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A Study on the Thermoelastic issues of a Steady-State Thin Annular Disc.

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Abstract: In this work, steady-state thermoelastic problems in a thin annular disc are studied. The coupling of mechanical and thermal effects in a material is known as thermoelasticity, and it is an important property in many engineering applications, including the construction of rotating parts like discs. This work aims to investigate the behavior of a thin annular disc under mechanical and thermal loads, with particular attention to the steady-state response. The analysis's findings provide crucial information on the thin annular disc's thermoelastic behavior. It is clear that the disc's internal temperature distribution significantly affects how the disc behaves mechanically. Over time, the structural integrity and performance of the disc may be impacted by the stresses and deformations caused by thermal gradients. Additionally, the investigation looks into how several factors, such material characteristics and disc geometry, affect the thermoelastic response. To sum up, this study offers important insights into the thermoelastic behavior of thin annular discs in steady-state circumstances.

Keywords: thermoelastic issue, thin annular disc, steady state

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INTRODUCTION

Thermal strains were identified by Deshmukh [2] through his study of the transient heat conduction problem in a thin hollow cylinder. The inverse problem of thermal stresses on a thin annular disc was studied by Gogulwar and Deshmukh [2]. The thick annular disc's quasi-static steady state thermal stresses have been determined by Kulkarni and Deshmukh [3]. Examine the annular fin's direct thermoelastic problem where the base is exposed to a heat flux that is a decaying exponential function of time. This work examines the thermoelasticity of thick annular discs using Michell's function and Goodier's thermoelastic displacement potential function [6]. Using the integral transform approach, the temperature distribution is obtained. The displacement function, stresses, and temperature change have all been estimated numerically and graphically, and the results are presented in series form using Bessel's functions. In this case, we calculate the impacts of internal heat generation in terms of radial strains. Using numerical examples, a mathematical model of a thick annular disc has been created by taking into account copper (pure) discs. No one has ever researched this kind of issue before. This is a fresh advancement in the sector.

History and Survey of Thermoelasticity

Thermoelasticity is the ability of a solid object to alter in size and shape in response to changes in temperature. Greater expansion and contraction is experienced by more elastic materials than by more inelastic ones. Thermoelasticity is a concept that scientists employ to create materials and items that can resist temperature changes without breaking.

Although the equations describing thermoelasticity have been studied by scientists for more than a century, stress testing materials to measure their thermoelastic properties has only lately started. Engineers can forecast how much a material will expand or contract at a given temperature by exposing it to rising and declining temperatures. When constructing machines or weight-bearing constructions with components that must fit together precisely, this knowledge is crucial. Engineering professionals can better design objects that retain their structural integrity over a range of temperatures by having a solid understanding of thermoelasticity.

The way engineers create a variety of products has been influenced by the concepts of thermoelasticity. For example, sidewalks are made with small intervals between slabs since it is known that heated concrete expands. The absence of these gaps would prevent the concrete from expanding, placing a tremendous amount of strain on the substance and increasing the likelihood of cracks, splits, or holes. Similarly, expansion joints are incorporated into the design of bridges to facilitate component expansion during heating.

Applied mathematicians and engineers frequently encounter difficult problems when trying to solve partial differential equations, which are a common type of equation in their fields of study. Nothing could be more ideal than to solve these equations precisely. However, one must use numerical approaches to find an approximate solution due to the intricacy of the exact solution. In applied mathematics, thermoelasticity is one such area. The renowned work "Memorie sur le calcul des actions moteculairs développées par les changements de température dans les corps solides" [7] served as the basis for the mathematical theory of thermoelasticity. He originally formulated the basic thermoelasticity equation in his memoire, taking into account the deformation brought about by temperature variations. According to Duhamel's analysis, it is possible to compute the stresses caused by a temperature gradient independently from the stresses caused by mechanical forces, and to derive the total stresses via superposition. It appears that superposition has never been used in stress analysis before. Duhamel applied his well-known fundamental equations to a number of particular situations, such as the subject to radially increasing temperature and the circular cylinder.

