Journal of Advances in Science and Technology
Vol. 22, Issue No. 2, April-2025, ISSN 2230-9659

DOI: https://doi.org/10.29070/q7hx7702

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Numerical modelling and mathematical modelling of two-phase blood flow

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Abstract: The study of two-phase blood flow through numerical and mathematical modeling has emerged as a crucial approach for understanding the complex behavior of blood, a non-Newtonian fluid comprising plasma and various cellular components, primarily red blood cells. This research aims to develop comprehensive models that capture the dynamics between these two distinct phases—fluid (plasma) and particulate (cells)—within the circulatory system. Mathematical modeling provides a framework for formulating the governing equations based on principles of fluid dynamics, mass conservation, and momentum transfer. These equations account for factors such as viscosity, density variations, interfacial interactions, and shear-thinning behavior. Numerical methods, including finite difference, finite element, and lattice Boltzmann techniques, are employed to simulate blood flow in various geometries representing arterial and microvascular networks. The models aid in predicting flow characteristics such as velocity profiles, pressure gradients, and hematocrit distribution under both physiological and pathological conditions. This dual-phase modeling enables the investigation of complex phenomena like cell aggregation, plasma skimming, and flow separation in stenosed or bifurcated vessels. The insights gained are instrumental in enhancing the understanding of cardiovascular health, optimizing biomedical device design, and improving clinical diagnostics. The study thus bridges theoretical fluid mechanics with practical applications in hemodynamics and medical research.

Keywords: Numerical, Mathematical, Modelling, Two-Phase, Blood Flow

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INTRODUCTION

Human circulatory blood flow is an incredibly complicated phenomenon governed by the finely balanced interplay of biological components and fluid dynamics. All of the body's tissues and organs receive oxygen, nutrition, hormones, and waste products through the circulatory system's blood. Blood is a non-Newtonian, heterogeneous, multiphase fluid that primarily consists of red blood cells (RBCs), white blood cells (WBCs), and platelets, which are liquid components of plasma. The majority of these cells are red blood cells (RBCs), which are crucial for rheological studies of blood, particularly in relation to the effects of varying flows in venous, arterial, and capillary systems (Giddens, D. P. 1983). Due to the presence of many phases—the most crucial of which are cellular components and plasma—accurately simulating blood flow requires a two-step procedure. Conventional single-phase models simplify the mathematics, but they fail to take into consideration the complexities of microcirculation, pathological conditions, cell deformability, and the impact of phase separation (Stone, H. A. 2006). It is now impossible to understand or predict the behaviour of blood in many physiological and pathological situations without using mathematical models and numerical simulations of two-phase blood flow. This field aims to improve and refine models of blood flow dynamics by integrating computational methods with biology, applied mathematics, and fluid dynamics (Doyle, P. S. 2009).

It is common practice to describe the plasma as a continuous Newtonian fluid, with the suspended cellular components, particularly red blood cells (RBCs), represented as either a scattered particulate phase or a deformable continuum with distinct viscoelastic characteristics. The main objective is to study the relationship between the two stages, how they affect one another's movement, and how they react to factors like pressure gradients, shear stress, and restrictions imposed by the vessel walls. Coupled partial differential equations (PDEs) describing the stress-strain behaviour of the individual phases are commonly supplemented by PDEs derived from the conservation of momentum and mass for each phase in these models (Zhao, X. 2014). Incorporating elements such as cell aggregation, deformability, axial movement of RBCs, Fahraeus and Fahraeus-Lindqvist effects, and plasma skimming into models is possible at varying levels of complexity. In capillaries and arterioles, where the scale is similar to that of RBCs, the mathematical description becomes more difficult and techniques based on micro-structural or particulate-based modelling are required. On the other hand, the bulk behaviour of blood flow is described using continuum models like two-fluid or mixture theory in bigger arteries and veins. The very coupled and nonlinear equations that emerge from two-phase models cannot be solved without numerical simulation (Schroter, R. C. 1971). The application of sophisticated numerical techniques such as the Finite Element Method (FEM), Finite Volume Method (FVM), Lattice Boltzmann Method (LBM), and Particle-In-Cell (PIC) methods becomes even more imperative when dealing with pulsatile blood flow, complex vascular geometry, and boundary conditions that vary both spatially and temporally. Many factors, including velocity profiles, shear rates, pressure distributions, and phase concentrations, may be visualised and quantitatively analysed using these models. Researchers have utilised computational fluid dynamics (CFD) platforms like ANSYS Fluent, OpenFOAM, and COMSOL Multiphysics to model two-phase blood flow (Kiesewetter, H. 1999).

