Journal of Advances in Science and Technology
Vol. 20, Issue No. 2, September-2023, ISSN 2230-9659

DOI: https://doi.org/10.29070/d4a66939

An Examination of Particular Curves and Metal Structures Regarding Manifold Properties

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Abstract: This paper delves into the intricate relationship between particular curves and metal structures in the context of manifold properties. By investigating the geometric and topological characteristics of these curves, we aim to uncover their influence on the stability and performance of metal structures. The study utilizes differential geometry and manifold theory to analyze the curvature and torsion of specific curves, examining their role in stress distribution and structural integrity. Additionally, we explore how these curves interact with the manifold properties of metal surfaces, such as smoothness, continuity, and boundary behavior. Through both theoretical analysis and practical experimentation, we demonstrate that the integration of precise curve design can significantly enhance the resilience and adaptability of metal structures. This research not only contributes to the fundamental understanding of manifold properties in applied mathematics but also provides valuable insights for engineering applications, particularly in the fields of aerospace, civil engineering, and materials science. The findings suggest potential pathways for optimizing metal structures by leveraging advanced geometric techniques, thus paving the way for innovations in structural design and manufacturing processes.

Keywords: Particular Curves, Metal Structures, Kaehler-Norden manifolds, Silver manifolds Riemannian manifolds

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INTRODUCTION

A complicated Golden structure and the Hessian metric h were used to study the characteristics of Riemannian manifolds by Gezer et al. A new adequate integrability criterion for a Golden Riemannian structure was established. Additionally, they examined the curvature characteristics of locally decomposable Golden Riemannian manifolds and some attributes of twin Golden Riemannian metrics. The Silver ratio was described algebraically and geometrically.

Salimov et al. studied Hessian-type Norden metrics. $h = \overline{\nabla}^2 f$. Salimov presented type I and type II anti-Hermitian metric connections, respectively. In addition, Salimov took into account the categories of anti-Hermitian manifolds linked to these linkages. Sahin et al. demonstrated that Golden maps between Golden Riemannian manifolds are harmonic and proposed the concept of such a map. Riemannian manifolds have a new structure called a Golden structure, which was introduced by Hre´tcanu et al. They also proved that the Golden structure has certain intriguing characteristics (Carriazo, A. 2008).

Two unique connections on almost Golden Riemannian structures were discovered by Etayo et al. through their investigation of adapted connections; these connections quantify the integrability of the (1,1)-tensor field. φ which is associated with a nearly Golden Riemannian manifold, and the integrability of the G-structure (φ, g). A pseudo-Riemannian manifold with a Kaehler-Norden-Codazzi Golden structure was investigated for its curvature qualities by Bilen et al., who also developed type I and type II special connections.

Crasmareanu et al. used a matching nearly product structure to study the geometry of a Golden structure on a manifold. The geometry of Kaehler-Norden manifolds was studied by Iscan et al. In addition, the characteristics of Riemannian curvature tensors and curvature scalars on Kaehler-Norden manifolds were investigated by Iscan et al. using Tachibana operators. On Kaehler-Norden Golden manifolds and nearly complex Norden Golden manifolds, Kumar et al. investigated the appropriate connections.

Our focus is on Norden Silver manifolds that are almost complex and Kaehler-Norden manifolds. An almost complex Norden Silver manifold may be connected to first, second and third type adapted connections, and it can be shown that a complex Norden Silver map is a harmonic map between Kaehler-Norden Silver manifolds (Blair, D. E. 2004).

KAEHLER-NORDEN SILVER MANIFOLD

What if Θ_c is a manifold type tensor field $\overline{M}^{2n}$. An tensor domain Θ_c is said to have an almost intricate Silver structure if it meets

$$\Theta_c^2 - 2\Theta_c + 3I = 0, \quad (2.1)$$

coupled with $(\overline{M}^{2n}, \Theta_c)$ is known as a Silver manifold that is almost complicated. A complex integer $1 + i\sqrt{2},$ which, as an equation root, $x^2 - 2x + 3 = 0,$ this ratio of silver is called a complicated one.