The traditional quasi-static method for solving thermoelastic problems under stationary loading conditions and with time-dependent temperature fields is based on the idea that the governing field equation's inertia terms may be ignored. It is well known that this theory, which dates back to [8], produces beneficial outcomes in a wide range of applications. It is obvious to everyone that the accuracy of the estimate must be based on the characteristics of the time fluctuations present in the temperature distribution as well as the magnitude of the pertinent intrinsic inertia parameters. Dynamic factors ignored in the standard treatment of the problem may become substantial if, in particular, the temperature field exhibits a sufficiently steep time-gradient. Furthermore, the problem's fundamental nature changes when the inertia terms are considered; the process of thermal stress transmission is no longer just diffusive but also involves the propagation of elastic waves.[9] appears to be the first attempt to investigate the inertia effects in a transient thermoelastic issue. The specific issue addressed in [9] is an elastic half space that is free of loading and is exposed to a sudden application of a uniform surface temperature change over its whole plane boundary, which is then maintained consistently thereafter. In the absence of thermoelastic coupling, it is assumed that the temperature distribution throughout the medium obeys the heatconduction equation. The resulting thermal stresses are precisely calculated in a classical framework that is limited by lateral displacement. Mura [10] independently obtained the same conclusions, while Danilovskaya [11–12] expanded on her earlier work to take convective boundary conditions into account. To yet, there have been no investigations referenced that determine the corresponding thermal displacements that lead to the conclusion that was obtained.

The Steady-State Scenario:

It is not unusual for engineering applications to require materials and components to work in steady-state conditions. A condition is said to be in a steady state when its properties do not change over the course of some amount of time. This indicates that the temperature distribution and mechanical stresses within the disc have established a stable equilibrium under the given thermal and mechanical loads in the context of thermoelasticity in thin annular discs. This is the case when the disc is subjected to a combination of thermal and mechanical loads.

Challenges and Implications:

It is essential, for a variety of reasons, to have a solid understanding of thermoelastic difficulties in thin annular discs:

Structural Integrity:

The interaction between the disc's temperature and the mechanical forces it experiences can have a significant bearing on the disc's ability to maintain its structural integrity. Thermal gradients have the ability to cause stresses and deformations, which could, in the long run, result in failures such as cracks.

Performance Optimization:

Performance enhancement is absolutely necessary for a variety of engineering applications, such as turbine rotors and braking discs. Thermoelastic analysis is helpful in the process of designing components that are able to function effectively despite being subjected to harsh circumstances.

Material Selection:

To prevent problems caused by thermoelastic behavior, it is essential to use the suitable materials, which should have the right thermal and mechanical properties. Engineers are able to select materials that are able to sustain the requisite temperature ranges and mechanical loads more effectively thanks to the research of thermoelasticity.

Statement of the problem-I

As suggested by [12], a complete solution to the thermoelastic problem displacement field is to be determined such that, for T = 0, there is zero traction on all surfaces of the solid shaft. Thus, to seek the solution, we assume the following:

(i) Zero traction conditions along the radial direction

$$\sigma_{rr} = 0, \sigma_{r\theta} = 0, \sigma_{rz} = 0 \; at \; r = b \ldots\ldots\ldots (1.1)$$

(ii) Zero normal force on z = 0, h

$$2\pi \int_0^b \sigma_{zz} r dr = 0 \text{ ....(1.2)}$$

(iii) Boundary conditions of the finite-length solid shaft be simply supported at the two longitudinal edges,

$$u_r = 0, \sigma_{zz} = 0, \sigma_{\theta z} = 0, \sigma_{rz} = 0 \; at \; z = 0, h \text{.....(1.3)}$$

Thus, in formulating these conditions, we have taken into account from equation (3), that $\sigma_{rz} = 0, \sigma_{r\theta} = 0, \sigma_{\theta z} = 0$ and the other stress components depend only on r as a special case.

