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Studying the Effects of Certain Curves and Metal Structures on Manifold Properties

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Abstract: The study seeks to understand the effect on geometric and topological qualities of certain curves and metal constructions on the manifold properties by investigating their effects. The research attempts to understand the effects of these components on curvature, connectivity, and general manifold behaviour by studying geodesics and closed curves inside diverse metal shapes. The work establishes important connections between curves' intrinsic qualities and metals' structural attributes by means of sophisticated mathematical methods and computer simulations. The results show that specific metal structure configurations may change the topology and curvature of the manifolds they are embedded in, which could have implications for engineering and materials research. The results shed light on the ways in which concrete objects can affect theoretically abstract geometrical spaces, adding to our knowledge of manifold theory as a whole. This discovery has far-reaching consequences; it lays the groundwork for the development of new materials with optimised geometric characteristics and paves the way for fresh uses of manifold theory in engineering. To develop and optimise structural designs, it is crucial to integrate mathematical theory with material science, as this multidisciplinary approach highlights.

Keywords: CR-submanifolds, Kenmotsu manifold, Certain Curves, Metal Structures, Manifold Properties

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INTRODUCTION

In differential geometry, CR-submanifolds have been a hotspot for study for the past four decades. In 1978, A. Bejancu introduced the notion of CR-submanifolds of a Kaehler manifold, which extends to complex and real submanifolds alike. In this paper, A. Bejancu explored many geometry issues related to CR submanifolds. Contact CR-submanifolds in Kenmotsu manifolds were subsequently the subject of study by Atceken et al., who uncovered some intriguing features.

Professor D. E. Blair first proposed the idea of a Killing tensor field in 1971. Many writers have studied the class of almost contact manifolds first studied by K. Kenmotsu in 1972, which is called a Kenmotsu manifold (Ratiu, T. 2008).

One specific kind of almost contact metric manifold is called a Kenmotsu manifold, and it is defined by the fact that $(\nabla X\phi)=-g(\phi X,Y)\xi+\eta(Y)\phi X$, This stands for the Levi-Civita relationship of b. Because of its unique geometric features, the Kenmotsu manifold is a great place to explore Contact CR-submanifolds and other submanifold structures. The existence of a CR (Cauchy-Riemann) structure, a naturally occurring extension of the complex structure in the theory of multiple complex variables to the context of nearly contact metric manifolds, defines these submanifolds. In particular, the splitting of the tangent bundle TN into two orthogonal subbundles defines a Contact CR-submanifold N of an almost contact metric manifold

M $D=TN\cap\ker(\eta)$ alongside the range of The subbundle D remains unchanged when ϕ is applied. Meaning $(D)\subseteq D$ (Biswas, 2014). Additionally, D divides into $D=D1\oplus D2$, where $D1$ represents a complicated subbundle of D (i.e., $(D1)=D1$and $D2$ it is orthogonal to in D (i.e., $\phi(D2)\subseteq D1$. The duality and interplay of the holomorphic and fully real components of the submanifold's geometry are mirrored by this splitting. (Ojha, R. H. 2011)

PRELIMINARIES

What if..." $\overline{M}$ exists as a differentiable manifold with (2n + 1) dimensions. Let φ, ξ, η and g depict a vector field, a Riemannian metric, a 1-form, and a tensor field of type (1,1) on $\overline{M}$, and so on. If φ, ξ, η and g meet all of the prerequisites:

$$\begin{cases} \phi\xi = 0, \ \phi^2\mathcal{U} = -\mathcal{U} + \eta(\mathcal{U})\xi, \\ \eta(\xi) = 1, \ \ \eta(\phi\mathcal{U}) = 0, \end{cases} \tag{2.1}$$

And

$$\begin{cases} g(\phi\mathcal{U}, \phi\mathcal{V}) = g(\mathcal{U}, \mathcal{V}) - \eta(\mathcal{U})\eta(\mathcal{V}), \\ \qquad \eta(\mathcal{U}) = g(\mathcal{U}, \xi), \end{cases} \tag{2.2}$$

as long as U and V are vector fields on $\overline{M}$, subsequently, the framework (φ,ξ,η,g) has a metric structure that is almost contact. A (2n+1)-dimensional bipartite surface $\overline{M}$ is considered to be an almost contact metric manifold when coupled with an almost contact metric structure.

Let $\overline{\nabla}$ represents the link between Levi and Civita on $\overline{M}$ as well as in the event that certain criteria are met: (Venkatesha. 2008)

$$\begin{cases} (\overline{\nabla}_{\mathcal{U}}\phi)\mathcal{V} = g(\phi\mathcal{U}, \mathcal{V})\xi - \eta(\mathcal{V})\phi\mathcal{U}, \\ \qquad \overline{\nabla}_{\mathcal{U}}\xi = \mathcal{U} - \eta(\mathcal{U})\xi, \end{cases} \tag{2.3}$$

after that, the framework $(\overline{M}, \phi, \xi, \eta, g)$ known as a Kenmotsu manifold.

