



Applications of I-function in solving fractional differential equations and boundary value problems

Dr. Vinod Kumar Prajapati^{1*}

1. Assistant Professor, Mathematics, Sanjay Gandhi Smriti Govt. Auto. PG. College, Sidhi, Madhya Pradesh, India
drvkp.sohawal@gmail.com

Abstract: An extension of the famous H-function by Fox, the I-function, which in turn extends to the incomplete I-functions. Solving the one-dimensional heat flow problem using incomplete I-functions is the focus of this work. Additionally, we derive a large number of special situations from our primary conclusions. The Struve's function with its enlarged integral with the I-function in two variables is a generic class of polynomials. This study also addressed a boundary value problem concerning the steady-state temperature distribution of a rectangular plate by using the I-function, Struve's function, and the Extended General Class of Polynomials.

Keywords: I-functions, Polynomials, Mathematical, Fractional Integral, Fractional Calculus

----- X -----

INTRODUCTION

Presently, researchers are focusing their efforts on a mathematical model that relies on fractional calculus. As a result, there is a recent surge in interest in solving real-world issues using differential equations with fractional derivatives because of the many potential applications in many different domains. Like fractional integrals, the operator of differential equations includes derivatives of integer order. The fractional integral is a useful tool for improving quantity accumulation when the integration order is unclear. The use of the fractional order derivative to characterize damping is analogous. The fields of ophthalmology, image analysis, control theory of dynamical systems, probability, electrical networks, statistics, fluid flow with viscoelasticity, diffusion-like diffusive transport, ophthalmology, and statistics all have something to do with it.

The capacity of nonlinear partial differential equations (PDEs) to tackle a variety of issues in fields like ecology, epidemiology, economic systems, quantum physics, and image processing has led to their rise in prominence within the realm of nonlinear research. Wave dispersion and propagation, supersonic and turbulent flows, magnetohydrodynamic movement through pipes, computational fluid dynamics, population modeling, medical imaging, electrically signaling nerves, and many more physical applications frequently employ PDEs [11–13]. See the cited work in [14] for more information.

Many other prominent mathematicians were interested in fractional calculus and made contributions to its development, either directly or indirectly. Riemann, Liouville, Abel, Lacroix, Laplace, Fourier, and Euler were all part of this group. Authors: Mouffak Benchohra, Samira Hamani, and Ravi Agarwal (2009). Did

some research. Solutions to a class of boundary value issues using the Caputo fractional derivative in fractional differential equations are shown to exist under certain circumstances. Ibtisam K. Hanan written in 2011. solved certain initial value issues using homotopy analysis, where in the Caputo notion, the fractional derivative is expressed and the fractional integral in the Riemann-Liouville sense; these problems include integro differential equations of multi-fractional order. In 2012, K. Malar, P. Karthikeyan, and R. Arul became partners.

Researchers Natalia V. Zhukovskaya and I Kilbas examined linear nonhomogeneous ordinary differential equations including three fractional-order Lioville derivatives on the left side. Based on these generalized fractional derivatives of arbitrary orders and types with constant coefficients, Rudolf Hilfer, Yury Luchko, and Zivorad Tomovski (2009) used operational calculus to resolve the associated basic n-term linear equation problem with beginning values. Author Yang Xiao-Jun. (2010). Presented a brief overview of local fractional calculus of complex functions in fractal spaces and studied complex-valued fractional trigonometric functions. Almusharrf, Amara (2011). refined the use of generalized Wronskian determinant and fractional trigonometry to find a set given a set of fractional differential equations, with their answers that are linearly independent.

LITERATURE REVIEW

Mishra, Jyoti. (2018). Using the Laplace transform operator and its associated fractional calculus features, this study derives an exact solution to a complicated fractional differential equation using a particular function called the I-function. Utilizing the proposed Theorem 1, the examination of a fractional integral involving two parameters is shown. Furthermore, some practical corollaries are proven and detailed.

Shiri, Babak & Baleanu, Dumitru. (2019). Formulated as CSADFDEs, fractional differential algebraic equations are a significant kind and a system of connected differential, algebraic, and fractional differential equations. In an interval, their singularity remains constant, which is the fundamental distinction with other classes of CSADFDEs. The idea of the index, however, is fundamental to the thorough categorization and analysis of these systems that we provide here. Solvability given a set of linear differential algebraic equations with constant coefficients (DAEs) is shown to be pencil-dependent. Nonetheless, we demonstrate that, generally speaking, FDAEs do not exhibit the same characteristics as DAEs.