Following the Exponential profile treatment, we will now consider the case, where Young's modulus E and the thermal expansion coefficient α varies radially according to a general exponential form, and thus governing differential equation become

$$r^2 u_r^n(r) + r[1 - kn(r/b)^k] u_r^i(r) - \left\{\frac{1 + v[-1 + kn(r/b)^k]}{1 - v}\right\} u_r(r)$$

$$= \left\{\frac{kvrn(r/b)^k}{1-v}\right\} \dot{u}_z(z) - \frac{e^{-n(r/b)^k}}{v(1-v)} \left\{-r^3 e^{-2n(r/b)^k} \times (2v^2 + v - 1)\rho\omega^2 E_0^{-1} + \right.$$

$$\left. 2\alpha_0 k(1 + v) rn(r/b)^k \Delta T\right\} \text{....(1.4)}$$

where E_0 and α_0 are arbitrary constant having the same dimension as E and α respectively, n and k are the material parameters whose combination forms wide range of nonlinear and continuous profiles to describe reasonable variation of material constants and thermal expansion coefficients, when the thermal effect is neglected. Equations (1.1) to (1.4) constitute the mathematical formulation of the problem under consideration.

Solution of the problem

The nonhomogenous differential equation (13) is of confluent hypergeometric type (see [1]) which may be solved by introducing a new variable $x = x(r) = n(r/b)^k$ and applying the transformations

$$u_r(r) = r u_r^*(x), u_\theta(\theta) = 0 \; and \; u_z(z) = G_z \text{.......(1.5)}$$

where G is the unknown constants and is to be determined, and the nonhomogenous term f(r) turns out to be

$$f(r) = \frac{kvrn(r/b)^k}{1-v} u_z^i - \frac{e^{-n(r/b)^k}}{v(1-v)} \left\{-r^3 e^{-2n(r/b)^k} (2v^2 + v - 1)\rho\omega^2 E_0^{-1} + \right.$$

$$\left. 2\alpha_0 k(1 + v) rn(r/b)^k \Delta T\right\} \text{......................(1.6)}$$

The term $u_r(r) = ru_r^*(x)$ represents a radial expansion or contraction in which, in general, the inner and outer radii change but angle remains constant, and $u_z(z) = G_z$ is a uniform axial extension or contraction. The standard form of the homogenous confluent hypergeometric differential equation using equation (14) can be obtained by setting right hand side of equation (13) to be zero, thus the homogeneous part is transformed into

$$xu_r^{n*}(x) + k^{-1}[2 + k - kx]u_r^{n*}(x) - [(1 + v)k]^{-1}u_r^*(x) = 0 \ldots\ldots\ldots\ldots(1.7)$$

The straightforward solution can be cast in the form

$$u_r^*(x) = \hat{C}_1 F_c(\alpha, \beta; x) + \hat{C}_2 x^{-\frac{2}{k}} F_c(\alpha - \beta + 1, 2 - \beta; x) \ldots\ldots.(1.8)$$

where $\hat{C}_i(i = 1,2)$ is an arbitrary integration constant to be determined and $F_c(\alpha.\beta; x)$ is the confluent hypergeometric function, which is defined as the analytic continuation of the so-called confluent hypergeometric series,

$$F_c(\alpha.\beta; x) = 1 + \frac{\alpha}{\beta}\frac{x}{1!} + \frac{\alpha}{\beta}\frac{(\alpha+1)}{(\beta+1)}\frac{x^2}{2!} + \cdots \ldots\ldots\ldots\ldots\ldots \frac{\alpha}{\beta}\frac{(\alpha+1)}{(\beta+1)}\frac{\ldots\ldots\ldots}{\ldots\ldots\ldots}\frac{(\alpha+j-1)}{(\beta+j-1)}\frac{x^3}{j!} + \cdots\ldots(1.9)$$