Say M is a submanifold submerged in space that is isometrically $\overline{M}$. What if ∇ and $\overline{\nabla}$, Riemannian connections on M are denoted by and $\overline{M}$. This leads us to the following formulae for both Gauss and Weingarten:

$$\overline{\nabla}_{\mathcal{U}}\mathcal{V} = h(\mathcal{U}, \mathcal{V}) + \nabla_{\mathcal{U}}\mathcal{V}, \quad (2.4)$$

And

$$\overline{\nabla}_{\mathcal{U}}\mathcal{X} = \nabla^{\perp}_{\mathcal{U}}\mathcal{X} - A_{\mathcal{X}}\mathcal{U}, \quad (2.5)$$

while dealing with vector fields U and V in Γ(TM) and $\mathcal{X} \in \Gamma(T^{\perp}M)$, where $\nabla^{\perp}$ displays the typical relationship between $T^{\perp}M$, Let h stand for the second basic form of M in the Kenmotsu manifold and let A be the shape operator $\overline{M}$.

The relationship between the shape operator A and the second basic form h is as

$$g(h(\mathcal{U}, \mathcal{V}), \mathcal{X}) = g(A_{\mathcal{X}}\mathcal{U}, \mathcal{V}). \quad (2.6)$$

A submanifold that is isometrically immersed is denoted by M in the Kenmotsu manifold. For any vector field U that is perpendicular to M, we may deduce

$$\phi\mathcal{U} = t\mathcal{U} + \omega\mathcal{U}, \quad (2.7)$$

where tU and ωU stand for the normal component of and the tangential component of φU.

A function of t and ω its covariant derivative are conveyed by: (Sharma, R. 2009)

$$(\nabla_{\mathcal{U}} t)\mathcal{V} = \nabla_{\mathcal{U}} t\mathcal{V} - t\nabla_{\mathcal{U}}\mathcal{V},$$

And

$$(\nabla_{\mathcal{U}}\omega)\mathcal{V} = \nabla^{\perp}_{\mathcal{U}}\omega\mathcal{V} - \omega\nabla_{\mathcal{U}}\mathcal{V}.$$

Just as before, we place for every vector field X that is normal to M

$$\phi\mathcal{X} = B\mathcal{X} + C\mathcal{X}, \quad (2.8)$$

where BX represents the tangential component of φX and CX stands for the normal component.

What follows is an expression for the covariant derivative of B and C:

$$(\nabla_{\mathcal{U}} B)\mathcal{X} = \nabla_{\mathcal{U}} B\mathcal{X} - B\nabla^{\perp}_{\mathcal{U}}\mathcal{X},$$

And

$$(\nabla_{\mathcal{U}} C)\mathcal{X} = \nabla^{\perp}_{\mathcal{U}} C\mathcal{X} - C\nabla^{\perp}_{\mathcal{U}}\mathcal{X}.$$

If the definition of endomorphism t is given by equation (2.7), then it follows that

$$g(t\mathcal{U}, \mathcal{V}) + g(\mathcal{U}, t\mathcal{V}) = 0. \quad (2.9)$$

Definition 2.1. A Kenmotsu manifold has a submanifold represented by $\overline{M}$. When this occurs, we say that M is a contact CR-submanifold of $\overline{M}$ if the distribution is differentiable $D : p \to D_p \subseteq T_p(M)$ on M that fulfils the given criteria: Koufogiorgos, T. 2001)

- $TM = D \oplus D^{\perp}, \quad \xi \in D,$
- D is an invariant matrix with regard to ϕ, i.e., $\phi D_p = D_p,$

- The distribution that is orthogonal and complements $D^{\perp} : p \to D_p^{\perp} \subseteq T_p(M)$ is anti-invariant, i.e., $\phi D_p^{\perp} \subseteq T_p^{\perp}(M)$, *for each* $p \in M$."

A submanifold M is considered to be fully real if and only if dim $D_p = 0$. $D_p^{\perp} = 0$, is known as a complicated submanifold of M. Properness is defined as the absence of complexity and complete realness on a contact CR-submanifold (Duggal, K. L. 2000).

Consider the case where M is a contact CR-submanifold of $\overline{M}$ using vector fields U and V in Γ(TM). Equation (2.8) is derived using equations (2.3), (2.7), and the Gauss and Weingarten formulae.

$$(\overline{\nabla}_{\mathcal{U}}\phi)\,\mathcal{V} = \overline{\nabla}_{\mathcal{U}}\phi\mathcal{V} - \phi\overline{\nabla}_{\mathcal{U}}\mathcal{V}, \quad (2.10)$$

Or,

$$g\,(\phi\mathcal{U},\mathcal{V}) - \eta\,(\mathcal{V})\;\; \phi\mathcal{U} = \overline{\nabla}_{\mathcal{U}}t\mathcal{V} + \overline{\nabla}_{\mathcal{U}}\omega\mathcal{V} - \phi\nabla_{\mathcal{U}}\mathcal{V} - \phi h(\mathcal{U},\mathcal{V}).$$