METHODOLOGY

Mathematical modelling

Following the phases of the mathematical model described below, this chapter presents the approach for the proposed study:

(I) Description of Bio-physical Problem

Malaria as a biophysical issue is the subject of this particular discussion. So far, mosquitoes are the vectors of the malaria virus. During malaria disease, the blood flow in blood vessels was evaluated using a two-phase blood flow model. Malaria lowers haemoglobin levels and makes red blood cells less elastic. Maintaining blood circulation via microcirculation relies on the aggregation and deformability of erythrocytes.

(II) Real Model

(a) Frame of Reference

We need to choose a frame of reference in order to mathematically represent the condition of moving

blood. Considering the complexity and ubiquity of the blood flow problem, we adopt a generalised three-dimensional orthogonal curvilinear coordinate system, abbreviated as E3, and from here on we refer to it as three-dimensional Euclidean space. If we assume that i = 1, 2, and 3 and that O is the origin, then the coordinate axes would be OXi.

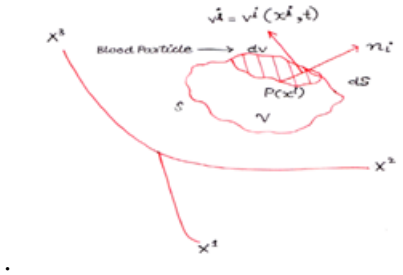

Figure 1: Controlled Volume

(b) Parameterization of the Bio-physical Problem

In three-dimensional space, let xi represent the coordinate of every given point P. The parameters for the research work's formulation have been chosen. Methods that quantify the dispersion of blood velocity in space $v^k = v^k(x^i, t)$ where k, i = 1, 2, 3. Even though blood is an incompressible fluid, these functions nonetheless explain two thermodynamic features of the fluid: its density (ρ) and pressure (p), which are dependent on both space (xi) and time (t). Traditionally, for each given blood vessel, the mass ratio, r, has been thought of as a constant metric. variable blood veins have variable concentrations of red blood cells, hence there is varying degree of fluctuation. Although temperature T is an additional parameter to think about, only isothermal flow has been examined so far when it comes to blood flow in humans.

(III) Formulation

Here, we connect the parameters and use the equations from Chapter 4 to obtain the solution.

Conservation Law

(a) Law of Conservation of Mass

Since there is no obvious source or sink in the human circulatory system and the heart is the only pumping station, it is reasonable to apply the principle of conservation of mass to haemodynamics. Blood venous return is equal to blood influx in all four types of blood vessels: arteries, arterioles, capillaries, and venules.

Figure 2: Mass of blood inflow and outflow

Flow rate of red blood cells in mass units of time

$$-\frac{\partial}{\partial t}\int_{v} X\rho_c DV$$

Plasma inflow mass/second

$$-\frac{\partial}{\partial t}\int_{v} (1-X)\rho_p dV$$

Red blood cell mass expelled in a certain time period

$\int_{s} X\rho_c v^i n_i dS$ or $\int_{v} \left(X\rho_c v^i\right)_i dV$ (by Gauss's Divergance Theorem)

Exhaust mass of plasma in a given time period

$\int_{s} (1-X)\rho_p dV\, v^i n_i dS$ or $\int_{v} \left(1-X\rho_c v^i\right)_i dV$ (by Gauss's Divergance Theorem)

The density of plasma is denoted by □p, the controlled volume limited by surface S is represented by V, and □c represents the density of red blood cells. The ratio of red blood cells to blood volume is denoted by X.

