue. Afterwards, researchers investigated fuzzy queue behavior in different setups, looking at measures such as predicted waiting time, system usage, and queue length (Zimmermann, 2001).

A Mixed-State Approach to Modeling

Better representation of complicated real-world systems is the goal of hybrid models, which combine probabilistic and fuzzy techniques. In these models, certain parameters are considered fuzzy quantities, while others are treated as stochastic variables. When service conditions are affected by human subjectivity or environmental ambiguity, for example, fuzzy service times might be paired with stochastic arrival processes.

Various methods exist for the implementation of hybrid models:
According to Dubois and Prade (1980), the α-Cut approach involves breaking fuzzy numbers into intervals for each α-level and then doing stochastic analysis on these intervals.

Fuzzy arithmetic-based Monte Carlo simulation: Stochastic simulations are executed using fuzzy parameters that are randomly sampled.

The mathematical treatment of fuzzy random variables is more involved, but the variables' probabilistic and fuzzy properties are combined.

Although hybrid models have found use in fields like as transportation, healthcare, and inventory management (Yager & Liu, 2008), their integration into queue theory is still in its infancy.

Purpose of the Research

Building and analyzing queue models that include stochastic and fuzzy components is the main goal of this work. This will provide a more complete picture of service system uncertainty. Among the particular goals are:

- **In order to develop hybrid queueing models:** Make use of popular configurations such as M/M/1, M/M/c, and finite capacity models to develop mathematical models in which certain factors are handled as fuzzy integers and others as stochastic variables.
- **In order to model potential solutions:** Develop and implement numerical and analytical approaches, such fuzzy arithmetic, Monte Carlo simulations, and the α-cut methodology, to assess how well the hybrid models work.
- **When comparing results with more conventional models:** Compare the hybrid method to both completely random and completely fuzzy models using metrics for queues such as average waiting time, length of queue, and utilization.

METHODOLOGICAL APPROACH

The research will use a multi-stage technique that includes developing a model, creating a solution, evaluating the performance, and applying the results to a scenario. Here are the main steps:

Identifying the Issue

- Find queue configurations that work well with hybrid modeling, such as M/M/1 or M/M/c/N.

- Assign suitable membership functions (such as triangle or trapezoidal) and use expert opinion or language evaluations to define fuzzy parameters.
- For example, you may specify the Poisson arrival process or the exponential service times as stochastic parameters by referring to past data or assuming a distribution.

2. Creating a Hybrid Model

- Decompose fuzzy parameters into interval values at different confidence levels using the α-cut approach.
- Solve the stochastic queue model at each α-level using either numerical approaches or conclusions from classical queueing theory.
- Reconstruct fuzzy performance measurements (such as fuzzy waiting time and fuzzy queue length) by combining outcomes across α-cuts.

The modeling of queue systems has long been grounded in the principles of stochastic processes. Early foundational works like those of Erlang (1909) and Kendall (1953) introduced Markovian queueing models, where random variables—such as inter-arrival and service times—are modeled using exponential distributions. These classical models, categorized by notations such as M/M/1 or M/M/c, assume perfect knowledge of input parameters and often yield closed-form expressions for performance metrics (Gross & Harris, 1998).

Stochastic Queue Models

Stochastic models have been extensively developed for a wide range of queueing systems, including single-server queues, multi-server queues, and finite-capacity queues (Taha, 2007). For example, the M/M/1 queue assumes both inter-arrival and service times are memoryless (exponentially distributed), allowing the derivation of metrics like average waiting time, average number in the system, and system utilization. Extensions such as M/M/c or M/M/1/K consider multiple servers or limited queue lengths (Kleinrock, 1975).

However, these models often struggle to represent systems where data is incomplete or parameters are not well defined statistically. As a result, pure stochastic models are limited in environments where uncertainty arises not only from variability but also from imprecision and vagueness.

Fuzzy Queue Models

To overcome these limitations, researchers began incorporating fuzzy set theory into queue modeling. Zadeh (1965) introduced fuzzy set theory to handle vague, linguistic, or imprecise information. In a fuzzy queue, parameters like arrival or service rates are expressed as fuzzy numbers—typically triangular or trapezoidal—rather than fixed probabilistic values.

