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DIFFERENTIABILITY OF COMPOSITE FUNCTIONS

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Differentiability of Composite Functions

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Abstract - Composite functions are so common that we usually don't think to think to label them as composite functions. However, they arise any time a change in one quantity produces a change in another which, in turn, produces a change in a third quantity. Does that sound confusing? Don't worry, an example will make things clear.

An Example - For this example, we'll assume that the number of humans living on the coast affects the number of whales in nearby coastal waters. Since whales eat plankton, the number of whales will affect the number of plankton in the waters.

Let's be more specific. Since whales don't like all the noise that people make, they move out of an area when too many people move in. If we denote the number of thousands of people by x and the number of whales by y, a simple model would be that.

Now since the whales are eating the plankton, more whales mean less plankton. If we measure the amount of plankton by z, then a simple model is that.

Now the end result is that the number of people influence the number of whales which influences the number of plankton. To see how the number of plankton depend on the number of people, we can compute that More generally, if we have two functions $y = f(x)$ ***and*** $z = g(y)$***, we call the new function*** $z = g(f(x))$ ***the composite of*** f ***and*** g ***and denote it by*** $g \circ f(x) = g(f(x))$***.***

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INTRODUCTION

Let *f*,*g*,*h* be continuous real functions such that:

$$\forall\ x \in \mathbb{R}: h(x) = f \circ g(x) = f(g(x))$$

Then:

$$h'(x) = f'(g(x))g'(x)$$

where *h′* denotes the derivative of *h*.
Using the *Dx* notation:

$$dx(f(g(x))) = Dg(x)(f(g(x)))Dx(g(x))$$

This is often informally referred to as the **chain rule (for differentiation)**.
Leibniz's notation for derivatives (d*y*d*x*) allows for a particularly elegant statement of this rule:

d*y*d*x*=d*y*d*u*·d*u*d*x*

where:

- d*y*d*x* is the derivative of *y* with respect to *x*
- d*y*d*u* is the derivative of *y* with respect to *u*
- d*u*d*x* is the derivative of *u* with respect to *x*

However, this must **not** be interpreted to mean that derivatives can be treated as fractions. It simply is a convenient notation.

Corollary

dy/d*x*=(d*y*d*u*)(d*x*d*u*)

for d*x*d*u*≠0.

Proof

Let $g(x)=y$, and let:

$g(x+\delta x) = y+\delta y$

$\Longrightarrow \quad \delta y \quad = g(x+\delta x)-g(x)$

Thus:

- $\delta y \to 0$ as $\delta x \to 0$, and
- $\delta y \delta x \to g'(x)(1)$

There are two cases to consider:

Case 1

Suppose $g'(x)\neq 0$ and that δx is small but non-zero.

$$\lim \delta x \to 0 h(x+\delta x)-h(x)\delta = \lim \delta x \to 0 f(g(x+\delta x))-f(g(x))g(x+\delta x)-g(x)g(x+\delta x)-g(x)$$

$x \quad \delta x$

$$= \lim \delta x \to 0 f(y+\delta y)-f(y)\delta y \delta y \delta x$$

$$= f'(y)g'(x)$$

Then $\delta y\neq 0$ from (1) above, and:

hence the result.

Case 2

Now suppose $g'(x)=0$ and that δx is small but non-zero.

Again, there are two possibilities:

Case 2a

If $\delta y=0$, then $h(x+\delta x)-h(x)\delta x=0$.

Hence the result.

Case 2b

If $\delta y\neq 0$, then $h(x+\delta x)-h(x)\delta x=f(y+\delta y)-f(y)\delta y\delta y\delta x$.

As $\delta y\to 0$:

(1): $f(y+\delta y)-f(y)\delta y\to f'(y)$

(2): $\delta y\delta x\to 0$

Thus:

$\lim \delta x\to 0 h(x+\delta x)-h(x)\delta x\to 0=f'(y)g'(x)$

Again, hence the result.

If f and g are two functions defined by y = f(u) and u = g(x) respectively then a function defined by y = f [g(x)] or fog(x) is called a composite function or a function of a function.

The theorem for finding the derivative of a composite function is known as the **CHAIN RULE**.

Theorem :

If f and g are differentiable and are defined by y = f (u) and u = g (x), then the composite function y = f [g (x)] is differentiable and we have $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Corollary :

If y = f (u), u = g (v) and v = h (x) where f, g and h are differentiable functions of u, v and x respectively, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$

IMPORTANCE OF COMPOSITE FUNCTION

Composite functions are much more common than you may realize. For instance, if you want to compute $e^{1.1^2}$ on your hand-held calculator, you will enter 1.1 and then press the button which squares the entry. After that, you will press the button which exponentiates the entry. Each of the buttons you press, to square and to exponentiate, represent a function. By feeding what comes out of the squaring function into the exponentiation function, you are really computing the composite of these two functions. Said in mathematical notation, you are first evaluating $y = f(x) = x^2$ at 1.1 and then putting the result into the function $z = g(y) = e^y$. By composing them, you obtain $z = g(f(x)) = e^{x^2}$ which you evaluate at 1.1.

If we have two functions f(x) and g(x) we can define a composite function h(x)≡f(g(x)). Thus if f(x)=x3 and g(x)=2x−1 we have h(x)=(2x−1)3=8x3−12x2+6x−1.

On the other hand if we define the composite function k(x)≡g(f(x)) we then have k(x)=2(x3)−1. Notice that h(x) and k(x) are different functions: when composing functions, the order matters. (If you think of functions as *procedures*, this makes intuitive sense: putting on your socks and then your shoes has a different result from doing this the other way round\dots).

In compounding functions such as h(x)=f(g(x)) you have to be a bit careful to ensure that the range of g is in the domain of f: that is, that g doesn't throw up any outputs for which f is undefined.

Composite functions are what you get when you take the output of one function and use it for the input of the next one. In this discussion, we will discuss the composition of functions which are $R^1 \rightarrow R^1$, i.e. a real number as an input and a real number as an output. The notation for this is $(f\circ g)(x)=f(g(x))$, where the output of g(x) becomes the input of f(x) and is described as $(f\circ g)(x)$. As a real example, let's use $f(x)=x^2+2*x-2$, and $g(x)=3*x+2$. By replacing all of the occurrences of x in f(x) by the formula for g(x) we can find the formula in x for the composite function. So:

$(f\circ g)(x) = f(g(x)) = f(3*x+2)$

$(f\circ g)(x) = (3*x+2)^2+2*(3*x+2)-2$

$(f\circ g)(x) = (9*x^2+12*x+4)+(6*x+4)-2$

$(f\circ g)(x) = 9*x^2+18*x+6$

We can also find $(g\circ f)(x)$:

$(g\circ f)(x) = g(f(x)) = g(x^2+2*x-2)$

$(g\circ f)(x) = 3*(x^2+2*x-2)+2$

$(g\circ f)(x) = 3*x^2+6*x-4$

That's basic composite functions. Compositing functions can do weird things to the domain and range, though, because your doing mappings onto mappings (i.e. mapping one domain to another domain to a range). One important point is that the composite of a function with its inverse yields an identity function. That is $f(f^{-1}(x))=f^{-1}(f(x))=x$. What's happening is that your mapping the original function to its range and then straight back into the domain.

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