Journal of Advances in Science and Technology
Vol. 21, Issue No. 1, March-2024, ISSN 2230-9659

DOI: https://doi.org/10.29070/roapzjo8

A Study on Intuitionistic Fuzzy Topological in Continuous Functions

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Abstract: From what Coker has said, the concepts of intutionistic sets and intutionistic points were first introduced long ago. His subsequent years were spent refining and presenting intuitionistic topological spaces , during which he outlined the fundamental characteristics of these spaces and provided an introduction to understanding them. Furthermore, he offered definitions for the concepts of intutionistic connectedness, intutionistic compactness, and intutionistic continuous functions. Deduced a number of characteristics that are associated with ji-open sets in intuitionistic topological spaces. In this work, we provide a new category of functions on intuitionistic topological space, which we refer to as ji-continuous functions. We explain these functions by referring to ji-closed sets, ji-closure, and ji-interior. In addition to this, we have defined ji-irresolute and articulated several properties that are present in intuitionistic topological spaces.

Keywords: Intuitionistic, Fuzzy, Topological, Continuous , Functions

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INTRODUCTION

Fuzzy set theory is necessary prior to investigating fuzzy topological spaces. In this context, "fuzzy subset" refers to a function that determines whether or not set X belongs to the interval [0, 1]. Imagine the set X as a fuzzy subset of X with a membership function that is the same as one on X.

This fuzzy subset may be represented by the symbols 1 or 1X or X. Though it's not always true, any subset of X is also a fuzzy subset of X. Thus, the concept of a fuzzy subset broadens the definition of the word "subset" to include additional situations.

The condition iff (x) (x) holds for every x in X. Always remember that all fuzzy subsets include the empty fuzzy subset and that all fuzzy subsets also contain themselves.

If, for every $y \in X$ save one, say $x \in X$, a fuzzy set in X takes the value 0, we call it a fuzzy point. We refer to this fuzzy point as xλ if its value at x is $\lambda(0 < \lambda \leq 1)$, and the point x is also known as its support. Denoted as Pt(X), this set contains all fuzzy points in X. Any fuzzy point xt that is in the set λ is defined as t being less than λ(x).

Be aware that while dealing with fuzzy set theory, the following characteristics that hold when dealing with set theory do not apply.

With the exception of where $\lambda = 0$ or $\lambda = X$, $\lambda \vee (1 - \lambda) = X$, and $\lambda \wedge (1 - \lambda) = 0$, $\lambda \wedge (1 - \lambda) = 0$.

X and Y are two nonempty sets. Let λ and μ be elements of X and Y, respectively. If f (λ): Y → is true, then f (λ) is a fuzzy subset of Y. (0, 1)

$$f(\lambda)(y) = \begin{cases} \sup_{x \in f^{-1}(\{y\})} \lambda(x) & \text{if } f^{-1}(\{y\}) \neq \phi \\ 0 & \text{if } f^{-1}(\{y\}) = \phi. \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.1)$$

One further thing to consider is that the set (X, τ) (or just X) is referred to as a fuzzy topological space, or f ts for short. The abbreviation for fuzzy open sets, which are individuals that belong to the set τ, is f o.

On the other hand, the abbreviation for fuzzy closed sets, which are the complements of fuzzy open sets, is f c.

The symbol λ is used to represent the fuzzy subset of space X. The following is a definition of the fuzzy closure (Cl(λ)) and the fuzzy interior (Int(λ)) of λ, one after the other.

For every value of λ ranging from c to o, if ε is an element of X and ⋁μ holds true, then Cl(λ) is equivalent to ⋁μ, and Int(λ) is also equal to ⋁μ within the same context.

E-open set

E-open set that includes intuitionistic fuzzy e-open set requirements. In this case, the outcome is derived from an intuitionistic fuzzy open set, which in turn leads to an intuitionistic fuzzy δ -semi open and an intuitionistic fuzzy δ -pre-open. Collectively, these two sets form an intuitionistic fuzzy e-open set.

