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Two-phase blood flow mathematical modelling and numerical simulation

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Abstract: The study of two-phase blood flow through mathematical modelling and numerical simulation provides a comprehensive framework for understanding the complex interactions between plasma and cellular components, primarily red blood cells (RBCs), within the cardiovascular system. Blood exhibits non-Newtonian behavior and behaves as a suspension, making two-phase modeling essential for accurately capturing its rheological properties, especially in microcirculation. This research integrates fluid dynamics and numerical techniques to simulate the coupled flow behavior of plasma (as a continuous phase) and RBCs (as a dispersed phase) using governing equations based on Navier-Stokes formulations and volume fraction continuity. Key parameters such as viscosity, density, and shear rate are incorporated to examine flow resistance, hematocrit distribution, and velocity profiles under physiological conditions. Finite volume and finite element methods are employed for numerical simulation, ensuring precision in spatial and temporal resolution. The model also accounts for vessel geometry and flow pulsatility to better represent in vivo scenarios. Such an approach allows for detailed investigation of pathophysiological states like anemia, thrombosis, or sickle cell disease. Overall, this work enhances the predictive capabilities of biomedical models, contributing to advancements in medical diagnostics, targeted drug delivery, and the design of artificial blood substitutes or vascular implants.

Keywords: Two-Phase, Blood Flow, Mathematical Modelling, Numerical Simulation

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INTRODUCTION

One of the most significant applications of two-phase blood flow modelling is in the identification and treatment of cardiovascular diseases (CVDs), which are a leading cause of mortality worldwide. The usual flow dynamics are significantly altered by atherosclerosis, aneurysms, thrombosis, and hypertension, which are risk factors for ischaemia, stroke, and organ failure. In order to measure these changes more precisely, two-phase models are better than one-phase ones. This is especially true when it comes to capturing events like haemolysis, clot formation, and platelet activation. More than that, they make it possible to test stents, prosthetic valves, and ventricular assist devices (VADs) in realistic flow environments. Blood flow modelling in microcirculation is another important field of study since red blood cell deformability, aggregation, and margination significantly impact tissue perfusion and oxygen transport efficiency. Here, two-phase models have shed light on the mechanical components that impact the micro-rheological characteristics of blood, which in turn impact disorders such as sickle cell anaemia, diabetes, and malaria. In order to make the problem more manageable, developing two-phase models requires making a number of assumptions and approximations. Some examples of these assumptions include the assumption of constant or fluctuating laminar flow, the omission of specific viscous or inertial effects, and the simplification of blood artery geometry to axisymmetric or cylindrical shapes. More accurate, data-driven models tailored to individual patients are, however, within reach because to advancements in HPC and ML.

For example, by utilising surrogate or reduced-order modelling approaches, as well as optimising boundary conditions, machine learning algorithms may help estimate unknown model parameters and decrease computing costs. Another current field of study that improves the physiological relevance of simulations is the integration of multiscale modelling frameworks. These frameworks connect mechanics at the cellular level with haemodynamics at the organ level (Timmis, J. 2020).

The non-Newtonian behaviour of blood must be taken into consideration in order to accurately depict two-phase flows. The shear-thinning properties of blood cause its viscosity to decrease with increasing shear rate in small channels and at low shear rates. Several rheological models have sought to account for this non-Newtonian behaviour; these include the Carreau, Herschel-Bulkley, and Casson models. By simulating a two-phase structure, these models illuminate crucial processes including the formation of a cell-free layer adjacent to the vessel walls, which modifies the effective viscosity and shear stress distribution. In addition, two-phase models that incorporate thixotropy and viscoelasticity may be used to explain the transient changes in flow properties that happen during various physiological processes like exercise, trauma, or surgery (Cai, Q., & Fan, Y. 2020).