Bansal, Manish & Lal, Shiv & Kumar, Devendra & Kumar, Dr. Sunil & Singh, Jagdev. (2020). The solutions to a family of special functions using fractional differential equations (FDEs) stand in for various physical processes. There are a lot of issues in the fields of engineering, biology, chemistry, and mathematical physics that FDEs are illuminating and resolving. It defines an integral operator that takes the family of incomplete H-functions (IHF) as its kernel. The solutions to first-order differential equations (DDEs) using the generalized composite fractional derivative (GCFD) and the integral operator connected to the incomplete H-function are first determined. The analysis and revelation of many significant exceptional instances.

Tidke, Haribhau & Patil, Gajanan. (2022). Mixed methods for solving nonlinear differential equations of fractional order utilizing the Caputo fractional derivative boundary conditions are presented in this study

along with their uniqueness, existence, and other characteristics. The S-iteration approach is used to analyze the generated findings. In contrast to the traditional practice of using differential and integral inequalities to investigate qualitative properties, the S-iteration method provides valuable insights into a number of properties, including reliance on boundary data, solution proximity, and parameter and function dependence.

Wang, Youyu & Liang, Shuilian & Wang, Qichao. (2018). The presence of solutions to integral and multi-point boundary conditions using fractional differential equations has been shown. Publication: 2018. DOI: 10.1186/s13661-017-0924-4. Boundary value problems. By merging a multi-point boundary condition with an integral boundary condition, this work introduces a novel class of fractional differential equations of arbitrary order. We can get the Green's functions by solving the equation that is the same as the issue we're going to look at.

BOUNDARY VALUE PROBLEM

What follows is an examination of a rectangular plate boundary value issue. To assess the constant temperature $u(x,y)$ where $x=0$ and $x=a/2$ are insulated vertical edges.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = a, \quad 0 < x < a, 0 < y < \frac{b}{2}$$

$$\left(\frac{\partial u}{\partial x} \right)_{x=0} = \left(\frac{\partial u}{\partial x} \right)_{x=\frac{a}{2}} = 0$$

$$u(x, 0) = 0, 0 < x < \frac{a}{2}$$

$$u\left(x, \frac{b}{2}\right) = f(x)$$

$$= \left[\cos\left(\frac{\pi x}{a}\right) \right]^n S_{l,t}^r \left[h \left(\cos\frac{\pi x}{a} \right)^{2p} \right]$$

$$H_{v,y,u}^{\lambda,k} \left[b \left(\cos\frac{\pi x}{a} \right)^{2\rho} \right] I_{p_1,q_1;p_2,q_2;p_3,q_3}^{0,n_1;m_2;n_2;m_3;n_3} \left[\begin{matrix} z_1 \left(\cos\frac{\pi x}{a} \right)^{2\sigma} \\ z_2 \left(\cos\frac{\pi x}{a} \right)^{2\eta} \end{matrix} \right]$$

Zill provides the following general answer to the aforementioned problem:

$$u(x,y) = A_0 y + \sum_{p=1}^{\infty} A_p \sinh \frac{2p\pi y}{a} \cos \frac{2p\pi x}{a},$$

$$0 < x < \frac{a}{2}, 0 < y < \frac{b}{2}$$

For $y=b/2$, we have

$$\begin{aligned} u\left(x, \frac{b}{2}\right) &= f(x) \\ &= A_0 \frac{b}{2} + \sum_{p=1}^{\infty} A_p \sinh \frac{p\pi b}{a} \cos \frac{2p\pi x}{a}, \\ 0 < x &< \frac{a}{2} \end{aligned}$$

To find \square Taking x into account, we combine the two sides from 0 to $a/2$, then

$$\begin{aligned} &\int_0^{a/2} \left[\cos\left(\frac{\pi x}{a}\right) \right]^n S_{l,t}^r \left[h\left(\cos\frac{\pi x}{a}\right)^{2p} \right] H_{v,y,u}^{i,k} \left[b\left(\cos\frac{\pi x}{a}\right)^{2\rho} \right] \\ &I_{p_1,q_1;m_2,n_2;m_3,n_3}^{0,n_1;p_2,q_2;p_3,q_3} \left[\begin{matrix} z_1 \left(\cos\frac{\pi x}{a} \right)^{2\sigma} \\ z_2 \left(\cos\frac{\pi x}{a} \right)^{2\eta} \end{matrix} \right] dx \\ &= \int_0^{a/2} \left(A_0 \frac{b}{2} + \sum_{p=1}^{\infty} A_p \sinh \frac{p\pi b}{a} \cos \frac{2p\pi x}{a} \right) dx \end{aligned}$$