Gauss's Divergence Theorem

Assuming the force field is continuously differentiable within and outside the volume V's enclosed surface S,

$$\oint_{s} F^i.n_i df = \int_{v} F^i, i\, dV$$

(b) Law of Conservation of Momentum

When no external forces are exerted on a fluid, its total energy, or momentum, stays constant. This phenomenon is known as conservation of momentum. Thereby, momentum conservation will be upheld by the hepatic circulation system. The hydrodynamic pressure in both the red and white blood cells remains constant.

Figure 3: Direction of pressure and viscous force

An expression for the momentum rate of change

$$\frac{dp}{dt}$$

The force that causes blood to flow as a result

$$-P + \tau$$

Where:

Pressure $P = p,j\, g^{i,j}$

Viscous force r = r^{ij}

The outcomeant force is computed in accordance with Newton's second law as

$$\rho \frac{dv^i}{dt}$$

The expression $\rho \frac{dv^i}{dt}$ stands for the pace at which a certain particle's velocity in blood is changing.

The force that causes Newtonian blood flow as a result

$$-p,j\, g^{ij} + \eta_m e^{ij}$$

The force that causes blood to flow in a non-Newtonian manner

$$-p,j\, g^{ij} + \eta_m \left(e^{ij}\right)^n$$

Force that results from non-Newtonian Herschel-Bulkley

blood flow

$$-p,j\, g^{ij} + \eta_m \left(e^{ij}\right)^n + \tau_0$$

Where: Strain rate $e^{ij} = g^{jk} v_k^i + g^{ik} v_k^f$

Two phase blood flow

(I) The hypothesis on the two-phase blood volume

A number of fully formed biological components are concentrated in human blood, according to Bessonov et al. While white blood cells and platelets make up a negligible fraction of the overall volume, human blood cells account for almost 99% of all red blood cells. The two distinct parts of blood, plasma and red blood cells, make a homogeneous mixture. Packets of semi-permeable red blood cells are found in a liquid known as plasma. Blood behaves Newtonianly at high shear rates but non-Newtonianly and experiences yield stress under low shear rates. The following state of blood flow behaviour forms the basis of our study.

- Plasma and red blood cells are the two main components of blood.
- The flow of blood is not turbulent, but rather streamline or steady.
- There is no outflow or inflow in the liver's circulatory system.
- At the vessel's axis, the blood flow velocity is zero, while it's maximal at other points.

(II) Mass Ratio

According to Upadhyay V., the presence of blood cells influences blood flow. The effect that has been noted is clearly related to the volume that blood cells occupy.

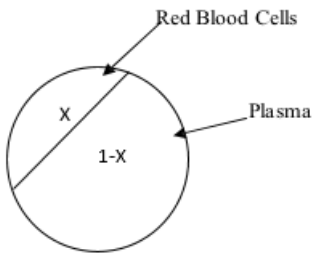

Figure 4: Unit volume of blood

H is the haematocrit, the percentage of blood volume that blood cells occupy; 1-X is the volume fraction that plasma occupies; and X is the ratio of H to 100; hence, X is the volume fraction that blood cells occupy in a given unit volume. We say that the mass ratio of blood cells to plasma is "r,"

$$r = \frac{X\rho_c}{(1-X)\rho_p}$$

Both the blood cell density and the plasma density are represented by the quantities □c and □p, respectively.

Let □m represent the homogeneous density of blood.

$$\rho_m = X\rho_c + (1 - X)\rho_p$$

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{q}{\rho_p}$$

Where:

RESULTS

Two Phase Blood Flow Equation

Equation of Continuity

A basic assumption in fluid dynamics is the equation of continuity, which states that the mass flow rate of an incompressible fluid stays constant along a streamline. Chapter 3's section (III) provides the basis for the two-phase blood flow continuity equation.