OVERVIEW OF HEMODYNAMICS

Blood flow and its characteristics and behaviour in the cardiovascular system are the subject of haemodynamics, a branch of applied science. How blood flows through arteries, how it contacts with the walls of those arteries, and how external factors like pressure gradients and resistance impact that flow are all part of this complex biophysical and physiological process. The fundamental goal of haemodynamics is to deduce the mechanisms by which blood carries oxygen and nutrients to various organs and tissues while simultaneously removing waste materials. When it comes to cardiovascular health, haemodynamics is essential for understanding the causes of conditions including high blood pressure, atherosclerosis, aneurysms, and heart failure. As more advanced mathematical and computational methods have been available, haemodynamics has progressed from an area based on observation and experimentation to one with strong analytical and predictive capabilities. Applied mathematics, biomedical engineering, physiology, and fluid dynamics now all meet at this point. The fundamental idea of haemodynamics is that blood is a fluid that follows the rules of motion when it flows. Blood was originally modelled as a Newtonian fluid, meaning that the connection between shear stress and shear rate was assumed to be linear. Smaller channels such as arterioles, capillaries, and venules exhibit more complicated blood behaviour, which is not captured by this assumption, but it is roughly correct for big arteries with high shear rates and fairly uniform flow. Because of its particle-like structure, blood really displays non-Newtonian properties; it consists of a liquid medium (plasma) containing red blood cells (RBCs), white blood cells (WBCs), platelets, and plasma proteins. The viscosity and flow responsiveness of blood are influenced by these suspended components, particularly under low shear situations. Therefore, while researching microcirculation or diseased states, it is essential to accurately simulate haemodynamics by taking into consideration the heterogeneous and multiphase character of blood (Quarteroni, A. 2002).

Blood pressure, flow rate, volume, resistance to flow, and vessel compliance are the basic characteristics that control haemodynamic processes. Physical rules, mainly those derived from fluid mechanics, link these factors. The relationship between blood flow (Q), the pressure gradient (ΔP), and vascular resistance (R),

which is represented as Q = ΔP / R, is akin to Ohm's law for fluid flow, the most fundamental and frequently cited connection in this context. Although this connection is helpful for measuring vascular resistance or blood flow in clinical practice, it oversimplifies the complex and ever-changing nature of actual blood flow in live beings. The pulsatile nature of blood flow in biological systems is a result of the heart's rhythmic pumping motion and other factors such as the elasticity of the vessels, the geometry of their branches, and the variation in their cross-sectional areas. There are three main parts of the circulatory system: the arterial, capillary, and venous branches. These branches exhibit different haemodynamic properties. The blood often moves quickly, pulsatilely, and under intense pressure through the arteries. Because of their elasticity, arterial walls can stretch and contract with every pulse, mitigating the effects of the heart's pulsatile pressure waves. For a somewhat constant flow in tissues, this phenomenon is crucial. The exchange of gases, nutrients, and metabolic waste mostly takes place in the capillaries, the smallest blood vessels (García, A. 2010).

BLOOD AS A MULTI-PHASE FLUID

Compared to classical, one-phase fluids, blood, which is both necessary and one of the body's most complicated fluids, has quite different properties. Blood is a two- or multi-phase fluid because it contains both a liquid (plasma) and a dispersed cellular component (mostly red blood cells, white blood cells, and platelets). Blood flow dynamics are exceptionally difficult to predict because of the many non-Newtonian behaviours and spatial heterogeneities introduced by their multi-phase nature. Microcirculation, pathological conditions, and medical device design all rely on precise mathematical and numerical models of blood's flow through the vascular system, which in turn requires an understanding of blood as a multi-phase fluid. Plasma, the liquid component of blood, is a straw-colored fluid that makes up approximately 55% of the total volume of blood. You can find water making up around 90–92 percent of it, along with proteins like albumin and fibrinogen, electrolytes, minerals, hormones, and waste products. The physical properties of plasma cause it to act like a Newtonian fluid. But erythrocytes (red blood cells, or RBCs) make up the bulk of the dispersed particulate matter that makes up the other 45% of blood (Kim, E. H. 2003).

Because it contains both plasma and cells, blood has non-Newtonian rheological characteristics such viscoelasticity and shear-thinning viscosity (the viscosity reduces as the shear rate increases). Because of the complexity of these behaviours, two-phase fluid dynamics must be applied in place of more traditional Newtonian fluid models. Based on their volumetric dominance and dynamic behaviour, red blood cells are typically the focus of two-phase modelling, which views blood flow as consisting of two interpenetrating continua: the fluid phase, which includes plasma, and the particulate phase, which includes cells. Mathematically speaking, two-phase flow models can represent either a single fluid or a combination of fluids. The homogeneous models integrate the interaction between the two phases into averaged parameters such as effective viscosity and density, and they assume that the two phases move with the same velocity field.

Modelling two-phase blood flow using a heterogeneous method requires a basic set of equations for each phase, which include mass and momentum conservation. To illustrate, let's say that the plasma phase is the first and the red blood cell phase is the second. Every phase's continuity equations are (Bagchi, P. 2007):

$$\frac{\partial(\alpha_i \rho_i)}{\partial t} + \nabla.(\alpha_i \rho_i u_i) = 0 \quad for \ i = 1,2$$

where:

- αi is the proportion of phase i by volume (with α1+α2=1),
- ρi density of the first phase
- ui represents the phase i velocity vector.
- t is time.