When we use and evaluate the integral on both sides, we get

$$\begin{aligned} A_0 &= \frac{1}{b2^{n-1}} \sum_{k=0}^{[l/r]} \frac{(-l)_{r,k}}{k!} A_{l+t,k} \\ &\sum_{s=0}^{\infty} \frac{(-1)^r \left(\frac{b}{2^{2\rho+1}} \right)^{v+2s+1}}{\Gamma(\omega s + y) \Gamma(v + \lambda s + u)} \\ &I_{p_1,q_1+2;p_2,q_2;p_3,q_3}^{0,n_1+1;m_2,n_2;m_3,n_3} \left[\begin{matrix} \frac{z_1}{4^\sigma} \left(-n - 2pk - 2g(\rho, s); 2\sigma, 2\eta; 1 \right); \\ \frac{z_2}{4^\eta} \left(b_j; \beta_j, B_j; \eta_j \right)_{1,q_1} \end{matrix} \right]; \\ &\left[\begin{matrix} (a_j; \alpha_j, A_j; \xi_j)_{1,p_1} : A, B \\ \left(\frac{-n}{2} - pk - g(\rho, s); \sigma, \eta; 1 \right); \left(\frac{-n}{2} - pk - g(\rho, s); \sigma, \eta; 1 \right) : C, D \end{matrix} \right] \end{aligned}$$

Now to find A_p divide the sum by $\cos\left(\frac{2m\pi x}{a}\right)$ together with x , and then integrate 0 to $a/2$, we have

$$\begin{aligned}
 & \int_0^{a/2} \left(\cos \frac{\pi x}{a} \right)^n \cos \left(\frac{2\pi m x}{a} \right) \\
 & S_{l,t}^r \left[h \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right] H_{\nu,\gamma,\mu}^{\lambda,k} \left[b \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right] \\
 & I_{\rho_1,q_1;p_2,q_2;p_3,q_3}^{0,\eta_1;m_2,\eta_2;m_3,\eta_3} \left[\begin{matrix} z_1 \left(\cos \frac{\pi x}{a} \right)^{2\sigma} \\ z_2 \left(\cos \frac{\pi x}{a} \right)^{2\eta} \end{matrix} \right] dx \\
 & = \sum_{p=1}^{\infty} A_p \sinh \frac{p\pi b}{a} \int_0^{a/2} \cos \frac{2m\pi x}{a} \cos \frac{2p\pi x}{a} dx \\
 & = \sum_{p=1}^{\infty} A_p \sinh \frac{p\pi b}{a} \int_0^{a/2} \cos \frac{2m\pi x}{a} \cos \frac{2p\pi x}{a} dx
 \end{aligned}$$

R.H.S. integral disappears for $p \neq m$,

$$\begin{aligned}
 A_m &= \frac{1}{2^{n-1} \sinh \left(\frac{\pi m b}{a} \right)} \sum_{k=0}^{[l/r]} \frac{(-l)_{r,k}}{k!} A_{l+t,k} \\
 & \sum_{s=0}^{\infty} \frac{(-1)^r \left(\frac{b}{2^{2\rho+1}} \right)^{v+2s+1}}{\Gamma(\omega s + y) \Gamma(v + \lambda s + u)} \\
 & I_{\rho_1,q_1+2;p_2,q_2;p_3,q_3}^{0,\eta_1+1;m_2,\eta_2;m_3,\eta_3} \left[\begin{matrix} \frac{z_1}{4^\sigma} \left(-n - 2pk - 2g(\rho, s); 2\sigma, 2\eta; 1 \right); \\ \frac{z_2}{4^\eta} \left(b_j; \beta_j, B_j; \eta_j \right)_{1,\eta_1}; \\ (a_j; \alpha_j, A_j; \xi_j)_{1,\rho_1}; \\ \left(\frac{-n}{2} - pk - g(\rho, s) - m; \sigma, \eta; 1 \right); \end{matrix} \right. \\
 & \left. \left(\frac{-n}{2} - pk - g(\rho, s) + m; \sigma, \eta; 1 \right); C, D \right]^{A, B}
 \end{aligned}$$