Fuzzy e-open set

This project aims to study intuitionistic fuzzy topological spaces with fuzzy e-open sets, e-continuities, and e-compactness. After explaining intuitionistic fuzzy sets and topological spaces, we explain intuitionistic fuzzy e-open sets, e-continuity, and other topological concepts related to our results. While δ-open sets are stronger than e-open sets, δ-pre-open and δ-semi-open sets are weaker.

OBJECTIVES OF THE STUDY

1. To study on Fuzzy e-open set
2. To study on Intuitionistic topological spaces

INTUITIONISTIC TOPOLOGICAL SPACES

Due to the fact that intuitionistic topological spaces are going to be used to a significant degree, we will now describe them and list the characteristics that they possess. An empty set X is not allowed to exist.

The set A is considered to be an intuitionistic set if it is represented as A = (A1, A2), which is the expression of two disjoint subsets of X.

People who are not members of set A make up set A2, whereas those who are members of set A make up set A1. If and only if it is a component of A, then an intuitionistic point is considered to be an if. X is a set that does not include any empty elements, and the intuitionistic sets A and B are defined as follows: A = (A1, A2) and B = (B1, B2), respectively. In the perspective of intuitionism, the subset of X that is denoted by the equation Ai = (A (1) i, A (2) i) is arbitrary. To further elaborate, let {Ai/i ∈ J} be an example of such a family. A is a subset of B if A1 is a subset of B1 and A2 is a subset of B2. In other words, A is B's subset.

Families of nonempty sets in X that contain X and are closed under finite infima and arbitrary suprema are examples of those that are considered to be intuitionistic topologies. It may be defined for any set X that is not empty.

In situations when this is the case, we refer to the space (X,) as an intuitionistic topological space, which is abbreviated as IT S. It is referred to as an intuitionistic open (Io) set if you are a member of any set identified by the letter X. An intuitionistic closed set, also known as an Ic set, is the complementary set to an intuitionistic open set, also known as an o set.

We shall use the non-empty set X and a subset of X called "A" to demonstrate intuition. . The intuitionistic closure of A is Cl(A) = T K/K if X is a closed set in K and A is an open set. For this explanation, suppose X is a gapless set and A is a subset. Int(A) = S K/K denotes an intuitionist open set of X and K A..

If A has an interior and X and K are open intuitionistic sets, this is true. Consider the function f: X → Y. If f_(A2) = (f((A2) c))c and A = (A1, A2) is an intuitionistic subset of X, then f(A) is an intuitionistic subset of Y.. The equation f(A) = (f(A1), f_(A2)) defines this subgroup. Consider the function f: X → Y. According to intuitionistic concepts, f 1 (A) is a subset of X.. This subset is determined by the equation f −1 (A) = (f −1 (A1), f −1 (A2)), where A = (A1, A2) is an intuitionist subset of Y

The function f: X → Y is said to be an intuitionistic open (intuitionistic closed) map if, for each set U of X that contains elements of type Ic, f(U) is an element of type Ic in Y. In this case, we identify the function as an intuitionistic open map. Give some thought to the function f: X → Y. A definition of the intuitionistic subset in X × Y, which is represented by the graph of f and is abbreviated as GR(f), is as follows: (\(x, f(x))/x ∈ X}, {(x, f(x))/x ∈ X} c). .b

A collection {Gα/α ∈Λ} is referred to as an intuitionistic open cover of an intuitionistic set A

(IOC(A)) of an IT S (X, τ) when A is a subset of S Gα/Gα and α is an element of the set X. Intuitionistic T0 (IT0) is defined as an IT S (X, τ) that exists in τ in such a way that for every pair of variables x and y in X, there exists a subset G of τ such that for every element ey in X, ey is less than G.

Foundational ideas in smooth topological domains

Crucial ideas including seamless mappings, seamless structures, and the concept of a seamless topological space, which expands upon fuzzy topological spaces by centring on the gradual opening of subsets.

Flat Topological Domain:

The definition of a "smooth" topology is τ: IX → I, which is a mapping that satisfies certain requirements. In order to generate a smooth topological space (X, τ), which is an extension of Chang's fuzzy topological space, a non-empty set X is used.

Smooth Mappings:

A continuous function that preserves the "smoothness" of two smooth topological spaces is known as a smooth mapping. This function ensures that the two topologies are compatible with one another within the context of the mapping.