Every phase's momentum equations are:

$$\frac{\partial(\alpha_i \rho_i u_i)}{\partial t} + \nabla.(\alpha_i \rho_i u_i \otimes u_i) = -\alpha_i \nabla p_i + \nabla T_i + \alpha_i \rho_i g + M_{ij}$$

where:

- pi is the initial pressure
- τi is the phase i stress tensor, which is typically determined by the viscosity and strain rate.
- One measure of acceleration due to gravity is g.
- Between phases I and J, there occurs a momentum exchange (Mij) that can occur for several reasons, such as drag, lift forces, or collision.

The transfer of momentum across interfaces A drag force component is usually included in Mij and is represented as:

$$M_{ij} = K_{ij}(u_j - u_i)$$

This is commonly affected by the relative velocity, Reynolds number, and local volume percentage, and where Kij is the interphase momentum exchange coefficient. Kij modelling is an important part of two-phase blood models because it controls the efficiency of momentum transfer between plasma and RBCs (Misbah, C. 2014).

The axial buildup of RBCs, particularly in small arteries, is a key aspect of two-phase blood flow. The migration of red blood cells (RBCs) towards the centre of the channel, caused by hydrodynamic interactions and deformability, results in a layer of cells that is depleted at the walls of the vessel. A less sharp velocity profile, higher core apparent viscosity, and lower wall shear stress are the outcomes of this process. In microcirculation, these effects have a significant impact on oxygen delivery and platelet margination, the latter of which is critical for clot formation.

The variables that determine the effective viscosity of the whole blood, μeff, are the haematocrit (the

volume proportion of red blood cells), temperature, and shear rate. Various empirical relations have been employed to elucidate this reliance, including the Quemada and Casson models. The Casson model, for example, specifies:

$$\sqrt{T} = \sqrt{T_y} + \sqrt{\eta^2 \gamma}$$

where:

- T the strain due to shear
- Ty where yield stress is
- η refers to the plasma's thickness
- γ is the rate of shear

On the other hand, two-phase models that are more physically driven incorporate microscale models for the dynamics of individual red cells and employ suspension theory to determine the effective viscosity based on particle contact and deformation. Phase separation at bifurcations is another feature unique to multiphase models; this happens when red blood cell (RBC) and plasma distributions are uneven in daughter vessels as a result of inertia, vessel shape, and haematocrit level.

GOVERNING EQUATIONS OF BLOOD FLOW

Fundamental rules of fluid dynamics, including the conservation laws of mass, momentum, and, in more complex models, energy, control the blood's movement through the circulation system. The mathematical representation of these physical rules is given by a system of partial differential equations. Because blood really consists of two phases—a liquid plasma phase and a cellular phase—these governing equations need to be modified to reflect this fact in order for two-phase flow models of blood to be realistic. distinct modelling approaches call for distinct formulations of the governing equations when dealing with blood, which may be thought of as either a one-phase non-Newtonian fluid or a multi-phase fluid. The Navier-Stokes equations specify the flow of a Newtonian, incompressible fluid in classical fluid dynamics; these equations also represent the conservation of momentum and mass. The non-Newtonian characteristics of blood and the interactions between its phases make its behaviour more complicated (Bluestein, D. 2008).

Continuity Equation (Conservation of Mass)

Every phase's mass is conserved according to the continuity equation. The two-step process known as the Euler-Euler method involves treating plasma (phase 1) and red blood cells (phase 2) as continuous media. The equation for continuity in each phase is expressed as Every phase's mass is conserved according to the continuity equation. The two-step process known as the Euler-Euler method involves treating plasma (phase 1) and red blood cells (phase 2) as continuous media. The equation for continuity in each phase is expressed as:

$$\frac{\partial(\alpha_1 \rho_1)}{\partial t} + \nabla.(\alpha_1 \rho_1 u_1) = \Gamma_i \; for \; 1 = 1,2$$

where:

- αi: proportion of phase i by volume,
- ρi: volume of the first phase,
- ui: the vector of velocity for phase i
- Γi: phase-to-phase mass exchange term (often 0 in the absence of mass transfer),
- t: time.

Assuming there are no voids or overlapping volumes, the constraint α1 + α2 = 1 must be true across the domain(Salikhov, K. M. 2020).

Momentum Conservation Equation (Navier–Stokes for Two-Phase Fluids)

Every phase's momentum equation takes into consideration the velocity of change of momentum, as well as internal stresses, external forces (like gravity), and the exchange of momentum between phases (such drag forces between cells and plasma) (Volpert, V. 2016).