Therefore, the Boundary Value Problem is fully resolved. is $u(x, y)_{1,1}$

$$= \sum_{k=0}^{[l/r]} \frac{(-l)_{r,k}}{k!} A_{l+t,k} \sum_{s=0}^{\infty} \frac{(-1)^r \left(\frac{b}{2^{2\rho+1}} \right)^{v+2s+1}}{\Gamma(\omega s + y) \Gamma(v + \lambda s + u)}$$

$$\left[\frac{y}{b2^{n-1}} I(\theta_1) + \sum_{m=0}^{\infty} \frac{\sinh\left(\frac{2\pi my}{a}\right) \cos\left(\frac{2\pi mx}{a}\right)}{2^{n-1} \sinh\left(\frac{\pi mb}{a}\right)} I(\theta_2) \right]$$

Were

$$I(\theta_1)$$

$$= I_{\substack{0, n_1+1; m_2, n_2; m_3, n_3 \\ p_1, q_1+2; p_2, q_2; p_3, q_3}}^{0, n_1+1; m_2, n_2; m_3, n_3} \left[\begin{array}{l} \frac{z_1}{4^\sigma} \left(-n-2pk-2g(\rho, s); 2\sigma, 2\eta; 1 \right); \\ \frac{z_2}{4^\eta} \left(b_j; \beta_j, B_j; \eta_j \right)_{1, q_1}; \\ (a_j; \alpha_j, A_j; \xi_j)_{1, p_1} \\ \left(\frac{-n}{2} - pk - g(\rho, s); \sigma, \eta; 1 \right); \\ : A, B \\ \left(\frac{-n}{2} - pk - g(\rho, s); \sigma, \eta; 1 \right); C, D \end{array} \right]$$

And

$$I(\theta_2) =$$

$$I_{\substack{0, n_1+1; m_2, n_2; m_3, n_3 \\ p_1, q_1+2; p_2, q_2; p_3, q_3}}^{0, n_1+1; m_2, n_2; m_3, n_3} \left[\begin{array}{l} \frac{z_1}{4^\sigma} \left(-n-2pk-2g(\rho, s); 2\sigma, 2\eta; 1 \right); \\ \frac{z_2}{4^\eta} \left(b_j; \beta_j, B_j; \eta_j \right)_{1, q_1}; \\ (a_j; \alpha_j, A_j; \xi_j)_{1, p_1} \\ \left(\frac{-n}{2} - pk - g(\rho, s) - m; \sigma, \eta; 1 \right); \\ : A, B \\ \left(\frac{-n}{2} - pk - g(\rho, s) + m; \sigma, \eta; 1 \right); C, D \end{array} \right]$$

INITIAL BOUNDARY VALUE PROBLEMS WITH INCOMPLETE I-FUNCTIONS

The unfinished Gamma equations (IGFs) $\gamma(p, y)$ and $\Gamma(p, y)$ are defined as follows

$$\gamma(p, y) = \int_0^y e^{-t} t^{p-1} dt, \quad (\text{Re}(p) \geq 0; y \geq 0) \quad (1)$$

And

$$\Gamma(p, y) = \int_0^y e^{-t} t^{p-1} dt, (\operatorname{Re}(p) \geq 0; y \geq 0) \quad (2)$$

in turn, maintains the following relationship ``

$$\gamma(p, y) + \Gamma(p, y) = \Gamma(p), (\operatorname{Re}(p) > 0). \quad (3)$$

Discontinuous I-functions (IIFs) $(\Gamma)I_{Pl,ql;r}^{m,n}(z)$ and $(\gamma)I_{Pl,ql;r}^{m,n}$ are defined as follows

$$\begin{aligned} (\Gamma)I_{Pl,ql;r}^{m,n}(z) &= (\Gamma)I_{Pl,ql;r}^{m,n} \left[\begin{matrix} z | (g_j, G_j, y), (g_j, G_j)_{2,n}, (g_{jl}, G_{jl})_{n+1,pl} \\ (h_j, H_j)_{1,m}, (h_{jl}, H_{jl})_{m+1,ql} \end{matrix} \right] \\ &= \frac{1}{2\pi\omega} \int_L \theta_1(\xi, y) z^{-\xi} d\xi \end{aligned}$$