Smooth Structures:

While studying smooth structures on topological spaces, which are collections of functions that are regarded "smooth" according to a specified criterion, it is feasible to explore differentiable structures within the topological framework. This is made possible by studying smooth structures on topological spaces.

Gradation of Openness:

Going beyond the traditional open/closed classification that is used in normal topology, smooth topological spaces include the concept of "gradations of openness," which means that different regions of a space may be assigned varying degrees of openness across the space.

Subspaces:

In the context of smooth topological spaces, the concept of a subspace is comparable to that of standard topology. A subspace is a subset of the original space that has an induced smooth topology.

Continuity:

To ensure that smooth functions maintain the "smoothness" of the spaces, continuity in smooth topological spaces is described in terms of smooth mappings and the gradation of openness.

Compactness:

After doing research on compact subsets within the context of smooth topology, it has been determined that compactness in smooth topological spaces may be defined in terms of the smooth structure and the gradation of openness.

Relationship to Fuzzy Topology:

Smooth topological spaces are an extension of Chang's fuzzy topological spaces, which allows for the exploration of spaces that have a "fuzzy" or graded notion of openness.

One of the most significant concepts in topology is the concept of a continuous function. Both intuitionistic sets and intuitionistic points were first conceived of by Coker, who was the pioneer in this field. After that, he proceeded to define certain fundamental characteristics, as well as to develop and present intuitionistic topological spaces. Furthermore, he offered definitions for the concepts of intutionistic connectedness, intutionistic compactness, and intutionistic continuous functions.

The authors Suganya et al. reported and determined a number of characteristics of j□-open sets in intuitionistic topological spaces. In this work, we provide a new category of functions on intuitionistic topological space, which we refer to as j□-continuous functions. We explain these functions by referring to j□-closed sets, j□-closure, and j□-interior. In addition to this, we have defined j

j□-irresolute and articulated several properties that are present in intuitionistic topological spaces.

Prelimnaries

Definition2.1.[1]

Let \(Y \) be a non-empty set. An intuitionistic set (IS, for brevity) < is an object defined as < = <Y, <1, <2>, where <1 and <2 are subsets of Y that fulfil <1 ∩ <2 = ∅. The set <1 is referred to as the set of members of <, while <2 is designated as the set of non-members of <.

Definition2.2.[2] In short, an intuitive topology is Onanon-emptyset Y Is the following axiom satisfied by a familyτ of intuitionistic sets in Y? ∅,Y∈τ, for all 1,2∈τ ∪α∈ τ for any arbitrary family {i :α/α c J}, where (Y,τ) is referred to as an intuitionistic topological space and any intuitionisticset is referred to as an intuitionisticopenset (for short$\mathscr{J}$OS) in Y. The intuitionistic closedset (abbreviated ,CS) in X is the complement <C of a $\mathscr{J}$OS of <.

Definition 2.3. [1] Let ($\mathcal{K}$,τ) → (>,τ1) be a function and let $\mathcal{K}$,> be two nonempty sets. The preimage of> under is denoted by−1(ji) is the intutionistic set in$\mathcal{K}$ described by−1($\mathscr{B}$)= <X,−1($\mathscr{B}$1),−1($\mathscr{B}$2)> if$\mathscr{B}$=<X,,1,.2> is an intutionistic set in>.

Definition 2.4. Consider two intuitionistic topological spaces, ($\mathcal{K}$,) and (>,τ1). It is argued that a mapping from ($\mathcal{K}$,) to (>,τ1) is continuity of instruction if the preimage of the instruction is open in >is open to intuition.

if for each intuitionistic open <of >,−1(<) is $\mathscr{J}\alpha$-openin $\mathcal{K}$, then $\mathscr{J}\alpha$-continuous.

If the inverse image of any intutionistic open set in > is also an intutionistic semi-open set in $\mathcal{K}$, then the set is semi-continuous.

If the inverse image of any intutionistic open set in (>, τ1) is intutionistic regular-open in ($\mathcal{K}$, τ), then the set is intutionistic regular continuous.