In this context, V stands for the volume of space, ρc for the density of red blood cells, and ρp for plasma. The proportion of red blood cells and plasma in the total volume of blood is represented by XX and (1 − X)V, respectively. The essential $\int X\rho_c \, dV$ calculates the mass of red blood cells in V and The essential $\int (1 - X)\rho_c \, dV$ ascertains the mass of plasma inside a unit volume V. For i = 1, 2, and 3, the integral can denote the mass of red blood cells and plasma passing through a defined surface element surrounding a given volume independently.

$$\int_s X\rho_c \, v^i n_i dS \; and \; \int_s (1 - X)\rho_p \, v^i n_i dS$$

It follows that the mass of RBCs and plasma ejected from volume V/s may be calculated, respectively.

$$\int_s X\rho_c \, v^i n_i dS \; and \; \int_s (1 - X)\rho_p \, v^i n_i dS$$

Rate of red blood cell inflow $-\frac{\partial}{\partial t}\int_v X\rho_c \, dV$

Plasma inflow rate per time interval $-\frac{\partial}{\partial t}\int_v (1 - X)\rho_p \, dV$

When studying blood flow, one might use the theory of conservation of mass.

Flow of mass equals flow of mass out

The input of RBCs and plasma is equal to the output of the same.

$$-\frac{\partial}{\partial t}\int_v^{\square} X\rho_c\, dV \;\; -\frac{\partial}{\partial t}\int_v^{\square}(1-X)\rho_p\, dV \;\; = \int_s^{\square} X\rho_c\, v^i n_i dS + \int_s^{\square}(1-X)\rho_p\, v^i n_i dS \;\; 1 \qquad 1$$

A surface integral may be converted to a volume integral using Gauss's theorem.

$$\int_s^{\square} X\rho_c\, v^i n_i dS = \int_v^{\square}\left(X\rho_c v^i\right)_{,i} dV \quad 2a$$

$$\int_s^{\square}(1-X)\rho_p\, v^i n_i dS = \int_v^{\square}\left((1-X)\rho_p v^i\right)_{,i} dV \quad 2b$$

By plugging in (2a) and (2b) into (1),

$$-\frac{\partial}{\partial t}\int_V \mathrm{X}\rho_c dV - \frac{\partial}{\partial t}\int_V (1-\mathrm{X})\rho_p dV = \int_V \left(\mathrm{X}\rho_c v^i\right)_{,i} dV + \int_V \left((1-\mathrm{X})\rho_p v^i\right)_{,i} dV$$

$$\frac{\partial}{\partial t}\int_V \mathrm{X}\rho_c dV + \int_V \left(\mathrm{X}\rho_c v^i\right)_{,i} dV + \frac{\partial}{\partial t}\int_V (1-\mathrm{X})\rho_p dV + \int_V \left((1-\mathrm{X})\rho_p v^i\right)_{,i} dV = 0$$

Or

$$\int_V \left[\left(\mathrm{X}\rho_c v^i\right)_{,i} + \frac{\partial \rho}{\partial t}\right] dV + \int_V \left[\left((1-\mathrm{X})\rho_p v^i\right)_{,i} + \frac{\partial (1-\mathrm{X})\rho_p}{\partial t}\right] dV = 0$$

Or

$$\int_V \left[\left(\mathrm{X}\rho_c v^i\right)_{,i} + \frac{\partial \mathrm{X}\rho_c}{\partial t} + \left((1-\mathrm{X})\rho_p v^i\right)_{,i} + \frac{\partial (1-\mathrm{X})\rho_p}{\partial t}\right] dV = 0 \quad 3$$

The choice of the basic volume is completely at random, hence it follows that we must have

Since

$$\rho_m = X\rho_c + (1-X)\rho_p 5 \qquad 5$$

We obtain by plugging relation (5) into equation (4).

$$\frac{\partial \rho_m}{\partial t} + \left(\rho_m v^i\right)_{,i} = 0 \quad 6$$

Because blood cannot be compressed, the density ρm stays the same. Therefore, it is possible to simplify equation (6).

$$v^i_{,i} = 0$$

Or,

$$\frac{\partial v^i}{\partial x^i} + \frac{v^i}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^i} = \frac{1}{\sqrt{g}}\left(\sqrt{g}v^1\right)_{,i} = 07$$

7

The two-phase blood flow equation is this.