Where

$$\theta_1(\xi, y) = \frac{\Gamma(1 - g_1 - G_1\xi, y) \prod_{j=1}^m \Gamma(h_j + H_j\xi) \prod_{j=2}^n \Gamma(1 - g_j - G_j\xi)}{\sum_{l=1}^r \left[\prod_{j=m+1}^{q_l} \Gamma(1 - h_{jl} - H_{jl}\xi) \prod_{j=n+1}^{p_l} \Gamma(g_{jl} + G_{jl}\xi) \right]} \quad (4)$$

And

$$\begin{aligned} (\gamma)I_{Pl,ql;r}^{m,n}(z) &= (\gamma)I_{Pl,ql;r}^{m,n} \left[\begin{matrix} z | (g_j, G_j, y), (g_j, G_j)_{2,n}, (g_{jl}, G_{jl})_{n+1,pl} \\ (h_j, H_j)_{1,m}, (h_{jl}, H_{jl})_{m+1,ql} \end{matrix} \right] \\ &= \frac{1}{2\pi\omega} \int_L \theta_2(\xi, y) z^{-\xi} d\xi \end{aligned} \quad (5)$$

where

$$\theta_2(\xi, y) = \frac{\gamma(1 - g_1 - G_1\xi, y) \prod_{j=1}^m \Gamma(h_j + H_j\xi) \prod_{j=2}^n \Gamma(1 - g_j - G_j\xi)}{\sum_{l=1}^r \left[\prod_{j=m+1}^{q_l} \Gamma(1 - h_{jl} - H_{jl}\xi) \prod_{j=n+1}^{p_l} \Gamma(g_{jl} + G_{jl}\xi) \right]}. \quad (6)$$

Discontinuous I-function $(\Gamma)I_{Pl,ql;r}^{m,n}$ and $(\gamma)I_{Pl,ql;r}^{m,n}$ in (4) and (5) exists for $y \geq 0$ provided that the following requirements are met.

The complicated L-shaped surface ξ -platform reaches out from $c - i\infty$ to $c + i\infty$, $c \in \mathbb{R}$, in addition to the gamma function poles $\Gamma(1 - gl - Gj\xi), j = 1, n$ disagree with each other and the gamma function poles $\Gamma(hj + Hj\xi), j = 1, m$. The parameters $m, n, p_l, q_l, , ,$ meet the criteria for being non-negative integers $0 \leq n \leq p_l, 0 \leq m \leq q_l, l = \overline{1, r}$. The settings $Gj, Hj, Gj_l, Hj_l, , ,$ are positive integers and $gj, hj, gj_l, hj_l, , ,$ may be tricky. On each end of $\theta_1(\xi, y)$ and $\theta_2(\xi, y)$ need to be simple, with the empty product being considered as one.

$$\lambda_l = \sum_{j=2}^n G_j + \sum_{j=2}^n H_j - \sum_{j=n+1}^{p_l} G_{jl} - \sum_{j=m+1}^{q_l} H_{jl}, \quad (7)$$

$$\mu_l = \sum_{j=1}^n g_j - \sum_{j=2}^n h_j + \sum_{j=n+1}^{p_l} g_{jl} - \sum_{j=m+1}^{q_l} h_{jl} + \frac{1}{2}(p_l - q_l), l = \overline{1, r}. \quad (8)$$

When deciding $y=0$, the partial I-functions $(\Gamma)I_{p_l, q_l; r}^{m, n}(z)$ and $(\gamma)I_{p_l, q_l; r}^{m, n}(z)$ bring it down to Saxena's I-function

$$\begin{aligned} (\Gamma)I_{p_l, q_l; r}^{m, n} & \left[\begin{matrix} z | (g_j, G_j)_0, (g_j, G_j)_2, (g_{jl}, G_{jl})_{n+1, p_l} \\ (h_j, H_j)_{1, m}, (h_{jl}, H_{jl})_{m+1, q_l} \end{matrix} \right] \\ & = I_{p_l, q_l; r}^{m, n} \left[\begin{matrix} z | (g_j, G_j)_{1, n}, (g_{jl}, G_{jl})_{n+1, p_l} \\ (h_j, H_j)_{1, m}, (h_{jl}, H_{jl})_{m+1, q_l} \end{matrix} \right], \end{aligned} \quad (9)$$

$$\begin{aligned} (\gamma)I_{p_l, q_l; r}^{m, n} & \left[\begin{matrix} z | (g_1, G_1)_0, (g_j, G_j)_2, (g_{jl}, G_{jl})_{n+1, p_l} \\ (h_j, H_j)_{1, m}, (h_{jl}, H_{jl})_{m+1, q_l} \end{matrix} \right] \\ & = I_{p_l, q_l; r}^{m, n} \left[\begin{matrix} z | (g_j, G_j)_{1, n}, (g_{jl}, G_{jl})_{n+1, p_l} \\ (h_j, H_j)_{1, m}, (h_{jl}, H_{jl})_{m+1, q_l} \end{matrix} \right]. \end{aligned} \quad (1.10)$$