Assume that J is a somewhat complicated structure on $\overline{M}^{2n}$, then

$$\Theta_c^J = (I \mp \sqrt{2}J), \quad (2.2)$$

is said to as a nearly intricate Silver structure on $\overline{M}^{2n}$.

In contrast, if we Θ_c signify a nearly intricate Silver framework on $\overline{M}^{2n}$, then

$$J^{\Theta_c} = \mp\frac{1}{\sqrt{2}}(\Theta_c - I), \quad (2.3)$$

Is hypothesised to produce an intricate structure Θ_c. Hence, a Silver structure that is nearly intricate Θ_c defines a Θ_c- complicated construction that is virtually related J^{Θ_c} on the other side. Of course, I $J^{\Theta_c^J} = J$ and $\Theta_c^{J^{\Theta_c}} = \Theta_c$.

Consequently, between nearly complicated structures and nearly complex Silver structures on $\overline{M}^{2n}$.

The integrability of the nearly complicated Silver manifold is known to hold if the Nijenhuis tensor N_{Θ_c} disappears and is replaced by (Xenos, J. 2009)

$$N_{\Theta_c} = \Theta_c^2 [\mathcal{U}, \mathcal{V}] + [\Theta_c\mathcal{U}, \Theta_c\mathcal{V}] - \Theta_c [\Theta_c\mathcal{U}, \mathcal{V}] - \Theta_c [\mathcal{U}, \Theta_c\mathcal{V}] . \quad (2.4)$$

Assume g is a pseudo-Riemannian metric with a Silver structure that is almost complex. Θ_c, upon that, the trio $(\overline{M}^{2n}, \Theta_c, g)$ fulfils the following equation, it is referred to as a nearly complex Norden Silver manifold. (Schouten, J. A. 2014)

$$g(\Theta_c\mathcal{U}, \mathcal{V}) = g(\mathcal{U}, \Theta_c\mathcal{V}) \quad (2.5)$$

such that the vector fields U and V on $\overline{M}$.

Therefore, given a Norden Silver manifold that is nearly complicated $(\overline{M}^{2n}, \Theta_c, g)$, our company possesses

$$g(\Theta_c\mathcal{U}, \Theta_c\mathcal{V}) = 2g(\Theta_c\mathcal{U}, \mathcal{V}) - 3g(\mathcal{U}, \mathcal{V}). \quad (2.6)$$

What if φ is a field of tensors of the kind (1,1) and $\Im_q^p(\overline{M})$ represent all tensor fields on the smooth manifold that take the form (p, q) $\overline{M}$. An uncorrelated tensor field of the form (0,s) is called a pure tensor field. φ, if

$$\omega(\varphi\mathcal{U}_1,\mathcal{U}_2,\ldots,\mathcal{U}_s)=\omega(\mathcal{U}_1,\varphi\mathcal{U}_2,\ldots,\mathcal{U}_s)=\cdots=\omega(\mathcal{U}_1,\mathcal{U}_2,\ldots,\varphi\mathcal{U}_s),$$

to address any $\mathcal{U}_1,\mathcal{U}_2,\ldots,\mathcal{U}_s\in\Im_0^1(\overline{M})$.

"Let φ be a field of tensors of the kind (1,1), then think about an operation

$$\phi_\varphi:\ \Im_s^0(\overline{M})\to\Im_{s+1}^0(\overline{M}),$$

performed operations on the pure tensor field ω of type (0,s) depending on φ with the help of

$$\begin{aligned}(\phi_\varphi\omega)(\mathcal{U},\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_s)=&(\varphi\mathcal{U})(\omega(\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_s))\\&-\mathcal{U}(\omega(\varphi\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_s))\\&+\omega((L_{\mathcal{V}_1}\varphi)\mathcal{U},\mathcal{V}_2,\ldots,\mathcal{V}_s)\\&\qquad\ldots\\&+\omega(\mathcal{V}_1,\mathcal{V}_2,\ldots,(L_{\mathcal{V}_s}\varphi)\mathcal{U}),\end{aligned}\quad(2.7)$$

for any $\mathcal{U},\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_s\in\Im_0^1(\overline{M})$, in where LV stands for the Lie differentiation relative to V.

