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An Analysis on Fuzzy E -Open sets and Fuzzy E - Continuity in Topological Spaces

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Abstract: Fuzzy set theory extends traditional topological concepts through Fuzzy E-Open Sets and Fuzzy E-Continuity. These sets, in conjunction with fuzzy e-open sets, are generalised variations of conventional topological sets such as β-, a-, and e-open sets. Fuzzy and intuitionistic fuzzy e-open sets exhibit topological patterns, however their interrelations and behaviours may not consistently be reciprocal.

Keywords: Fuzzy E -Open Sets , Fuzzy E -Continuity , Topological Spaces

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INTRODUCTION

Fuzzy set theory allows for membership degrees to range from 0 (non-member) to 1 (member), and it was originally introduced a few decades ago. This methodology is advantageous for simulating ambiguity and uncertainty across several fields, including computer science, engineering, and mathematics.

An open set in classical topology is a subset of a topology, which is a collection of sets that satisfies particular axioms. We extend the notion of open sets to fuzzy sets by using fuzzy membership values.

Fuzzy E-Open Set: A set A in a fuzzy topological space is termed a fuzzy E-open set if for every element x in A, , there exists a degree of membership $\mu_A(x)$ such that $\mu_A(x)$ is larger than or equal to a specified threshold ε (where ε is a real number between 0 and 1). A set is considered fuzzy E-open if, for every point within the set, its degree of membership exceeds a specified fuzzy threshold.

A fuzzy set A is E-open if for each $x \in A$, , the following condition holds:

$$\mu_A(x) \geq \varepsilon$$

where $\mu_A(x)$ is the membership function of A, and ε is a threshold in the interval [0,1].

This definition generalizes the idea of open sets to account for partial membership, where elements in a set might only partially belong to it.

Fuzzy E-Continuity

Continuity in classical topology is defined in terms of the preimage of open sets under a function. A function $f : X \to Y$ is continuous if the preimage of every open set in Y is open in X. . Fuzzy E-continuity is a similar concept, but it extends this idea to fuzzy topological spaces.

Fuzzy E-Continuity: A function $f : X \to Y$Y between fuzzy topological spaces $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$is defined as fuzzy E-continuous if, for every fuzzy E-open set $V \subseteq Y$ in the codomain, the preimage $f^{-1}(V)$ is a fuzzy E-open set in the domain.

A function f is considered fuzzy E-continuous if, for any fuzzy E-open set $V \subseteq Y$Y, we have: :

$$f^{-1}(V) \text{ is a fuzzy E-open set in } X.$$

This indicates that the preimage of fuzzy open sets maintains a fuzzy degree of openness beyond the threshold..

Fuzzy E-Openness and E-Continuity

1. **Generalization**: They are beneficial in contexts where exact categorisation or membership is unattainable, allowing items to possess different degrees of affiliation to a set or a continuous mapping..
2. **Thresholding**: The primary distinction from classical topology is the incorporation of the threshold ε, which enables a fuzzy definition of "closeness" or "openness," as opposed to the rigid classification of sets as either open or closed..
3. **Applications**: These notions are applicable in contexts including ::

Artificial Intelligence and Machine Learning: Modelling uncertainty or ambiguity in decision-making processes.

- Fuzzy Control Systems: Systems that function amidst ambiguity and necessitate adaptable decision thresholds.
- Image Processing: In instances when pixel values do not distinctly belong to a single category (e.g., object vs backdrop), fuzzy sets can effectively express partial memberships.

OBJECTIVES OF THE STUDY

1 To study on Fuzzy topological space

2. To study on Fuzzy e-continuity and separation axioms for open sets

METHODOLOGY

We examined the methodology for fuzzy E-continuity in topological spaces and fuzzy E-open sets. The analysis of fuzzy E-continuity was also performed using severance axioms.