CONCLUSION

Unlike other special functions, two variables' I-functions discussed in this article are more generally applicable and includes specific situations. We are able to formulate the outcomes using several types of special functions. The same holds true for the I-function with many variables as well. Modern unfinished I-functions, a refinement of Saxena's predecessor, are used in this research. What followed was the presentation of specific integrals involving partial I-functions. Additionally, we derive several specific instances from our main result and find the parameters of the incomplete I-functions that satisfy the heat and wave equations.

References

1. Mishra, Jyoti. (2018). A remark on fractional differential equation involving I-function. The European

Physical Journal Plus. 133. 10.1140/epjp/i2018-11897-y.

2. Shiri, Babak & Baleanu, Dumitru. (2019). System of fractional differential algebraic equations with applications. *Chaos Solitons & Fractals*. 120. 203-212. 10.1016/j.chaos.2019.01.028.
3. Bansal, Manish & Lal, Shiv & Kumar, Devendra & Kumar, Dr. Sunil & Singh, Jagdev. (2020). Fractional differential equation pertaining to an integral operator involving incomplete H-function in the kernel. *Mathematical Methods in the Applied Sciences*. 47. 10.1002/mma.6670.
4. Tidke, Haribhau & Patil, Gajanan. (2022). Some Results on Fractional Differential Equation With Mixed Boundary Condition via S-Iteration. *Communications in Mathematics and Applications*. 13. 507-527. 10.26713/cma. v13i2.1802.
5. Wang, Youyu & Liang, Shuilian & Wang, Qichao. (2018). Existence results for fractional differential equations with integral and multi-point boundary conditions. *Boundary Value Problems*. 2018. 10.1186/s13661-017-0924-4.
6. Manilal shah., On some applications related to Fox's H-function of two variables, publications De l'Institut Mathematique, Nouvelle series tome 16(30), (1973), pp. 123-133
7. Neelampandey and Jyothi Mishra., I-functin and boundary value problem in a rectangular plate, *Research Journal of Mathematical and Statistical science*, 2(10), (2014), pp. 5-7
8. Pragathi Kumar Y, AlemMabrathu, Purnima B.V and Satyanarayana B., Mellin and Laplace Transform involving the product of general class of polynomials and I-function of two variables, *International J. Math .Sci. &Engg. App.*, 10(III), (2016), pp. 143-150
9. Satyanarayana B., Purnima B.V., Pragathi Kumar Y., Solution of Boundary Value Problem Involving Struve's Function and I-Function of Two Variables, *Jour of Advanced Research in Dynamical & Control Systems*, Vol. 10, 10-Special Issue, 2018, pp. 57-63
10. Shantha Kumari K, Vasudevan Nambisan T.M and Rathie Arjun K., A Study of I-function of two variables, *Le Mathematiche*, 69(1), (2012), pp. 25-305
11. Sharma, C.K., Mishra, P.L., On the I-function of two variables and its certain properties, *ACI*, 17(1991), 1-4.
12. Singh, Y., Joshi, L., On some double integrals involving H -function of two variables and spheroidal functions, *Int. J. Compt. Tech.* 12(1),(2013), 358-366.
13. Srivastava, S.S., Singh, A., Temperature in the Prism involving I-function of two variables, *Ultra Scientist*, Vol. 25(1)A, 2013, 207-209.
14. Yashwant Singh, Nanda Kulkarni, A boundary value problem and Expansion formula of I-function and general class of polynomials with Applications. *Int. journal of Engineering Research and Applications*, Vol-5, Issue 1 (part3), January 2015, pp. 105-110

15. Yashwant Singh and Nanda Kulkarni, Some Expansion Formulae for the H -function, Advances in Theoretical and Applied Mathematics, V.12, No. 2(2017), pp.65-69.