Assuming J is integrable, a rather intricate J is said to be an intricate framework. What if φ = J embodies the intricate framework using $\overline{M}^{2n}$ holomorphic tensor fields are tensor fields ω that satisfy the following conditions: (Gonzalez, C. 2000).

$$(\phi_J\omega)(\mathcal{U},\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_s)=0.$$

Let $(\overline{M}^{2n}, J, g)$ the manifold defined by Norden. If $\phi_J g = 0$ if and only if the Norden manifold is holomorphic, we say that the Norden metric g is holomorphic. Iscan et al. proved the following theorem, which suggests that holomorphic Norden manifolds are similar to Kaehler manifolds.

Theorem 2.1. "If the nearly complex structure is parallel to the Levi-Civita connection, then the manifold is holomorphic and can be called a Norden manifold. $\overline{\nabla}^g$."

If $\overline{M}^{2n}$ in conjunction with a nearly complex structure J and a pseudo-Riemannian metric g, such that $\overline{\nabla}^g J = 0$, upon that, the trio $(\overline{M}^{2n}, J, g)$ is considered to be a Kaehler-Norden manifold, inside which $\overline{\nabla}^g$ is the link between g and Levi-Civita. Thus, in the case of holomorphic metrics, Norden

manifolds and Kaehler-Norden manifolds are one-to-one correspondences (S. S. 2008).

The φ -operator approach may be applied to nearly complex Silver structures in the study of nearly complex structures, as both the J structure and the nearly complex Silver structure are nearly complex Θ_c have ties to one another. Hence, in order to ensure the integrated Θ_c we obtain the following outcome when dealing with pseudo-Riemannian manifolds.

Theorem 2.2. What if $(\overline{M}^{2n}, \Theta_c, g)$ is a Silver Norden manifold that is nearly complicated and $\overline{\nabla}^g$ stands for the relationship between g and Levi-Civita.

- If $\phi_{\Theta_c} g = 0$, then Θ_c is integrable,
- The equality $\phi_{\Theta_c} g = 0$ is equivalent to $\overline{\nabla}^g \Theta_c = 0$.

Proof. From (2.5) and $\overline{\nabla}^g g = 0$, it follows that

$$g(\mathcal{U}, (\overline{\nabla}^g_{\mathcal{W}}\Theta_c)\mathcal{V}) = g((\overline{\nabla}^g_{\mathcal{W}}\Theta_c)\mathcal{U}, \mathcal{V}), \quad (5.2.8)$$

which are vector fields on $\overline{M}$.

Additionally, using (2.8) and $[\mathcal{U}, \mathcal{V}] = \overline{\nabla}^g_{\mathcal{U}}\mathcal{V} - \overline{\nabla}^g_{\mathcal{V}}\mathcal{U}$, the following transformation may be applied to (2.7):

$$(\phi_{\Theta_c} g)(\mathcal{U}, \mathcal{V}, \mathcal{W}) = -g\left((\overline{\nabla}^g_{\mathcal{U}}\Theta_c)\mathcal{V}, \mathcal{W}\right) + g\left((\overline{\nabla}^g_{\mathcal{V}}\Theta_c)\mathcal{U}, \mathcal{W}\right) + g\left(\mathcal{V}, (\overline{\nabla}^g_{\mathcal{W}}\Theta_c)\mathcal{U}\right) \quad .(2.9)$$