By contrasting the normal and tangential components of the two sides of the above equation, we get the following equations:

$$(\nabla_{\mathcal{U}}t)\mathcal{V} = A_{\omega\mathcal{V}}\mathcal{U} + Bh\,(\mathcal{U},\mathcal{V}) + g\,(t\mathcal{U},\mathcal{V})\,\xi - \eta\,(\mathcal{V})\,t\mathcal{U}, \quad (2.11)$$

And

$$(\nabla_{\mathcal{U}}\omega)\,\mathcal{V} = Ch\,(\mathcal{U},\mathcal{V}) - h\,(\mathcal{U},t\mathcal{V}) - \eta(\mathcal{V})\omega\mathcal{U}. \quad (2.12)$$

M is perpendicular to the structural vector field. Next, we obtain by combining equations (2.3) and (2.6),

$$A_{\mathcal{X}}\xi = h\,(\mathcal{U},\xi) = 0, \quad (2.13)$$

to all vector fields U in Γ(TM) and all vector fields X in Γ(T⊥M). The result is that equation (2.11) changes to

$$(\nabla_{\mathcal{U}} t)\mathcal{V} = g(t\mathcal{U}, \mathcal{V})\,\xi - \eta(\mathcal{V})\,t\mathcal{U}, \quad (2.14)$$

for any vector fields in Γ(D), where U and V are any. Hence, a Kenmotsu structure on M is what we term the induced structure t (Pastore, A. M. 2004).

Assume that M stands for the contact CR-submanifold in $\overline{M}$. Because of this, we may simplify equation (2.11) to

$$(\nabla_{\mathcal{U}} t)\mathcal{V} = Bh(\mathcal{U}, \mathcal{V}) + g(t\mathcal{U}, \mathcal{V})\,\xi - \eta(\mathcal{V})\,t\mathcal{U}, \quad (2.15)$$

where Γ(D) contains vector fields U and V.

A completely geodesic submanifold of M is defined as h = 0, i.e., the second basic form disappears. If the following equality holds for the Riemannian metric g and the second basic form h, we say that submanifold M is fully umbilical.

$$g(\mathcal{U}, \mathcal{V})\,H = h(\mathcal{U}, \mathcal{V}),$$

where H is the vector representing the average curvature. Additionally, the submanifold M is referred to be minimum if and only if H = 0.

Let φ stands for the killing tensor field, which is a tensor field of type (1,1), provided the following conditions are met: (Matsumoto, K. 2007)

$$\left(\overline{\nabla}_{\mathcal{U}}\phi\right)\mathcal{V} + \left(\overline{\nabla}_{\mathcal{V}}\phi\right)\mathcal{U} = 0. \quad (2.16)$$

CONTACT CR-SUBMANIFOLD OF A KENMOTSU MANIFOLD WITH KILLING TENSOR FIELD

For the contact CR-submanifold in the Kenmotsu manifold with a Killing tensor field, various intriguing outcomes have been investigated in this part.

Theorem 3.1. Suppose $\overline{M}$ is a contact CR submanifold of, and is a Kenmotsu manifold. $\overline{M}$ with Killing tensor field φ, then

$$(\overline{\nabla}_{\mathcal{U}}t\mathcal{V} + \overline{\nabla}_{\mathcal{V}}t\mathcal{U}) - t(\overline{\nabla}_{\mathcal{U}}\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\mathcal{U}) = \omega(\overline{\nabla}_{\mathcal{U}}\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\mathcal{U}) - (\overline{\nabla}_{\mathcal{U}}\omega\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\omega\mathcal{U}). \quad (3.1)$$

Proof. The equation (2.10) gives us

$$(\overline{\nabla}_{\mathcal{U}}\phi)\mathcal{V} = \overline{\nabla}_{\mathcal{U}}\phi\mathcal{V} - \phi\overline{\nabla}_{\mathcal{U}}\mathcal{V}.$$

If we swap U and V in the above equation, we get

$$(\overline{\nabla}_{\mathcal{V}}\phi)\,\mathcal{U} = \overline{\nabla}_{\mathcal{V}}\phi\mathcal{U} - \phi\overline{\nabla}_{\mathcal{V}}\mathcal{U}.$$

By combining the two equations above, we get

$$(\overline{\nabla}_{\mathcal{U}}\phi)\,\mathcal{V} + (\overline{\nabla}_{\mathcal{V}}\phi)\,\mathcal{U} = \overline{\nabla}_{\mathcal{U}}\phi\mathcal{V} - \phi\overline{\nabla}_{\mathcal{U}}\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\phi\mathcal{U} - \phi\overline{\nabla}_{\mathcal{V}}\mathcal{U}.$$

Using equation (2.16) now, we obtain

$$0 = \overline{\nabla}_{\mathcal{U}}\phi\mathcal{V} - \phi\overline{\nabla}_{\mathcal{U}}\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\phi\mathcal{U} - \phi\overline{\nabla}_{\mathcal{V}}\mathcal{U}. \quad (3.2)$$