Equation of Motion

In order to derive momentum, as described in section (III) of Chapter 3, the rate at which the linear momentum departs from the control volume must equal the external forces acting on the control volume.

Allow V to stand for the regulated volume inside the S-denoted surface that encompasses a certain geographical area. Think of a made-up fluid particle with a volume dV and a surface area $dS_i = n_i dS$ at the location *x*i in time t. Let $p = p(x^i, t)$ stands for the stress, $v^i = v^i(x^i, t)$ stand for the speed and *n*i for the surface's normal vector dS. One way to represent the pressure-induced force on this fluid element is as. Hence, the all-surface force due to the regulated volume may be written as

$$-\int pn_i dS$$

One way to convert the provided formula into a volume integral is by applying Gauss's theorem:

$$-\int pn_i dS = -\int p_{,i} dV 1$$

1

You may express the covariant derivative of pressure with respect to *x*i as the symbol *k*,i.

$$-p_{,i} = -p_{,j} g^{ij}\ 2$$

2

The conjugate metric tensor is represented by ij.

Both red blood cells and plasma have the same equation of motion, which is:

$$X\rho_c \frac{dv^i}{dt} = -Xp_{,j}g^{ij} \quad 3$$

$$(1-X)\rho_p \frac{dv^i}{dt} = -(1-X)p_{,j}g^{ij} \quad 4$$

$$\{X\rho_c + (1-X)\rho_p\}\frac{dv^i}{dt} = -p_{,j}g^{ij} \quad 5$$

$$\rho_m \frac{dv^i}{dt} = -p_{,j}g^{ij} \quad 6$$

Considering the formula for speed as $v^i = v^i(x^i, t)$

$$\frac{dv^i}{dt} = \frac{\partial v^i}{\partial t} + \frac{\partial v^i}{\partial t^1}\frac{dx^1}{dt} + \frac{\partial v^i}{\partial t^2}\frac{dx^2}{dt} + \frac{\partial v^i}{\partial t^3}\frac{dx^3}{dt}$$

$$\frac{dv^i}{dt} = \frac{\partial v^i}{\partial t} + \left(\frac{dx^1}{dt}, \frac{dx^2}{dt}, \frac{dx^3}{dt}\right).\left(\frac{\partial v^1}{\partial x^1}, \frac{\partial v^2}{\partial x^2}, \frac{\partial v^3}{\partial x^3}\right) = \frac{\partial v^i}{\partial t} + v^i\frac{\partial v^i}{\partial x^j}$$

$$\frac{dv^i}{dt} = \frac{\partial v^i}{\partial t} + v^j\left(\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right)v^i$$

$$\frac{dv^i}{dt} = \frac{\partial v^i}{\partial t} + v^j v^i_{,j}$$

In relation to the coordinate xj, the covariant derivative of the contravariant vector ai is denoted by the sign $a$ij.

It may be deduced from the facts given in (6) and (7) that

$$\frac{\partial v^i}{\partial t} + v^j v^i_{,j} = -\frac{1}{\rho_c}p_{,j}g^{ij} \quad 8a$$

$$\frac{\partial v^i}{\partial t} + v^j v^i_{,j} = -\frac{1}{\rho_p}p_{,j}g^{ij} \quad 8b$$

Now

$$\frac{\partial(X\rho_c v^i)}{\partial t} = X\rho_c\frac{\partial v^i}{\partial t} + \frac{\partial X\rho_c}{\partial t}v^i \quad 9a$$

And

$$\frac{\partial((1-X)\rho_p v^i)}{\partial t} = (1-X)\rho_p\frac{\partial v^i}{\partial t} + \frac{\partial(1-X)\rho_p}{\partial t}v^i \quad 9b$$

Red blood cells and plasma are both determined by the equation of continuity.