DATA ANALYSIS

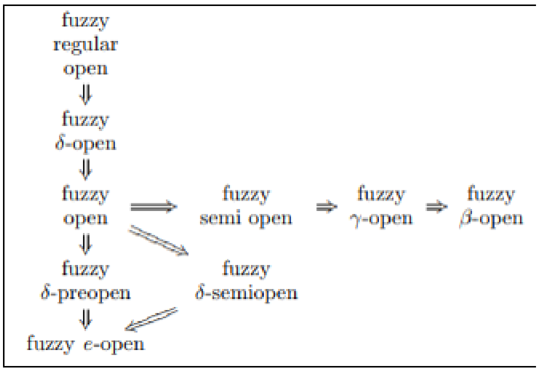
Definition: A fuzzy e-open, or fe open, is a fuzzy subset μ of a space X if

$$\mu \leq cl(\text{int}_\delta \mu) \vee \text{int}(cl_\delta \mu),$$

Fuzzy e-closure (abbreviated as fe-closure) if

$$\mu \geq cl(\text{int}_\delta \mu) \wedge \text{int}(cl_\delta \mu).$$

From the definitions, we derive the subsequent diagram.



As this example demonstrates, none of these consequences are reversible..

Consider the set X = {a, b, c} and the fuzzy sets ϕ1, υ2, υ3 that are defined as subsets of X.

$$\upsilon_1(a) = 0.2,\ \ \upsilon_2(a) = 0.1,\ \ \upsilon_3(a) = 0.2$$
$$\upsilon_1(b) = 0.3,\ \ \upsilon_2(b) = 0.1,\ \ \upsilon_3(b) = 0.4$$
$$\upsilon_1(c) = 0.4,\ \ \upsilon_2(c) = 0.4,\ \ \upsilon_3(c) = 0.4$$

Using τ = {0, u1, u2, u3, u4, 1} and λ = 0.7, 0.6, 0.4, we obtain a fuzzy set λ. Thus, λ can be fuzzy γ-open, semi-open, or γ-open instead of e-open..

The fifth lemma. Consider μ as a fuzzy subset of X.

$$\text{(i)}\ \ pcl_\delta(\mu) = \mu \vee cl(\text{int}_\delta(\mu))\ \ and\ \text{pint}_\delta(\mu) = \mu \wedge \text{int}(cl_\delta(\mu))$$
$$\text{(ii)}\ \ scl_\delta(\mu) = \mu \vee \text{int}(cl_\delta(\mu))\ \ and\ \text{sint}_\delta(\mu) = \mu \wedge cl(\text{int}_\delta(\mu))$$

Proof of Theorem 6:: If μ = pintδ(μ) ∨ sintδ(μ), then μ is a fuzzy e-open subset of space X.

Proof. Let μ be fuzzy e-open. Then $\mu \leq cl(\text{int}_\delta\mu) \vee \text{int}(cl_\delta\mu)$. By lemma 3.5, we have $\text{pint}_\delta(\mu) \vee \text{sint}_\delta(\mu) = (\mu \wedge \text{int}(cl_\delta(\mu)) \vee (\mu \wedge cl(\text{int}_\delta(\mu)) = \mu \wedge (\text{int}(cl_\delta(\mu)) \vee cl(\text{int}_\delta(\mu)))$ $= \mu$.
Conversely, if $\mu = \text{pint}_\delta(\mu) \vee \text{sint}_\delta(\mu)$ then, by lemma 3.5, $\mu = \text{pint}_\delta(\mu) \vee \text{sint}_\delta(\mu) = \mu \wedge \text{int}(cl_\delta(\mu)) \vee \mu \wedge cl(int_\delta(\mu)) = \mu \wedge (\text{int}(cl_\delta(\mu)) \vee cl(\text{int}_\delta(\mu))) \leq \text{int}(cl_\delta(\mu)) \vee cl(\text{int}_\delta(\mu))$ and hence μ is fuzzy e-open. □

Any intersection of fuzzy e-closed sets is also a fuzzy e-closed set and any union is a fuzzy e-open set.

Given that τ = {0, υ1, υ2, 1}, δ-preopen is not it, though.

Consider the set X = {a, b, c}and the fuzzy sets ν1, ν2, ν3, and ν4 that are defined as subsets of X.

$$\nu_1(a) = 0.3,\ \nu_2(a) = 0.4,\ \nu_3(a) = 0.4,\ \nu_4(a) = 0.3$$
$$\nu_1(b) = 0.5,\ \nu_2(b) = 0.2,\ \nu_3(b) = 0.5,\ \nu_4(b) = 0.5$$
$$\nu_1(c) = 0.5,\ \nu_2(c) = 0.6,\ \nu_3(c) = 0.6,\ \nu_4(c) = 0.4$$

Assume τ = {0, ν1, ν2, ν3, ν1 ∧2, 1}. The fuzzy set ν4 is e-open but not δ-semi open, γ-open, or semi-open.