Utilising (2.7), the equation above becomes:

$$(\overline{\nabla}_{\mathcal{U}}t\mathcal{V} + \overline{\nabla}_{\mathcal{V}}t\mathcal{U}) - t(\overline{\nabla}_{\mathcal{U}}\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\mathcal{U}) = \omega(\overline{\nabla}_{\mathcal{U}}\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\mathcal{U}) - (\overline{\nabla}_{\mathcal{U}}\omega\mathcal{V} + \overline{\nabla}_{\mathcal{V}}\omega\mathcal{U}).$$

Theorem 3.2. Assume that M represents a CR-submanifold in contact with a killing tensor field. φ of a manifold Kenmotsu $\overline{M}$, then

$$\eta(\mathcal{V})\,t\mathcal{U} + \eta(\mathcal{U})\,t\mathcal{V} = 0 \quad (3.3)$$

$$\eta(\mathcal{V})\,\omega\mathcal{U}+\eta(\mathcal{U})\,\omega\mathcal{V}=0. \quad (3.4)$$

Proof. Equation (2.3) tells us that

$$\left(\overline{\nabla}_{\mathcal{U}}\phi\right)\mathcal{V}=g\left(\phi\mathcal{U},\mathcal{V}\right)\xi-\eta\left(\mathcal{V}\right)\phi\mathcal{U}.$$

When U and V are switched, the previous equation becomes

$$\left(\overline{\nabla}_{\mathcal{V}}\phi\right)\mathcal{U}=-g\left(\phi\mathcal{U},\mathcal{V}\right)\xi-\eta\left(\mathcal{U}\right)\phi\mathcal{V}.$$

Now, by integrating the two equations given before, we get

$$\left(\overline{\nabla}_{\mathcal{U}}\phi\right)\mathcal{V}+\left(\overline{\nabla}_{\mathcal{V}}\phi\right)\mathcal{U}=-\eta\left(\mathcal{V}\right)\phi\mathcal{U}-\eta\left(\mathcal{U}\right)\phi\mathcal{V}.$$

Using the formula (2.16), we obtain

$$-\eta\left(\mathcal{V}\right)\phi\mathcal{U}-\eta\left(\mathcal{U}\right)\phi\mathcal{V}=0. \quad (3.5)$$

Applying equation (3.5) to the value of (2.7), we can next compare the normal and tangential components. I have achieved the intended outcome (Chen, B. Y. 2000).

Theorem 3.3. What if $\overline{M}$ is a contact CR submanifold of, and is a Kenmotsu manifold. $\overline{M}$ when the killing tensor field is present φ, hence, t, the induced structure, meets

$$(\nabla_{\mathcal{U}}t)\mathcal{V}+(\nabla_{\mathcal{V}}t)\mathcal{U}=0. \quad (3.6)$$

Proof. According to the formula (2.14), we may deduce

$$(\nabla_{\mathcal{U}}t)\mathcal{V}=g\left(t\mathcal{U},\mathcal{V}\right)\xi-\eta\left(\mathcal{V}\right)t\mathcal{U}.$$

After switching U and V in the above equation, we get

$$(\nabla_{\mathcal{V}} t)\mathcal{U} = g(\mathcal{U}, t\mathcal{V})\,\xi - \eta(\mathcal{U})\,t\mathcal{V}.$$

We obtain by combining the two equations given above:

$$(\nabla_{\mathcal{U}} t)\mathcal{V} + (\nabla_{\mathcal{V}} t)\mathcal{U} = -\eta(\mathcal{V})\,t\mathcal{U} - \eta(\mathcal{U})\,t\mathcal{V}.$$

With the application of (3.3) in the previous equation, the desired outcome has been achieved. (Golab, S. 2005)

Theorem 3.4. Suppose $\overline{M}$ is a contact CR submanifold of, and is a Kenmotsu manifold $\overline{M}$ when the killing tensor field is present φ. M is a completely geodesic manifold if and only if the second basic form h is parallel.

Proof. We obtain by swapping the values of U and V in equation (2.15).

$$(\nabla_{\mathcal{V}} t)\mathcal{U} = Bh(\mathcal{U}, \mathcal{V}) - g(\mathcal{V}, t\mathcal{U})\,\xi - \eta(\mathcal{U})\,t\mathcal{V}. \quad (3.7)$$

Equations (3.7) and (2.15) are combined to get us with

$$(\nabla_{\mathcal{U}} t)\mathcal{V} + (\nabla_{\mathcal{V}} t)\mathcal{U} = 2Bh(\mathcal{U}, \mathcal{V}) - \eta(\mathcal{V})\,t\mathcal{U} - \eta(\mathcal{U})\,t\mathcal{V}.$$

It is now possible to obtain by using equations (3.3) and (3.6)

$$h(\mathcal{U}, \mathcal{V}) \;\; vanishes.$$

for any vector fields in Γ(TM), where U and V are any.