$$\frac{\partial X\,\rho_c}{\partial t} = -\left(X\rho_c v^i\right)_{,i} \quad 10a$$

$$\frac{\partial (1-X)\,\rho_c}{\partial t} = -\left((1-X)\rho_p v^i\right)_{,i} \quad 10b$$

We get the following by plugging (8a) and (10a) into equation (9a):

$$\frac{\partial\left(X\rho_c v^i\right)}{\partial t} = -X\rho_c v^i v^i_{,j} - Xp_{,j}g^{ij} - v^i\left(X\rho_c v^j\right)_{,j}$$

$$\frac{\partial\left(X\rho_c v^i\right)}{\partial t} = -Xp_{,j}g^{ij} - \left(X\rho_c v^i v^j\right)_{,j}$$

$$\frac{\partial\left(X\rho_c v^i\right)}{\partial t} = -\prod^{ij}_{c,j} \quad 11a$$

In a similar vein, by plugging (8b) and (10b) into (9b) we obtain

$$\frac{\partial\left((1-X)\rho_p v^i\right)}{\partial t} = -\prod^{ij}_{p,j} \quad 11b$$

The tensors $\prod^{ij}_{c,j}$ and $\prod^{ij}_{p,j}$ Red blood cells and plasma both display symmetry.

For the regulated volume V, integrate equations (11a) and (11b) in the following way:

$$\left\{\mathrm{X}\rho_c v^i + (1-\mathrm{X})\rho_p v^i\right\}_{,i} + \frac{\partial}{\partial t}\left\{\mathrm{X}\rho_c + (1-\mathrm{X})\rho_p\right\} = 0 \quad 12a$$

$$\frac{\partial}{\partial t}\int_V \left((1-X)\rho_p v^i\right)dV = -\int_V \Pi^{ij}_{p,j}\,dV \quad 12b$$

Using the divergence theorem of Gauss, one can

$$\frac{\partial}{\partial t}\int_V \left(X\rho_c v^i\right)dV = -\int \Pi^{ij}_c n_j\,dS \quad 13a$$

$$\frac{\partial}{\partial t}\int_V \left(X\rho_c v^i\right)dV = -\int \Pi^{ij}_p n_j\,dS \quad 13b$$

The integral of the *i*th momentum component for plasma and red blood cells in the regulated volume V is represented by equations (13b) and (13a). Π^{ij}_c *and* Π^{ij}_p represent the i ѣ$h$momentum component that is moving via a unit area perpendicular to the xj axis of plasma and red blood cells, respectively. The tensible

Π_c^{ij} *and* Π_p^{ij} are called the rank two momentum flux density tensor and Π_c^{ij} *and* Π_p^{ij} signifies the movement of the i ѣh momentum component across a unit surface area for plasma and red blood cells, respectively. Z❙ stands for the unit surface normal at surface dSi.

Equations (11a) and (11b) in the formulation by Euler state that the tensor Π_c^{ij} *and* Π_p^{ij} represent the total momentum flux density of plasma and red blood cells, including both the reversible change in momentum due to the movement of fluid particles and the force of pressure. So, a component must be included $\tau_c^{ij} = X\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right)$ and $\tau_p^{ij} = (1-X)\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right)$ found in equations (13a) and (13b), which represent the momentum flow density tensor for plasma and red blood cells, respectively.

$$\Pi_c^{ij} = Xpg^{ij} + X\rho_c v^j v^i \text{ and } \Pi_p^{ij} = (1-X)pg^{ij} + (1-X)\rho_c v^j v^i$$

$$\tau_c^{ij} = Xpg^{ij} + X\rho_c v^j v^i - X\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right)$$

$$\tau_p^{ij} = (1-X)pg^{ij} + (1-X)\rho_c v^j v^i - (1-X)\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right)$$

Where η stands for the coefficient of viscosity, $e^{ij}\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right)$ is the strain rate tensor and represents the shear stress tensors of red blood cells and plasma, respectively.