In the first stage, the momentum equation is:

$$\frac{\partial(\alpha_i \rho_i u_i)}{\partial t} + \nabla.(\alpha_i \rho_i u_i \otimes u_i) = -\alpha_i \nabla p + \nabla T_i + \alpha_i \rho_i g + M_{ij}$$

where:

- p: shared pressure across phases (taking mechanical equilibrium into consideration),
- τi: for the first phase, the viscous stress tensor
- g: the force of gravity, which might not be very strong in a horizontal stream,
- Mij: energy is transferred from phase j to phase i as a result of drag or lift.

The drag force, really an expression for the interphase momentum transfer term Mij, is:

$$M_{ij} = K_{ij}(u_j - u_i)$$

Viscous Stress Tensor and Non-Newtonian Behavior

The word "viscous stress tensor" (τi) means:

$$T_i = \mu_i(\nabla u_i + \nabla u_i^T) - \frac{2}{3}\mu_i(\nabla.u_i)I$$

where:

- μi: dynamic phase i viscosity,
- I: identity tensor.

Having said that, the viscosity of blood varies. In the cellular phase, non-Newtonian behaviour is caused by

viscosity, which is commonly affected by haematocrit (Hct) and shear rate γ.

The Casson model is a popular rheological tool for this purpose (Kalluri, R. M. 2009):

$$\sqrt{T} = \sqrt{T_y} + \sqrt{\eta^2 \gamma}$$

that more closely matches the results of the experiments, or the Carreau-Yasuda model:

$$\mu(\gamma) = \mu_0 + (\mu_\infty - \mu_\infty)[1 + (\lambda\gamma)^a]^{(n-1)/a}$$

where:

- μ0: viscosity at zero shear,
- μ∞: the viscosity at infinite shear,
- λ: duration fixed,
- n: force-law indices,
- a: suitable parameter.

Single-Phase Simplification and Limitations

An effective viscosity μeff that is dependent on haematocrit and shear rate is occasionally used in big arteries as part of a simplified one-phase Navier-Stokes model:

$$\rho\left(\frac{\partial u}{\partial t} + (u.\nabla)u\right) = -\nabla p + \nabla.\left[\mu_{eff}(\nabla u + \nabla u^T)\right] + \rho g$$

Despite its computational efficiency, this model fails to account for some occurrences, such as cell separation at bifurcations, platelet margination, or the creation of a layer devoid of cells.

ROLE OF VISCOSITY AND DENSITY IN TWO-PHASE MODELING

To accurately portray the system's mechanical behaviour and physiological dynamics, two-phase blood flow models rely on viscosity and density. The presence of red blood cells (RBCs) floating in plasma causes blood, a composite fluid, to differ significantly from single-phase Newtonian fluids in terms of its properties. Any analytical or computational framework must accurately account for the phase-specific viscosity and density due to the non-uniform distribution, deformability, and aggregation of RBCs, as well as the different rheological characteristics of the continuous phase (plasma) and the dispersed phase (RBCs). All along the vascular network, but especially in the microcirculatory domain, these characteristics have an effect on momentum transfer, pressure distribution, shear stress, phase interaction, and flow resistance. Equations for two-phase flows are based on the density and viscosity of the two phases, which are not static qualities but rather dynamic values that interact with flow rate, haematocrit, shear rate, vessel shape, and density (Wang, X. 2015).

The two-phase flow modelling in the Euler-Euler formulation is done by treating each phase as an

interpenetrating continuous medium and applying conservation equations to each phase separately. The internal resistance to deformation caused by shear stress is controlled by the viscosity (μ), whereas the inertial properties of the fluid are related to the density (ρ). Blood plasma is often represented as a Newtonian fluid with a constant viscosity μ1 and density ρ1, however red blood cell (RBC) phase, because of its particulate and deformable nature, can display non-Newtonian behaviour with an effective viscosity μ2 and density ρ2. Taking these characteristics into consideration, the following are the momentum and conservation of mass equations for each phase:

Continuity equation for phase i:

$$\frac{\partial(\alpha_i \rho_i)}{\partial t} + \nabla.(\alpha_i \rho_i u_i) = 0$$