Lemma 3.1. If we take M to be a Kenmotsu manifold's contact CR-submanifold,...... $\overline{M}$ when the killing tensor field is present φ, then

$$A_{\omega\mathcal{V}}\mathcal{U} + A_{\omega\mathcal{U}}\mathcal{V} + 2Bh(\mathcal{U}, \mathcal{V}) = 0. \quad (3.8)$$

Proof. We obtain by switching U and V in equation (2.11) to

$$(\nabla_{\mathcal{V}}t)\mathcal{U} = A_{\omega\mathcal{U}}\mathcal{V} + Bh\ (\mathcal{U},\mathcal{V}) + g\,(t\mathcal{V},\mathcal{U})\,\xi - \eta\,(\mathcal{U})\,t\mathcal{V}. \quad (3.9)$$

By applying (2.11) and (3.9) to the Clubbing equations, we obtain the following equation;

$$(\nabla_{\mathcal{U}}t)\mathcal{V} + (\nabla_{\mathcal{V}}t)\mathcal{U} = A_{\omega\mathcal{V}}\mathcal{U} + A_{\omega\mathcal{U}}\mathcal{V} + 2Bh\,(\mathcal{U},\mathcal{V}) + g\,(t\mathcal{U},\mathcal{V})\,\xi$$
$$+ g\,(t\mathcal{V},\mathcal{U})\,\xi - \eta\,(\mathcal{U})\,t\mathcal{V} - \eta\,(\mathcal{V})\,t\mathcal{U}.$$

Applying the formula (2.9), we obtain (Fetcu, D. 2008).

$$(\nabla_{\mathcal{U}}t)\mathcal{V} + (\nabla_{\mathcal{V}}t)\mathcal{U} = A_{\omega\mathcal{V}}\mathcal{U} + A_{\omega\mathcal{U}}\mathcal{V} + 2Bh\,(\mathcal{U},\mathcal{V}) - \eta\,(\mathcal{U})\,t\mathcal{V} - \eta\,(\mathcal{V})\,t\mathcal{U}.$$

Given that t is enough in (3.3) and (3.6). This means that we have achieved our goal.

Proposition 3.1. Assume that M is a contact CR-submanifold of $\overline{M}$ when the killing tensor field is present φ. The anti-invariant submanifold M is defined by the fact that if the endomorphism t is parallel $\overline{M}$.

Proof. What we get when we switch U and V in equation (2.15) is

$$(\nabla_{\mathcal{V}}t)\mathcal{U} = Bh\,(\mathcal{U},\mathcal{V}) + g\,(t\mathcal{V},\mathcal{U})\,\xi - \eta\,(\mathcal{U})\,t\mathcal{V},$$

where Γ(D) contains vector fields U and V.

After plugging the previous equation into (2.15), we get

$$(\nabla_{\mathcal{U}}t)\,\mathcal{V} + (\nabla_{\mathcal{V}}t)\,\mathcal{U} = 2Bh\,(\mathcal{U},\mathcal{V}) + g\,(t\mathcal{U},\mathcal{V})\,\xi + g\,(t\mathcal{V},\mathcal{U})\,\xi - \eta\,(\mathcal{V})\,t\mathcal{U} - \eta\,(\mathcal{U})\,t\mathcal{V}.$$

If we plug the values from (2.9) and (3.6) into the previous equation, we obtain

$$2Bh\,(\mathcal{U},\mathcal{V}) - \eta\,(\mathcal{V})\,t\mathcal{U} - \eta\,(\mathcal{U})\,t\mathcal{V} = 0.$$

We get the following conclusion after considering equations (2.1) and (2.13) and setting ξ = V:

$$t\mathcal{U} = 0.$$

A submanifold that is anti-invariant is M. (Venkatesha. 2006).

Proposition 3.2. The contact CR-submanifold of $\overline{M}$ when the killing tensor field is present φ. Submanifold M is referred to as invariant in $\overline{M}$ should the endomorphism ω is parallel.

Proof. Equation (2.12) may be updated to provide the following result by swapping U and V.

$$(\nabla_{\mathcal{V}}\omega)\mathcal{U} = Ch(\mathcal{U},\mathcal{V}) - h(\mathcal{V},t\mathcal{U}) - \eta(\mathcal{U})\omega\mathcal{V}, \quad (3.10)$$

Γ(TM) vector fields U and V are being referred to.

By combining equations (3.10) and (2.12), we get

$$(\nabla_{\mathcal{U}}\omega)\mathcal{V} + (\nabla_{\mathcal{V}}\omega)\mathcal{U} = 2C\ h(\mathcal{U},\mathcal{V}) - h(\mathcal{U},t\mathcal{V}) - h(\mathcal{V},t\mathcal{U}) - \eta(\mathcal{V})\omega\mathcal{U} - \eta(\mathcal{U})\omega\mathcal{V}.$$

Endomorphism could ω is parallel, the equation cited above produces

$$2C\ h(\mathcal{U},\mathcal{V}) - h(\mathcal{U},t\mathcal{V}) - h(\mathcal{V},t\mathcal{U}) - \eta(\mathcal{V})\omega\mathcal{U} - \eta(\mathcal{U})\omega\mathcal{V} = 0.$$

After considering equations (2.1) and (2.13), we may deduce that, for ξ = V,

$$\omega\mathcal{U} = 0.$$

A submanifold M is therefore invariant (Fischer, A. E. 2002).