Hence, the following equations describe the flow of plasma and the motion of red blood cells:

$$\frac{\partial\left(X\rho_c v^j\right)}{\partial t} = -\Pi^{ij}_{c,j} + \tau^{ij}_{c,j} \quad 14a$$

$$\frac{\partial\left((1-X)\rho_p v^j\right)}{\partial t} = -\Pi^{ij}_{p,j} + \tau^{ij}_{p,j} \quad 14b$$

$$\frac{\partial\left((1-X)\rho_p v^j\right)}{\partial t} = Xp_{,j}g^{ij} + \left(X\rho_c v^j v^j\right)_{,j} + X\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right)_{,j} \quad 15a$$

$$\tau_p^{ij} = (1-X)pg^{ij} + (1-X)\rho_c v^j v^i - (1-X)\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right) \quad 15b$$

Since blood cannot be compressed, the equation for $\left(\rho_c v^j\right)_{,i} = 0$ *and* $\left(\rho_p v^i\right)_{,i} = 0$ is able to work. Hence, the following form may be used to describe equations (15a) and (15b):

According to Newton, blood flow

$$\tau_p^{ij} = (1-X)pg^{ij} + (1-X)\rho_c v^j v^i - (1-X)\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right) \quad 16a$$

$$\frac{\partial((1-X)\rho_p v^i)}{\partial t} = -(1-X)p_{,j}g^{ij} - \left((1-X)\rho_p v^i v^j\right)_{,j} + (1-X)\eta\left(g^{jk}v^i_{,k} + g^{ik}v^j_{,k}\right)_{,j} \quad 16b$$

Relating to blood flow that is not Newtonian

$$\frac{\partial(X\rho_c+(1-X)\rho_p)v^i}{\partial t} + \left\{\left(X\rho_c + (1-X)\rho_p\right)v^j\right\}v^i_{,j} = -p_{,j}g^{ij} + \eta\left(g^{jk}v^i_{,k}\right)^n_{,j} \quad 17$$

Or

$$\frac{\partial(\rho_m v^i)}{\partial t} + \left(\rho_m v^j\right)v^i_{,j} = -p_{,j}g^{ij} + \eta\left(g^{jk}v^i_{,k}\right)^n_{,j} \quad 18$$

$$\rho_m\frac{\partial v^i}{\partial t} + \rho_m v^j v^i_{,j} = \tau^{ij}_{,j} \quad 19$$

Where: $\tau^{ij}_{,j} = -p_{,j}g^{ij} + \eta_m\left(g^{jk}v^i_{,k}\right),j$

$$\tau^{ij} = -pg^{ij} + \eta_m\left(e^{ij}\right) = -pg^{ij} + \tau^{ij}; \tau^{ij} = \eta_m\left(e^{ij}\right)$$

All symbols have their standard meaning in this context.

The Navier-Stokes equation, which is used to describe the flow of blood, is the one that is presented above (19).

CONCLUSION

The numerical and mathematical modeling of two-phase blood flow provides a comprehensive framework for understanding the complex rheological behavior of blood, which is inherently a suspension of cells in plasma. By representing blood as a two-phase fluid—typically treating plasma as the continuous phase and red blood cells (RBCs) as the dispersed phase—researchers can accurately simulate physiological and pathological flow conditions across different vessel sizes. These models help in capturing critical phenomena such as phase separation, cell migration, aggregation, and the Fahraeus-Lindqvist effect. Numerical simulations based on finite element, finite volume, or lattice Boltzmann methods allow the visualization and quantification of hemodynamic parameters like velocity, pressure, and shear stress distribution. The integration of non-Newtonian properties and deformability of cells further enhances model fidelity. Such simulations have proven vital in predicting cardiovascular disorders, evaluating the impact of medical devices, and optimizing drug delivery mechanisms. However, challenges remain in accounting for complex vessel geometries, pulsatile flow, and patient-specific conditions. Overall, the synergy of mathematical formulations with robust computational methods offers a powerful tool for advancing biomedical research, guiding clinical decisions, and designing therapeutic interventions by providing insights into the intricacies of microcirculatory and macrocirculatory blood flow behavior.

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