The arguments α and β are real, and the dimensionless scalars of the confluent hypergeometric function FC in equation (1.8) have the following meanings:

Finally back transforming, using $ru_r^*(x) = u_r(r)$ we obtain

$$u_r(r) = \hat{C}_1 F_c(\alpha, \beta; n(r/b)^k) + \hat{C}_2 F_c(n(r/b)^k)^{-2/k} r F_c(\alpha - \beta + 1, 2 - \beta; n(r/b)^k)$$

Or

$$u_r(r) = C_1 P(r) + C_2 Q(r), \ldots\ldots\ldots(1.10)$$

where P(r) and Q(r) are the fundamental solutions of the reduced differential equation as

$$P(r) = rF_c(\alpha, \beta; n(r/b)^k) \ldots\ldots.(1.11)$$

$$Q(r) = r^{-1}F_c(\alpha - \beta + 1, 2 - \beta; n(r/b)^k) \ldots..(1.12)$$

STATEMENT OF THE PROBLEM-II

Consider thick circular plate of thickness 2h occupying the space $D: 0 \le r \le a, -h \le z \le h:$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\emptyset(r,z,t)$ as Nowacki is

$$\frac{\partial^2\emptyset}{\partial r^2}+\frac{1}{r}\frac{\partial\emptyset}{\partial r}+\frac{\partial^2\emptyset}{\partial z^2}=\left(\frac{1+v}{1-v}\right)\alpha_t\,T\ldots\ldots\ldots(2.1)$$

andνwhere at are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as Noda et al. is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=0\ \ldots\ldots\ldots(2.2)$$

Subject to the boundary conditions

$$M_r(T,0,1,a)=g(z) \qquad -h\le z\le h,$$

$$M_z(T,1,k_1,h)=f_1(r) \qquad 0\le r\le a,\ \ldots..(2.3)$$

$$M_z(T,1,k_2,-h)=\frac{Q_0}{\lambda}f_2(r) \qquad 0\le r\le a,\ \ldots..(2.4)$$

where k is thermal diffusivity of material of the plate.

The displacement function in the cylindrical coordinate system are represented by Love's function as Khobragade are

Thermoelastic Response of a Thick Circular Plate due to Heat Generation…

$$u_r=\frac{\partial\emptyset}{\partial r}-\frac{\partial^2 L}{\partial r\partial z}\ldots\ldots..(2.5)$$

$$u_z=\frac{\partial\emptyset}{\partial z}+2(1-v)\nabla^2 L-\frac{\partial^2 L}{\partial z^2}\ldots..(2.6)$$

The Love's function must satisfy

$$\nabla^2\nabla^2 L=0\ldots\ldots(2.7)$$

Where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the thermoelastic displacement potential and Love's functionϕ L as Noda et al. are

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \emptyset}{\partial r^2} - \nabla^2\emptyset\right) + \frac{\partial}{\partial z}\left(v\nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \ldots\ldots\ldots (2.8)$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \emptyset}{\partial r} - \nabla^2\emptyset\right) + \frac{\partial}{\partial z}\left(v\nabla^2 L - \frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right)\right\} \ldots (2.9)$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \emptyset}{\partial z^2} - \nabla^2\emptyset\right) + \frac{\partial}{\partial z}\left((z - v)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\} \ldots (2.10)$$

For traction free surface stress function $\sigma_z = \sigma_{r\theta} = 0 \; at \; z = \pm h$ for thick plate.

$$\sigma_{rz} = 2G\left\{\left(\frac{\partial^2 \emptyset}{\partial r \partial z}\right) + \frac{\partial}{\partial z}\left\{(1 - v)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right\}\right\} \ldots\ldots (2.11)$$

Equations (2.1) to (2.11) constitute the mathematical formulation of the problem under consideration.