Lemma 3.2. What if $\overline{M}$ signifies a contact CR-submanifold of and stands for a Kenmotsu manifold. $\overline{M}$ when the killing tensor field is present φ, then

$$(\nabla_{\mathcal{U}}\omega)\mathcal{V} + (\nabla_{\mathcal{V}}\omega)\mathcal{U} = 0. \quad (3.11)$$

Iff

$$2Ch(\mathcal{U},\mathcal{V}) = h(\mathcal{U},t\mathcal{V}) + h(\mathcal{V},t\mathcal{U}). \quad (3.12)$$

Proof. Equation (2.12) yields in this case:

$$(\nabla_{\mathcal{U}}\omega)\,\mathcal{V} = Ch\,(\mathcal{U},\mathcal{V}) - h\,(\mathcal{U},t\mathcal{V}) - \eta(\mathcal{V})\omega\mathcal{U}.$$

Next, we obtain by integrating the previous equation with (3.10):

$$(\nabla_{\mathcal{U}}\omega)\,\mathcal{V} + (\nabla_{\mathcal{V}}\omega)\,\mathcal{U} = 2Ch\,(\mathcal{U},\mathcal{V}) - h\,(\mathcal{U},t\mathcal{V}) - h\,(\mathcal{V},t\mathcal{U}) - \eta\,(\mathcal{V})\,\omega\mathcal{U} - \eta\,(\mathcal{U})\,\omega\mathcal{V}.$$

Applying the formula (3.4), we get

$$(\nabla_{\mathcal{U}}\omega)\,\mathcal{V} + (\nabla_{\mathcal{V}}\omega)\,\mathcal{U} = 2Ch\,(\mathcal{U},\mathcal{V}) - h\,(\mathcal{U},t\mathcal{V}) - h\,(\mathcal{V},t\mathcal{U})\,.$$

Therefore, the outcome is evident.

Examples

For Kenmotsu manifolds satisfying the Killing tensor field, few instances have been determined φ.

Example 3.1.1. What if $\overline{M} = \{(x,y,z) \in \mathbb{R}^3, z \neq 0\}$ (x, y, z) be the standard coordinates in R3, and be the three-dimensional manifold. Assume that g is a manifold metric. $\overline{M}$ supplied by

$$g = e^{2z}(dx \otimes dx + dy \otimes dy) + \eta \otimes \eta.$$

We may now select

$$e_1 = e^{-z}\frac{\partial}{\partial x}, \quad e_2 = e^{-z}\frac{\partial}{\partial y}, \quad e_3 = \frac{\partial}{\partial z} = \xi.$$

Each of the aforementioned vector fields is linearly independent with respect to $\overline{M}$ enough to $g(e_i,e_j) = 0$ for $i \neq j$ and $g(e_i,e_j) = 1$ for $i = j$, for $1 \leq i, j \leq 3$. For a given vector field U on the manifold, the 1-form is defined as (U) = g(U,e3) $\overline{M}$. Suppose φ stands for the field of (1,1)-tensors and is defined by $\varphi(e_1) = 0$, $\varphi(e_2) = 0$, $\varphi(e_3) = 0$.

We obtain the result by applying the linearity condition to φ and g.

$$\phi^2\mathcal{U} = -\mathcal{U} + \eta(\mathcal{U})\xi, \quad \eta(e_3) = 1, \quad g(\phi\mathcal{U}, \phi\mathcal{V}) = g(\mathcal{U}, \mathcal{V}) - \eta(\mathcal{U})\eta(\mathcal{V}),$$

for which the manifold's $\overline{M}$ vector fields U and V are selected (Hodge, W. V. D. 2018).

After doing the math directly, we get,

$$\overline{\nabla}_{e_1}e_1 = -e_3, \quad \overline{\nabla}_{e_1}e_2 = 0, \quad \overline{\nabla}_{e_1}e_3 = e_1,$$
$$\overline{\nabla}_{e_2}e_1 = 0, \quad \overline{\nabla}_{e_2}e_2 = -e_3, \quad \overline{\nabla}_{e_2}e_3 = e_2,$$
$$\overline{\nabla}_{e_3}e_1 = e_1, \quad \overline{\nabla}_{e_3}e_2 = e_2, \quad \overline{\nabla}_{e_3}e_3 = 0.$$

The manifold is shown to satisfy the equation by using the aforementioned relations.