Momentum conservation for phase i:

$$\frac{\partial(\alpha_i \rho_i u_i)}{\partial t} + \nabla.(\alpha_i \rho_i u_i \otimes u_i) = -\alpha_i \nabla p + \nabla T_i + \alpha_i \rho_i g + M_{ij}$$

the vector of velocity ui, the stress tensor τi, the volume fraction αi, the often-negligible gravitational acceleration g, and the interphase momentum transfer term Mij. Newtonian fluids have a linear relationship between viscosity and the stress tensor τi, while non-Newtonian fluids have a nonlinear relationship. When considering the plasma phase, the stress tensor is:

$$T_1 = \mu_1(\nabla u_1 + \nabla u_1^T) - \frac{2}{3}\mu_1(\nabla.u_1)I$$

Varying with shear rate $\dot{\gamma}$, haematocrit Hct, and aggregation behaviour, μ2 is not constant for the RBC phase. This non-Newtonian viscosity is defined using a number of models, including the Casson, Carreau-Yasuda, and Quemada models. A generalised expression is given, for instance, by the Carreau-Yasuda model:

$$\mu(\gamma) = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\lambda\gamma)^a]^{(n-1)/a}$$

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model:

$$\mu(\gamma) = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\lambda\gamma)^a]^{(n-1)/a}$$

This is where the zero-shear viscosity (μ0) and infinite-shear viscosity (μ∞) come into play, together with a time constant (λ), the power-law index (nnn), and the adjustment of the transition between shear rates (a). In particular, this model accounts for the shear-thinning behaviour of RBCs, which has a major impact on flow resistance in capillaries and arterioles.

When it comes to controlling momentum dynamics, density is just as important. Density is a direct proportional variable to the inertial component of the Navier-Stokes equations. Plasma has a density of around 1025 kg/m3, while RBCs have a density of about 1125 kg/m3 under normal physiological circumstances. In high-speed flows, centrifugation, or oscillatory motion (e.g., the heart cycle or pulsatile pumping), the seemingly insignificant difference becomes non-negligible. Phase separation events, such as plasma skimming and axial migration of RBCs in tiny channels, are also influenced by the density contrast. A cell-free plasma layer forms at the walls of the vessels as a result of these processes; this layer changes the wall shear stress and decreases the apparent viscosity.

In addition to relative velocities and viscosities, the drag between RBCs and plasma is described by the interphase momentum exchange term Mij. An example of a typical portrayal is (Meinhart, C. D. 2011):

$$M_{ij} = K_{ij}(u_j - u_i)$$

the drag coefficient, denoted as Kij, may be represented mathematically as:

$$K_{ij} = \frac{9\mu_1}{2r^2}\emptyset$$

In this case, the local volume fraction is denoted by ϕ, and r is the radius of the RBC. The Stokes drag law, which has been adjusted for multiphase flow, is the source of this statement, which demonstrates how the plasma viscosity and the size and concentration of RBCs directly affect drag. Every phase's velocity fields and pressure gradients, and the flow dynamics as a whole, are profoundly impacted by the resulting drag force.

In addition, the flow regime is characterised by the Reynolds number (Re), which is determined by both density and viscosity (Yuan, F. 2000):

$$M_{ij} = K_{ij}(u_j - u_i)$$

the drag coefficient, denoted as Kij, may be represented mathematically as:

$$K_{ij} = \frac{9\mu_1}{2r^2}\emptyset$$

the characteristic diameter (D) and the average velocity (uuu) are defined. In the aorta and other macrocirculations, inertial effects are dominant and Re is large (~1000-2000). Re is extremely low (<1) in microcirculation (such as capillaries), and viscous effects are the main factor. To properly characterise the interphase dynamics and flow regime transitions in two-phase models, it is necessary to calculate different Reynolds numbers for each phase (Norouzi, M. 2019).

CONCLUSION

In conclusion, the mathematical modeling and numerical simulation of two-phase blood flow provide a deeper understanding of the complex hemodynamic behavior occurring within the human circulatory system. By treating blood as a suspension of plasma and cellular elements—particularly red blood cells—two-phase models enable the capture of key physiological phenomena such as cell-plasma interactions, phase separation, and the non-Newtonian nature of blood flow, especially in microcirculation. These models help to identify how variations in viscosity, density, and shear stress influence blood flow patterns and wall interactions, which are critical for predicting disease progression such as atherosclerosis or thrombosis. Numerical simulations based on finite element, finite volume, or lattice Boltzmann methods provide the computational framework to solve the governing partial differential equations under realistic physiological conditions and boundary constraints. These simulations allow for visualization and analysis of blood flow under various pathological and geometric configurations, offering insights that are not easily achievable through experimental techniques alone. Overall, two-phase modeling and simulation represent a powerful tool for advancing biomedical research, improving diagnostic precision, optimizing medical device designs, and supporting clinical decision-making in cardiovascular health and disease management. Future advancements integrating patient-specific data and multi-scale modeling approaches will further enhance the predictive capabilities of such simulations.

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