In Example 4, X = {a, b, c} and four fuzzy sets (u1, u2, u3, u4) are defined as

$$u_1(a) = 0.3, u_2(a) = 0.6, u_3(a) = 0.6, u_4(a) = 0.3$$
$$u_1(b) = 0.4, u_2(b) = 0.5, u_3(b) = 0.5, u_4(b) = 0.4$$
$$u_1(c) = 0.5, u_2(c) = 0.5, u_3(c) = 0.4, u_4(c) = 0.4$$

Proof. (i) Let λ_α be a collection of fuzzy e-open sets. Then for each α, $\lambda_\alpha \leq (cl(\text{int}_\delta(\lambda_\alpha))) \vee (\text{int}(cl_\delta(\lambda_\alpha))) \leq (cl(\text{int}_\delta(\vee\lambda_\alpha))) \vee (\text{int}(cl_\delta(\vee\lambda_\alpha)))$. Thus $\vee\lambda_\alpha$ is a fuzzy e-open set.
(ii) Since $\mu_\alpha = 1 - \lambda_\alpha$ is fuzzy closed set, from (i) we have $\mu_\alpha = 1 - \lambda_\alpha \geq 1 - [(cl(\text{int}_\delta(\vee\lambda_\alpha))) \vee (\text{int}(cl_\delta(\vee\lambda_\alpha)))]$. From this we have $\mu_\alpha \geq [1 - (cl(\text{int}_\delta(\vee\lambda_\alpha)))] \wedge [1-(\text{int}(cl_\delta(\vee\lambda_\alpha)))]$. This implies $\mu_\alpha \geq [(int(\text{cl}_\delta(1-(\vee\lambda_\alpha))))] \wedge [(\text{cl}(int_\delta(1-(\vee\lambda_\alpha))))]$.

As $1-(\vee\lambda_\alpha) = \wedge(1-\lambda_\alpha)$ we get $\mu_\alpha \geq [(int(\text{cl}_\delta(\wedge(\mu_\alpha))))] \wedge [(\text{cl}(int_\delta(\wedge(\mu_\alpha))))]$.

Distinct axioms of fuzzy e-continuity and separation

(vi) For any fuzzy set u < X, the constraint function f(clintδ(u) ∧ int clδ(u)) is less than or equal to cl(f(u)

Proof. $(i) \Rightarrow (ii)$: Let the singleton set x_p in X and every fuzzy open set v in Y such that $f(x_p) \leq v$. Since f is fuzzy e-continuous. Then $x_p \in f^{-1}(f(x_p)) \leq f^{-1}(v)$. Let $u = f^{-1}(v)$ which is a fuzzy e-open set in X. So, we have $x_p \leq u$. Now $f(u) = f(f^{-1}(v)) \leq v$.

$(ii) \Rightarrow (iii)$: Let λ be any fuzzy open set in Y. Let x_p be any fuzzy point in X such that $f(x_p) \leq \lambda$. Then $x_p \in f^{-1}(\lambda)$. By(ii), there exists a fuzzy e-open set $u \leq X$ such that $x_p \leq u$ and $f(u) \leq \lambda$. Therefore, $x_p \in u \leq f^{-1}(f(u)) \leq f^{-1}(\lambda) \leq \text{int}(\text{cl}_\delta f^{-1}(\lambda)) \vee cl(int_\delta f^{-1}(\lambda))$.

$(iii) \Rightarrow (iv)$: Let λ be any fuzzy closed set in Y. Then $1-\lambda$ be a fuzzy open set in Y. By (iii), $f^{-1}(1-\lambda) \leq \text{int}(cl_\delta f^{-1}(1-\lambda)) \vee cl(\text{int}_\delta f^{-1}(1-\lambda))$. This implies $1 - f^{-1}(\lambda) \leq \text{int}(cl_\delta(1-f^{-1}(\lambda))) \vee cl(\text{int}_\delta(1-f^{-1}(\lambda))) \leq \text{int}(1-\text{int}_\delta f^{-1}(\lambda)) \vee cl(1-cl_\delta f^{-1}(\lambda)) = 1 - cl(\text{int}_\delta f^{-1}(\lambda)) \vee 1 - \text{int}(cl_\delta f^{-1}(\lambda))$ and hence $1 - f^{-1}(\lambda) =$