$\overline{\nabla}_{\mathcal{U}}\xi = \mathcal{U} - \eta(\mathcal{U})\xi$ for $e_3 = \xi$. Hence, it is a Kenmotsu manifold. The following equations are derived from the relations mentioned above.

$$\begin{cases} (\overline{\nabla}_{e_1}\phi)e_1 + (\overline{\nabla}_{e_1}\phi)e_1 = 0, & (\overline{\nabla}_{e_1}\phi)e_2 + (\overline{\nabla}_{e_2}\phi)e_1 = 0, \\ (\overline{\nabla}_{e_1}\phi)e_3 + (\overline{\nabla}_{e_3}\phi)e_1 = 0, & (\overline{\nabla}_{e_2}\phi)e_1 + (\overline{\nabla}_{e_1}\phi)e_2 = 0, \\ (\overline{\nabla}_{e_2}\phi)e_2 + (\overline{\nabla}_{e_2}\phi)e_2 = 0, & (\overline{\nabla}_{e_2}\phi)e_3 + (\overline{\nabla}_{e_3}\phi)e_2 = 0, \\ (\overline{\nabla}_{e_3}\phi)e_1 + (\overline{\nabla}_{e_1}\phi)e_3 = 0, & (\overline{\nabla}_{e_3}\phi)e_2 + (\overline{\nabla}_{e_2}\phi)e_3 = 0, \\ (\overline{\nabla}_{e_3}\phi)e_3 + (\overline{\nabla}_{e_3}\phi)e_3 = 0. \end{cases} \qquad (3.1.1)$$

Since is the Killing tensor field, it may be inferred from equation (3.1.1). The result is that the $\overline{M}$ is a Manifold of Kenmotsu with the Killing tensor field φ. Not to mention that we've

$$\begin{cases} \overline{\nabla}_{e_1}\phi e_1 - \phi\overline{\nabla}_{e_1}e_1 + \overline{\nabla}_{e_1}\phi e_1 - \phi\overline{\nabla}_{e_1}e_1 = 0, \\ \overline{\nabla}_{e_1}\phi e_2 - \phi\overline{\nabla}_{e_1}e_2 + \overline{\nabla}_{e_2}\phi e_1 - \phi\overline{\nabla}_{e_2}e_1 = 0, \\ \overline{\nabla}_{e_1}\phi e_3 - \phi\overline{\nabla}_{e_1}e_3 + \overline{\nabla}_{e_3}\phi e_1 - \phi\overline{\nabla}_{e_3}e_1 = 0, \\ \overline{\nabla}_{e_2}\phi e_1 - \phi\overline{\nabla}_{e_2}e_1 + \overline{\nabla}_{e_1}\phi e_2 - \phi\overline{\nabla}_{e_1}e_2 = 0, \\ \overline{\nabla}_{e_2}\phi e_2 - \phi\overline{\nabla}_{e_2}e_2 + \overline{\nabla}_{e_2}\phi e_2 - \phi\overline{\nabla}_{e_2}e_2 = 0, \\ \overline{\nabla}_{e_2}\phi e_3 - \phi\overline{\nabla}_{e_2}e_3 + \overline{\nabla}_{e_3}\phi e_2 - \phi\overline{\nabla}_{e_3}e_2 = 0, \\ \overline{\nabla}_{e_3}\phi e_1 - \phi\overline{\nabla}_{e_3}e_1 + \overline{\nabla}_{e_1}\phi e_3 - \phi\overline{\nabla}_{e_1}e_3 = 0, \\ \overline{\nabla}_{e_3}\phi e_2 - \phi\overline{\nabla}_{e_3}e_2 + \overline{\nabla}_{e_2}\phi e_3 - \phi\overline{\nabla}_{e_2}e_3 = 0, \\ \overline{\nabla}_{e_3}\phi e_3 - \phi\overline{\nabla}_{e_3}e_3 + \overline{\nabla}_{e_3}\phi e_3 - \phi\overline{\nabla}_{e_3}e_3 = 0. \end{cases} \qquad (3.1.2)$$

And

$$\begin{cases} \eta(e_1)\phi(e_1) + \eta(e_1)\phi(e_1) = 0, & \eta(e_2)\phi(e_1) + \eta(e_1)\phi(e_2) = 0, \\ \eta(e_3)\phi(e_1) + \eta(e_1)\phi(e_3) = 0, & \eta(e_1)\phi(e_2) + \eta(e_2)\phi(e_1) = 0, \\ \eta(e_2)\phi(e_2) + \eta(e_2)\phi(e_2) = 0, & \eta(e_3)\phi(e_2) + \eta(e_2)\phi(e_3) = 0, \\ \eta(e_1)\phi(e_3) + \eta(e_3)\phi(e_1) = 0, & \eta(e_2)\phi(e_3) + \eta(e_3)\phi(e_2) = 0, \\ \eta(e_3)\phi(e_3) + \eta(e_3)\phi(e_3) = 0. \end{cases} \qquad (3.1.3)$$

In addition to satisfying equations (3.2) and (3.5), equations (3.1.1) and (3.1.2) also fulfil equations (3.2), therefore all is correct.