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Definition2.5. If there is an intutionistic open set <≠∅˜ and Y˜ such that $\mathcal{D}$⊆□cl($\mathcal{D}$ ∩ <), then an intutionistic set$\mathcal{D}$ of an intuitionistic topological space (,) is said to be an intuitionistic$i$-openset (shortly ji-openset).

$\mathcal{J}i$-Continuity

Definition 3.1. Let ($\mathcal{K}$,) and (>, τ1) be two intuitionistic topological spaces. A mapping: ($\mathcal{K}$,τ) → (>,τ1) is an intuitionistic i-continuous function if the preimage of every intuitionistic open set in (>,τ1) is intuitionistic i-open in ($\mathcal{K}$,τ).

Example3.2 Let $\mathcal{K}$ = {17, 19, 21} with τ = {$\mathcal{K}$˜, ∅˜, 1, 2, 3} where 1 = <$\mathcal{K}$, {17}, {19}>, 2 = <$\mathcal{K}$, ∅, {19}>, 3 = <$\mathcal{K}$, {17, 21}, ∅> and4 = <$\mathcal{K}$, {17}, ∅>. Let > = {r, &, t} with a family τ1 = {>˜, ∅˜, <1, <2, <3} where <1 = <>, {r}, {t and <3 = < >, ∅, {r, t} >. Provide a definition: ($\mathcal{K}$,τ) transforms to (>,τ1) as (17) equals &, (19) equals t, (21) equals r, and ($\mathcal{K}$) equals >. The inverse image of any intuitionistic open set in (>, 1) is intuitionistic ally i-open in ($\mathcal{K}$, τ), hence it is $\mathcal{J}i$-continuous.

Theorem3.3.

Every intuitionistic continuous function is ji-continuous. Demonstration: Let ($\mathcal{K}$, τ) be a space that is intuitionistically continuous to (>, τ1). The inverse image of any intuitionistic open set in (>, τ1) is intuitionistically open in ($\mathcal{K}$, τ). Every intuitionistic open set is a ji-open set. Consequently, the inverse image of any intuitionistic open set in (>, 1) is ji-open in ($\mathcal{K}$, τ). Consequently, is ji-continuous.

Corollary 3.4. All regular functions that are based on intuition are ji-continuous. All intutionistic regular continuous functions are i-continuous because, according to the previous theory, every intutionistic regular open set is an intutionistic open set.

Remark 3.5. This is shown by the following example, which disproves the reverse of the theorem and corollary stated before.

Example3.6. Assume that $\mathcal{K}$={i,y} and that >={<˜,∅˜,,1,.2}, where .1 =<1,2,{y}> and >2= 3,{2},3 >. We have <1 =< >,ζ,4> and 2=<>,4,1>, where <1 is defined as <>˜,∅˜,<1,>2}.

"Define" means The function (4,3) is transformed into (>,11) when (1)=, (y)= 4, and (<)=>. The set jt is continuous, but the set −1(<>,1,2>) is not exhaustively open in (1,2). thus, is not inherently continuous.

CONCLUSION

The fuzzy topology of X is comprised of the fuzzy subsets of X that satisfy the three conditions that are listed below, provided that X is a set that is not empty. Assuming that both g and h are true, it follows that g h is also true for all values of 0 and 1 in the set τ. According to the same logic, if fi is true for each and every element in I, then W I in I fi must also be true. In the event that this is the case, we refer to the set (X,) or simply X as a fuzzy topological space (or f ts for short). Pt. (I am X) is a representation of the whole collection of fuzziness indices for the variable X. A fuzzy point (x) is defined whenever the condition iff t xt is true. If the two fuzzy subsets are comparable for each x in a set X, then they are stated as equal, which is denoted by the symbol =. Remember that the answer is (x) if and only if there is an element x in X that is such that (x) exists. This condition must be met. A fuzzy subset of X is equal to 1 (x) for each x that is included inside X. In addition, the number 1 is used to represent the complement of a fuzzy subset in X. According to Coker, the concepts of intuitionistic sets and intuitionistic points were first introduced. After that, he proceeded to define certain fundamental characteristics, as well as to develop and present intuitionistic topological spaces. Furthermore, he offered definitions for the concepts of intutionistic connectedness, intutionistic compactness, and intutionistic continuous functions..

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