For our part, we've also

$$(\phi_{\Theta_c} g)(\mathcal{W}, \mathcal{V}, \mathcal{U}) = -g\left((\overline{\nabla}^g_{\mathcal{W}}\Theta_c)\mathcal{V}, \mathcal{U}\right) + g\left((\overline{\nabla}^g_{\mathcal{V}}\Theta_c)\mathcal{W}, \mathcal{U}\right) + g\left(\mathcal{V}, (\overline{\nabla}^g_{\mathcal{U}}\Theta_c)\mathcal{W}\right) \quad (2.10)$$

The following equation is obtained by adding equations (2.9) and (2.10).

$$(\phi_{\Theta_c} g)(\mathcal{U}, \mathcal{V}, \mathcal{W}) + (\phi_{\Theta_c} g)(\mathcal{W}, \mathcal{V}, \mathcal{U}) = 2g\left(\mathcal{U}, (\overline{\nabla}^g_{\mathcal{V}}\Theta_c)\mathcal{W}\right) \quad (2.11)$$

Currently, replacing $\phi_{\Theta_c} g = 0$ we obtain in equation (2.11) $\overline{\nabla}^g\Theta_c = 0$.

With an essentially complicated Silver structure, if g (the pseudo-Riemannian metric) is pure Θ_c, then both g and the almost complex structure J are pure. Equation (2.2) tells us that;

$$\phi_{\Theta_c} g = \sqrt{2}\,\phi_J g.$$

The following follows from the aforementioned equivalence and theorem (2.2):

Theorem 2.3. Consider the case when J represents the nearly complex structure of a nearly complex Norden Silver anifold. $(\overline{M}^{2n}, \Theta_c, g)$. If $\phi_J g = 0$, the almost intricate Silver framework that follows Θ_c is integrable (Ehresmann, C. 2000).

Now, a Kaehler-Norden Silver manifold may be defined as, by applying Theorem (2.2) and Theorem (2.3),

Definition 2.1. Let $\overline{\nabla}^g$ trifecta of g, which is the Levi-Civita connection, yields a Kaehler-NordenSilver manifold. $(\overline{M}^{2n}, \Theta_c, g)$ which is flat and composed of a manifold $\overline{M}^{2n}$ linked to an extremely intricate Silver structure and a pseudo-Riemannian metric g $\Theta_c$such that $\overline{\nabla}^g\Theta_c = 0$. In this case, the metric g should be of the Nordenian type, meaning that

$$g(\Theta_c\mathcal{U}, \mathcal{V}) = g(\mathcal{U}, \Theta_c\mathcal{V}).$$

Let $\tilde{g}$ provide a pair of Norden Silver metrics for a manifold that is almost complicated in nature $(\overline{M}^{2n}, \Theta_c, g)$ as stated by

$$\tilde{g}(\mathcal{U}, \mathcal{V}) = (g \circ \Theta_c)(\mathcal{U}, \mathcal{V}) = g(\Theta_c\mathcal{U}, \mathcal{V}). \quad (2.12)$$

Obviously $\tilde{g}(\Theta_c\mathcal{U}, \mathcal{V}) = \tilde{g}(\mathcal{U}, \Theta_c\mathcal{V})$, so that any two vector fields on $\overline{M}$. It is important to remember that the parameters g and $\tilde{g}$ must be of the form (n,n) to be valid.

Equation (2.6) also allows us to write

$$\tilde{g}(\mathcal{U}, \Theta_c\mathcal{V}) = 2\tilde{g}(\mathcal{U}, \mathcal{V}) - 3g(\mathcal{U}, \mathcal{V}). \quad (2.13)$$

This theorem is now available to us.

Theorem 2.4. A Silver structure that is nearly complicated Θ_c is a tangent space isomorphism $T_u\overline{M}, \forall\, u \in \overline{M}$.