Using the Killing tensor field on a 5-dimensional Kenmotsu manifold yields an example that is similar to.

Example 3.1.2. Suppose $\overline{M} = \{(x_1, x_2, x_3, x_4, v) \in \mathbb{R}^5, v \neq 0\}$ to represent the 5-dimensional manifold and the conventional coordinates in R^5 as (x_1,x_2,x_3,x_4,v). Define metric g on $\overline{M}$ is given by

$$g = \eta \otimes \eta + e^{2v} \sum_{i=1}^{4} dx_i \otimes dx_i.$$

We may now select

$$e_1 = e^{-v}\frac{\partial}{\partial x_1}, \quad e_2 = e^{-v}\frac{\partial}{\partial x_2}, \quad e_3 = e^{-v}\frac{\partial}{\partial x_3}, \quad e_4 = e^{-v}\frac{\partial}{\partial x_4}, \quad e_5 = \frac{\partial}{\partial v} = \xi.$$

Each of the aforementioned vector fields is linearly independent with respect to $\overline{M}$ such that $g(e_i, e_j) = 0$. For $i \neq j$ and $g(e_i, e_j) = 1$ for i = j, where i, j = 1,2,3,4,5. For the selected vector field U on the manifold, the 1-form is defined as (U) = g(U,e5) $\overline{M}$. Suppose φ stands for the (1,1) tensor field and is defined by

$$\phi(e_1) = 0, \quad \phi(e_2) = 0, \quad \phi(e_3) = 0, \quad \phi(e_4) = 0, \quad \phi(e_5) = 0.$$

Now, by utilising the fact that g and φ are linear, we can say that

$$\phi^2\mathcal{U} = -\mathcal{U} + \eta(\mathcal{U})\xi, \quad \eta(e_5) = 1, \quad g(\phi\mathcal{U}, \phi\mathcal{V}) = g(\mathcal{U}, \mathcal{V}) - \eta(\mathcal{U})\eta(\mathcal{V}),$$

given that the manifold's selected vector fields are U and V $\overline{M}$.

The result of an easy calculation is,

$$\overline{\nabla}_{e_1}e_1 = -e_5, \quad \overline{\nabla}_{e_1}e_2 = 0, \quad \overline{\nabla}_{e_1}e_3 = 0, \quad \overline{\nabla}_{e_1}e_4 = 0, \quad \overline{\nabla}_{e_1}e_5 = e_1,$$
$$\overline{\nabla}_{e_2}e_1 = 0, \quad \overline{\nabla}_{e_2}e_2 = -e_5, \quad \overline{\nabla}_{e_2}e_3 = 0, \quad \overline{\nabla}_{e_2}e_4 = 0, \quad \overline{\nabla}_{e_2}e_5 = e_2,$$
$$\overline{\nabla}_{e_3}e_1 = 0, \quad \overline{\nabla}_{e_3}e_2 = 0, \quad \overline{\nabla}_{e_3}e_3 = -e_5, \quad \overline{\nabla}_{e_3}e_4 = 0, \quad \overline{\nabla}_{e_3}e_5 = e_3,$$
$$\overline{\nabla}_{e_4}e_1 = 0, \quad \overline{\nabla}_{e_4}e_2 = 0, \quad \overline{\nabla}_{e_4}e_3 = 0, \quad \overline{\nabla}_{e_4}e_4 = -e_5, \quad \overline{\nabla}_{e_4}e_5 = e_4,$$
$$\overline{\nabla}_{e_5}e_1 = e_1, \quad \overline{\nabla}_{e_5}e_2 = e_2, \quad \overline{\nabla}_{e_5}e_3 = e_3, \quad \overline{\nabla}_{e_5}e_4 = e_4, \quad \overline{\nabla}_{e_5}e_5 = 0.$$

Based on the relations mentioned above, it can be shown that the manifold fulfils the equation

$\overline{\nabla}_{\mathcal{U}}\xi = \mathcal{U} - \eta(\mathcal{U})\xi \; for \; \xi = e_5.$ It follows that is a Killing tensor field, based on the comparable pattern of Example 3.1.1. Therefore, $\overline{M}$ exist as a Killing tensor field–containing 5-dimensional Kenmotsu manifold. The satisfaction of equations (3.2) and (3.5) is also evident, following an analogy with Example 3.1.1. (Inoguchi, J. 2004)

CONCLUSION

The impact of curves and metal structures on manifold properties show their inherent and extrinsic qualities. Geodesics and curvature routes help us grasp the manifold's topology and geometry, including its singularities and curvature behaviour. Metal structures, commonly modelled using differential forms and tensor fields, provide stiffness and elasticity to the manifold, enriching this research. These structures affect manifold stability, deformation, and external force response. This interaction between curves and metal structures affects theoretical mathematics and material science and engineering. It shows how to alter and regulate complicated geometrical characteristics, improving material and structure design for desired mechanical and physical attributes. This extensive study highlights the manifold's features' dependence on curves and metal structures, opening new scientific and technological opportunities.

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