Solution of the problem

Applying Marchi-Fasulo transform to the equation (2.2), we get

$$\frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{T}}{\partial r} - \lambda_n^2 \bar{T} = \psi \ldots\ldots\ldots\ldots\ldots 2.12$$

$$where, \psi = \frac{-P_n(h)}{k_1} f_1(r) \frac{-P_n(-h)}{k_2} \frac{Q_0}{\lambda} f_2(r) \ldots\ldots\ldots 2.13$$

Equation (2.12) is a Bessel's equation whose solution gives

$$\bar{T} = A I_0(\lambda_n r) + B K_0(\lambda_n r) + \bar{F}(r) \ldots\ldots\ldots (2.14)$$

Where $\bar{F}(r)$ is the P.I.

As $r \to 0, K_0 \to \infty$, But $\bar{T}$ is finite

$$\therefore B = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.15)$$

$$A = \frac{\bar{g} - \overline{F'}(a)}{I'_0(\lambda_n a)} \ldots\ldots\ldots\ldots\ldots (2.16)$$

$$\bar{T} = \frac{\bar{g} - \overline{F'}(a)}{I'_0(\lambda_n a)} I_0(\lambda_n r) + \bar{F}(r) \ldots\ldots\ldots\ldots\ldots (2.17)$$

Applying inverse Marchi-Fasulo transform to the equation (3.6) we get

$$T = \sum_{n=1}^{\infty}\left[\frac{P_n(z)}{\lambda_n}\frac{\bar{g}-\overline{F'}(a)}{I'_0(\lambda_n(a)}I_0(\lambda_n r) + \bar{F}(r)\right] \ldots\ldots\ldots\ldots\ldots 2.18$$

And

$$\phi = \frac{r^2}{r}\left(\frac{1+v}{1-v}\right)a_1\sum_{n=1}^{\infty}\left[\frac{P_n(z)}{\lambda_n}\frac{\bar{g}-\overline{F'}(a)}{I'_0(\lambda_n(a)}I_0(\lambda_n r) + \bar{F}(r)\right] \ldots\ldots\ldots\ldots 2.19$$

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$$L = \frac{r^2}{4}\left(\frac{1+v}{1-v}\right)a_1\sum_{n=1}^{\infty}\left[\frac{P_n(z)}{\lambda_n}\frac{\bar{g}-\overline{F'}(a)}{I'_0(\lambda_n(a)}I_0(\lambda_n r)\right] \ldots\ldots 2.20$$

DETERMINATION OF THERMOELASTIC DISPLACEMENT

Referring to the fundamental equation and its solution for the heat conduction problem, the solution for the displacement function are represented by the Goodier's thermoelastic displacement potential ϕ governed by equation are represented by

$$\phi(r,z,t) = \left(\frac{1+v}{1-v}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{-1}{\lambda_m\Lambda_{n,m}}\left[\wp_{n,m}exp(-\omega t) + \left(\bar{T}_0^* - \wp_{n,m}\right)exp\left(-k\Lambda_{n,m}t\right)\right)\right]P_m(z)\right\} \times S_0(k_1,k_2,\mu_n r)exp\left[\int_0^t \psi(\varsigma)d\varsigma\right] \quad \ldots(3.1)$$

CONCLUSION

Within the scope of this investigation, we investigated the intricate thermoelastic problems that are connected to the use of thin annular discs under steady-state settings. We have gained insights into the interplay between thermal and mechanical effects within the material by linking the governing equations of heat conduction and elasticity. These equations govern heat conduction and elasticity, respectively. This research makes a contribution to a better understanding of thermoelastic difficulties in thin annular discs, which lays the foundation for engineering solutions that are more resilient and efficient. In the future, study in this field may encompass further investigation of specific applications as well as the establishment of realistic guidelines for engineers and designers to follow in order to avoid thermoelastic-related issues that occur in thin annular disc structures.

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