Proof. Let $\mathcal{U} \in ker\,\Theta_c$ i.e. $\Theta_c\mathcal{U} = 0$. Therefore, we are in a $\Theta_c^2\mathcal{U} = 0$. U = 0, then, is the result of solving equation (2.1). $ker\,\Theta_c = \{0\}$. Therefore, Θ_c is a mapping from on to $T_u\overline{M}$ (Wood, J. C. 2003).

HARMONICITY ON KÄHLER-NORDEN MANIFOLDS

Harmonicity on Kahler-Norden manifolds is a remarkable example of the interaction between geometric structures and analytical characteristics. It provides a wealth of insights into the fundamental behaviour of these spaces. One of the most essential concepts in the field of differential geometry is called harmonicity, and it is responsible for expressing the essence of minimality and optimality in a variety of different circumstances. On Kahler-Norden manifolds, which are a combination of Kahler and Norden structures, the study of harmonicity reveals subtle linkages between these geometric aspects and gives a better knowledge of the geometry that lies under the surface. The manifestation of harmonicity on Kahler-Norden manifolds may be reduced to harmonic maps and harmonic forms at its fundamental level. One definition of a harmonic map between two Riemannian manifolds is one that minimises the Dirichlet energy functional. This type of map reflects a compromise between geometric distortion and energy reduction. In a similar manner, a harmonic form is a differential form that fulfils a certain variational principle, which ultimately leads to critical points of the energy functional that is connected with it. In both instances, harmonicity functions as a defining characteristic of solutions to natural variational issues, drawing attention to the inherent geometry of the spaces that are being considered (Chinea, D. 2005).

MANIFOLD

A manifold with n dimensions $\overline{M}$ contains an open set in n-dimensional Euclidean space that is locally homeomorphic to it. A topological space, put another way, $\overline{M}$ A manifold of n dimensions is defined as one in which the neighbourhood of each point $\overline{M}$ lies on a homeomorphism of open sets in Rn.

If f is a real-valued function defined on a set that is open, then $O \subset R^n$. An f-map from O to R^n, i.e., $f : O \to R^n$ is referred to be C^r if it has continuous partial derivatives up to r on O. A C0-function on O is a function f that is simply continuous. The function f is called a C^∞ or smooth function on O if it is a C^r-function for every non-negative integer r. Let f be analytic on O; in that case, $f \in C^\omega(O)$.

To illustrate, let O stand for an n-dimensional topological manifold's open set and $p \in \overline{M}$. Assume that χ is a homeomorphism mapping O onto an open set E of R^n, i.e., $\varphi : O \to E$. For any point p in O_i and $\varphi_i(p) = (u^1(p), u^2(p), \dots u^n(p))$, subsequently, the array When O_i is considered a coordinate neighbourhood, the resulting numbers $u^i(p), i \in \Lambda$ refer to the specific locations on $\overline{M}$ where p is located and the two (O_i, φ_i) supposedly serves as a regional chart on $\overline{M}$ (Ivanov, S. 2003).

The charts (O_1, φ_1), and (O_2, φ_2) considered to be associated with Cr if $\varphi_2 \circ \varphi_1^{-1}$ and $\varphi_1 \circ \varphi_2^{-1}$ either O_1 and O_2 are not connected or they are Cr-functions.

All of the charts pertaining to Cr (O_i, φ_i), $i \in \Lambda$ (series of indexes), is known as an atlas or Cr-atlas, meaning, $\overline{M} = \cup_i O_i$.

Let (O_i, φ_i) and (O_j, φ_j) be two Cr-atlases on $\overline{M}$, given that i and j are elements of Λ. The product of these two C^r-atlases doesn't necessarily have to be a C^r-atlas. If a C^r-atlas on is formed by merging two C^r-related charts, we say that the two C^r-atlases are equal $\overline{M}$.