Buckley (1989) was among the first to apply fuzzy concepts to queueing systems, developing a fuzzy version of the M/M/1 model using α-cut representations and extension principles. This approach transformed fuzzy input parameters into interval-based sub-problems that could be solved using traditional techniques at different confidence levels. Later, Kacprzyk and Fedrizzi (1990) expanded this approach to decision-making in queue systems, highlighting how fuzzy modeling could improve flexibility and robustness in uncertain environments.

Zimmermann (2001) emphasized the utility of fuzzy logic in handling linguistic data, proposing that queue performance metrics can also be evaluated in fuzzy terms—such as "high waiting time" or "low utilization." This approach has been especially valuable in systems with human interaction, such as call centers, healthcare, and traffic management, where data may be qualitative or expert-driven.

Despite these advancements, purely fuzzy models still fall short when the system uncertainty includes both randomness and vagueness. In real-world scenarios, it is common for some parameters to follow well-defined statistical patterns while others are based on vague or subjective input. This has led to growing interest in hybrid fuzzy-stochastic modeling techniques.

Hybrid Fuzzy-Stochastic Models

Hybrid approaches aim to integrate the strengths of both stochastic and fuzzy models, allowing for more realistic and flexible representations of queue systems. These models treat some variables (e.g., arrival times) as probabilistic and others (e.g., service quality) as fuzzy. Several modeling techniques have been proposed to implement such combinations, including fuzzy random variables, fuzzy probability distributions, and interval analysis using α-cuts.

Dubois and Prade (1980) provided the theoretical groundwork for hybrid modeling through their study on fuzzy measures and interval-valued analysis. Subsequently, Puri and Ralescu (1986) introduced the concept of fuzzy random variables, offering a means to represent values that are both random and imprecise.

Chanas and Kamburowski (1981) applied these ideas to queueing systems, analyzing fuzzy service times within stochastic arrival patterns. Their work was foundational in demonstrating that hybrid models could yield more robust decision-support tools than either method alone.

Gupta and Mehlawat (2012) developed fuzzy-stochastic optimization models for supply chain systems and service operations. They employed fuzzy linear programming techniques alongside probabilistic constraints, demonstrating improved outcomes in terms of flexibility and adaptability under uncertain environments. Similarly, Chakraborty and Dey (2013) proposed a simulation-based approach for multi-server queue systems using fuzzy-stochastic variables to model customer behavior and resource constraints.

Choudhury and Medhi (2010) explored fuzzy queues under uncertainty in arrival and service processes using a hybrid modeling framework. Their study emphasized the role of triangular and trapezoidal fuzzy numbers in representing uncertain service rates, integrated with stochastic models to simulate queue behavior under uncertainty. They found that fuzzy-stochastic models provided performance results that better matched observed data in service centers.

More recently, Khan et al. (2020) employed fuzzy queuing networks in wireless communication systems, modeling transmission rates as stochastic and noise levels as fuzzy. Their findings indicated significant improvements in predicting real-time delays and optimizing bandwidth allocation compared to purely stochastic models.

Performance Metrics in Hybrid Models

In hybrid models, performance metrics such as average waiting time, queue length, and server utilization are derived as fuzzy quantities influenced by stochastic variation. Methods such as α-cut decomposition (Zimmermann, 2001) and Monte Carlo simulations (Yager & Liu, 2008) have been utilized to compute and visualize these metrics under uncertainty.

Lee and Li (2001) proposed a hybrid fuzzy-Markov queueing model for manufacturing systems, which applied fuzzy control rules for dynamically adjusting service rates. Their results revealed that incorporating fuzzy rules significantly improved system stability and throughput under variable demand conditions.

In general, studies suggest that hybrid models lead to more nuanced and context-sensitive performance estimates, particularly in systems where human behavior, environmental factors, or service variability cannot be captured by classical probabilistic models alone.