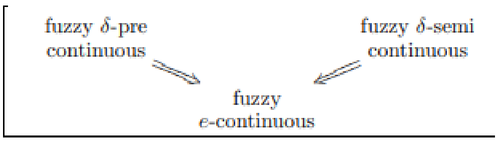
$1-(cl(\text{int}_\delta f^{-1}(\lambda)) \wedge \text{int}(cl_\delta f^{-1}(\lambda)))$. Hence $f^{-1}(\lambda) \geq cl(\text{int}_\delta f^{-1}(\lambda)) \wedge \text{int}(cl_\delta f^{-1}(\lambda))$ and this implies $f^{-1}(\lambda)$ is fuzzy e-closed in X.

$(iv) \Rightarrow (v)$: Let $\nu \leq Y$. Then $f^{-1}(cl(\nu))$ is fuzzy e-closed in X. (i.e)$\text{int}(cl_\delta f^{-1}(\nu)) \wedge cl(\text{int}_\delta f^{-1}(\nu)) \leq \text{int}(cl_\delta f^{-1}(cl(\nu))) \wedge cl(\text{int}_\delta f^{-1}(cl(\nu))) \leq f^{-1}(cl(\nu))$.

$(v) \Rightarrow (vi)$: Let $u \leq X$. Put $\nu = f(u)$ in (v). Then, $\text{int}(cl_\delta f^{-1}(f(u))) \wedge cl(\text{int}_\delta f^{-1}(f(u))) \leq f^{-1}(cl(f(u)))$. This implies that $\text{int}(cl_\delta(u)) \wedge cl(\text{int}_\delta(u)) \leq f^{-1}(cl(f(u)))$ $f(\text{int}(cl_\delta(u)) \wedge cl(\text{int}_\delta(u))) \leq cl(f(u))$

$(vi) \Rightarrow (i)$: Let $v \leq Y$ be fuzzy open set. Put $u = I_Y - v$ and $u = f^{-1}(v)$ then $f(\text{int}(cl_\delta(f^{-1}(v))) \wedge cl(\text{int}_\delta(f^{-1}(v)))) \leq cl(f(f^{-1}(v))) \leq cl(v) = v$. That is, $f^{-1}(v)$ is fuzzy e-closed in X, so f is fuzzy e-continuous. □

We acquire the subsequent diagram:



The following example demonstrates that these implications cannot be reversed. In Example 4, we have X = {a, b, c} and four fuzzy sets ϕ1, υ2, υ3 and υ4 that are defined as

$$\upsilon_1(a) = 0.4,\ \upsilon_2(a) = 0.6,\ \upsilon_3(a) = 0.6,\ \upsilon_4(a) = 0.4$$
$$\upsilon_1(b) = 0.6,\ \upsilon_2(b) = 0.4,\ \upsilon_3(b) = 0.4,\ \upsilon_4(b) = 0.5$$
$$\upsilon_1(c) = 0.5,\ \upsilon_2(c) = 0.4,\ \upsilon_3(c) = 0.5,\ \upsilon_4(c) = 0.5$$

The mappings f: (X, τ1) → (X, τ2) and g: (X, τ1) → (X, τ3) are specified as f(a) = a, f(b) = b, and f(c) = c. Define τ1 = {0, υ1, υ2, υ1 ∨ υ2, υ1 ∧ υ2, 1}, τ2 = {0, υ3, 1}, and τ3 = {0, υ4, 1}. Both f and g may be classified as fuzzy e-continuous functions; however, f fails to satisfy the conditions for fuzzy δ-pre-continuity, whereas g fulfils the requirements for fuzzy δ-semi-continuity.

Theorem 5 defines fuzzy topological spaces X, Y, and Z.

(i) *If* $f : X \to Y$ *fuzzy e-continuous and* $g : Y \to Z$ *is fuzzy continuous. Then* $g \circ f : X \to Z$ *is fuzzy e-continuous.*

(ii) *If* $f : X \to Y$ *fuzzy e-irresolute and* $g : Y \to Z$ *is fuzzy e-continuous. Then* $g \circ f : X \to Z$ *is fuzzy e-continuous.*

CONCLUSION

An open set in classical topology is a subset of a topology, which is a collection of sets that satisfies particular axioms. Fuzzy sets by using fuzzy membership values. Fuzzy e-continuous mappings are presented and studied, focussing on separation axiom characteristics..

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