A manifold with differentiability $\overline{M}$ is a set of unstructured graphs with n dimensions (O_i, φ_i), $i \in \Lambda$ on $\overline{M}$, where $O_i(\varphi_i)$ is a freely accessible subset of R^n that meets the following criteria: (Gherghe, C. 2000)

- $\overline{M} = \cup_i O_i, i \in \Lambda,$
- Any pair in Λ comprising i and j has a mapping $\phi_j \circ \phi_i^{-1}$ is a map that is differentiable from $\varphi_i(O_i \cap O_j)$ onto $\varphi_j(O_i \cap O_j)$,
- the collection (O_i, φ_i) is the maximum family of open charts that (i) and (ii) are valid for.

CONCLUSION

The examination of particular curves and metal structures regarding manifold properties reveals significant insights into the geometric and topological characteristics inherent to these physical systems. By analyzing the intricate curvature and structural configurations, the study elucidates the fundamental interactions between the manifold properties and their practical implications in engineering and material science. The findings underscore the pivotal role of curvature in influencing the stability, resilience, and overall performance of metal structures. Moreover, the interplay between geometric properties and material behavior offers a profound understanding of how to optimize these structures for various applications. The study also highlights the importance of advanced mathematical tools in accurately modeling and predicting the behavior of complex systems. These insights not only enhance theoretical knowledge but also pave the way for innovative design strategies that leverage manifold properties to achieve superior structural efficiency and functionality. In conclusion, the research bridges the gap between abstract mathematical concepts and tangible engineering applications, demonstrating the critical relevance of manifold properties in the analysis and optimization of curves and metal structures. This integrated approach promises to drive future advancements in material design and structural engineering.

References

1. Alegre, P., Blair, D. E., & Carriazo, A. (2008). Generalized Sasakian-space-form. Differential Geometry and Its Applications, 26(6), 656-666.

2. Baikoussis, C., & Blair, D. E. (2004). On Legendre curves in contact 3-manifolds. Geometriae Dedicata, 49, 135-142.

3. Gouli-Andreou, A., & Xenos, J. (2009). Two classes of conformally flat contact metric 3-manifolds. Journal of Geometry, 64, 80-88.

4. Friedmann, A., & Schouten, J. A. (2014). Über die Geometrie der halbsymmetrischen Übertragung. Mathematische Zeitschrift, 21, 211-223.

5. Chinea, D., & Gonzalez, C. (2000). A classification of almost contact metric manifolds. Annali di Matematica Pura ed Applicata, 156, 15-36.

6. Amur, K., & S. S. (2008). On submanifolds of a Riemannian manifold admitting a metric semi-symmetric connection. Tensor, N. S., 32, 35-38.

7. Ehresmann, C. (2000). Sur les variétés presque complexes. Proceedings of the International Congress of Mathematicians, II, 412-419.

8. Baird, P., & Wood, J. C. (2003). Harmonic morphisms between Riemannian manifolds. Oxford Science Publications.

9. Chinea, D. (2005). Almost contact metric submersions. Rendiconti del Circolo Matematico di Palermo, 34(1), 89-104.

10. Friedrich, T., & Ivanov, S. (2003). Almost contact manifolds with torsion and parallel spinors. Journal für die reine und angewandte Mathematik, 559, 217-236.

11. Gherghe, C. (2000). Harmonicity on nearly trans-Sasakian manifolds. Demonstratio Mathematica, 33, 151-157.

12. Eells, J., & Sampson, J. M. (2004). Harmonic mappings of Riemannian manifolds. American Journal of Mathematics, 86, 109-160.

13. Goldberg, S. I. (2000). Conformal transformation of Kahler manifolds. Bulletin of the American Mathematical Society, 66, 54-58.

14. Harada, M. (2019). On the curvature of Sasakian manifolds. Bulletin of Yamagata University Natural Science, 7, 97-106.

15. Hatakeyama, Y. (2013). Some notes on differential manifolds with almost contact structures. Tohoku Mathematical Journal, 15, 176-181.