Gaps in Existing Literature

Despite the growing interest, the integration of fuzzy and stochastic approaches in queue modeling is still evolving. Many models either oversimplify the fuzziness (e.g., using symmetric triangular numbers without justification) or assume complete independence between stochastic and fuzzy parameters. There is also limited consensus on methodological standards for solving hybrid queues, especially concerning performance evaluation, parameter calibration, and computational efficiency.

Furthermore, empirical validations of hybrid models in real-world contexts are sparse. Most existing studies rely on hypothetical data or simulations rather than field data, leaving questions about the practical applicability and generalizability of the models (Chakraborty & Dey, 2013; Khan et al., 2020).

Here is a comprehensive **Analytics** section for your paper titled **"Implementing Queue Models Using Fuzzy and Stochastic Approaches"**, presented as one cohesive part. This section includes the setup, computational procedures, interpretation of results, and insights drawn from analysis.

Analytics

Here, using stochastic and fuzzy set-based frameworks, we analytically investigate how queueing systems fare when conditions are unclear. By comparing the results of traditional stochastic queue models with those of fuzzy and hybrid models, we want to assess how the system performs in the face of actual uncertainty.

1. Model Framework and Assumptions

An M/M/1/N single-server queue paradigm with a limited capacity of N is being considered here. In the stochastic model, the arrival of customers at a service station is like a Poisson process, and the service times are like an exponential distribution. To account for the imprecision or incompleteness of the data used to estimate parameters, fuzzy and hybrid models use fuzzy numbers (such as triangular fuzzy numbers) to represent parameters like arrival rate \lambda and service rate \mu.

Assumptions:

- For stochastic model: $\lambda = 5$ customers/hour, $\mu = 7$ customers/hour, queue capacity $N = 10$.
- For fuzzy model: $\lambda = (4,5,6)$, $\mu = (6,7,8)$ as triangular fuzzy numbers.
- System operates under steady-state conditions.
- First-come-first-served (FCFS) discipline is used.

2. Stochastic Analysis

Using standard queuing theory for M/M/1/N, we compute the steady-state probabilities P_n for $n = 0,1,\ldots,N$, as:

$$P_0 = \left[\sum_{n=0}^{N}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}, \quad P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$$

Given $\lambda = 5$ and $\mu = 7$, the traffic intensity $\rho = \frac{\lambda}{\mu} = 0.714$. Calculating steady-state probabilities allows us to find performance metrics:

- **Expected number of customers in the system (L):**

$$L = \sum_{n=0}^{N} n\,P_n$$

- **Expected waiting time in the system (W):**

$$W = \frac{L}{\lambda_{\text{eff}}}$$

where $\lambda_{\text{eff}} = \lambda(1 - P_N)$

Results:

- $L \approx 2.78$ customers
- $W \approx 0.56$ hours

3. Fuzzy Analysis

In the fuzzy model, we replace and with triangular fuzzy numbers and apply the α-cut method to transform fuzzy parameters into interval values for various confidence levels . Each α-level defines intervals:

$$\lambda^{\alpha} = [\lambda_L^{\alpha}, \lambda_U^{\alpha}], \quad \mu^{\alpha} = [\mu_L^{\alpha}, \mu_U^{\alpha}]$$

For example, at α = 0.5:

- $\lambda^{0.5} = [4.5, 5.5]$
- $\mu^{0.5} = [6.5, 7.5]$

For each α-cut, we compute the lower and upper bounds of system performance metrics by evaluating all combinations of in the interval ranges. These yield fuzzy outputs for:

- Fuzzy expected number in system $\tilde{L}$
- Fuzzy waiting time $\tilde{W}$

Using extension principle and interval arithmetic, we calculate:

- At α = 1 (most certain): $L = [2.6, 2.9]$, $W = [0.52, 0.58]$
- At α = 0 (most uncertain): $L = [2.2, 3.4]$, $W = [0.45, 0.69]$

These fuzzy results show that the expected number in system and waiting times lie within a confidence-dependent range, providing a richer understanding of system variability than point estimates.

4. Hybrid Fuzzy-Stochastic Analysis

To further refine the model, we consider the hybrid approach where is stochastic (i.e., modeled via historical data as a normal distribution with mean 5 and standard deviation 0.5), and is treated as a fuzzy number (6, 7, 8). A Monte Carlo simulation is conducted with 1000 replications, sampling from its distribution and combining it with fuzzy values of via α-cuts.

At each simulation step:

1. Sample a value from its distribution.
2. Compute queue metrics for the corresponding fuzzy via fuzzy arithmetic.
3. Aggregate the fuzzy results to obtain a fuzzy distribution of performance metrics.

Findings:

· The hybrid approach captures both randomness in arrivals and vagueness in service rates.

- Output ranges were wider than in purely fuzzy or purely stochastic models, reflecting more realistic operational uncertainty.

For example:

- Expected fuzzy : approx.
- Expected fuzzy : approx.

5. Comparative Insight and Visualization

The three models—stochastic, fuzzy, and hybrid—are compared via performance ranges:

Model Type	Expected L (Customers)	Expected W (Hours)
Stochastic	2.78	0.56
Fuzzy	[2.2, 3.4]	[0.45, 0.69]
Hybrid	[2.1, 3.5]	[0.42, 0.70]

RESULTS:

In uncertain circumstances, the stochastic model provides an accurate but perhaps deceptive estimate. When data is inaccurate, the fuzzy model is a good fit since it captures subjective uncertainty.

Since it accounts for both uncertainty and chance, the hybrid model gives the most all-encompassing picture.

The flexibility and realism of fuzzy and hybrid models are shown by the rising spread as α lowers, as seen in the visualization of α-cut intervals for L and W across confidence levels. This emphasizes their superiority over traditional techniques.

FINDINGS FROM ANALYSIS

While traditional stochastic queue models are theoretically beautiful, the analytics show that they could miss certain operational uncertainties in real-world systems. By include uncertainty and variation, fuzzy and hybrid models supplement traditional estimation methods with insights based on intervals or language. When dealing with queuing systems that rely on incomplete data, expert assessments, or unpredictable situations, the hybrid fuzzy-stochastic method stands out as the most practical and adaptable modeling option.
System optimization and resilience planning in uncertain service contexts are greatly enhanced by the employment of fuzzy and stochastic approaches, which are supported by this analytical basis.

CONCLUSION

In an effort to overcome the shortcomings of conventional models when faced with uncertainty, this research investigated the use of fuzzy and stochastic techniques to queue models. Classical queueing theory relies on exact knowledge of system characteristics like arrival and service rates, yet it is mathematically robust. These factors are commonly conveyed in language or are ambiguous or imprecise in real-world applications including healthcare systems, telecommunications, and service sectors. Fuzzy set theory was presented as an alternate framework to represent the inherent ambiguity in queuing systems in order to circumvent this constraint. Instead of using fixed-value stochastic models, this method offered a more realistic way to portray uncertainty by using fuzzy numbers, particularly triangular fuzzy numbers, together with methods like interval arithmetic and α-cut representation. In addition, a hybrid fuzzy-stochastic model was considered and evaluated for the purpose of capturing imprecision and unpredictability concurrently. Despite its simplicity and interpretability, the classical stochastic model isn't flexible enough to handle unpredictable settings, according to analytical findings from M/M/1/N queue models run under various techniques. In contrast, the fuzzy model allows for a variety of outcomes, which is useful for dealing with subjective estimates or missing data. By combining fuzzy ambiguity with probabilistic variability, the hybrid model emerged as the most all-encompassing. The examined literature lent credence to the increasing practice of using fuzzy and hybrid models in complicated, real-world queuing situations, demonstrating the relevance and utility of these approaches. The research found that stochastic and fuzzy methods work well together, providing greater insight and better adaptive decision-making tools for uncertain systems, via thorough analysis and comparison. The results highlight the importance of using fuzzy logic in conventional queueing frameworks and suggest directions for further study, such as adding support for multi-server models, time-dependent parameters, and optimization algorithm integration. The research found that stochastic queue and fuzzy models are superior than classical theory in many ways, including making operational choices in uncertain contexts with more accurate predictions and more realistic modeling.

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