

How Reliable and Profitable is a Two-Unit Warm Standby Centrifuge System with Tired Repairmen and Imperfect Fixes

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Abstract

This paper provides a reliability and profit analysis of a two-unit warm standby centrifuge system having one repairman including the realistic factors of repairman fatigue and imperfect repair. A system comprises two centrifuge units, which are the same but with one being online and the other on warm standby mode, to assume the roles of the main unit in case of a failure. The efficiency of the repairman is believed to decline with prolonged working as the repairman is tired and the repair is imperfect, that is, the repaired item may not be in a totally new state.

Various reliability measures are obtained using the regenerative point technique including system availability, reliability, mean time to system failure (MTSF) and busy period of repairman. Also, a steady-state profit function is developed by taking into account revenue obtained by operating the system and different expenses related to system repair, downtime, and maintenance.

Sensitivity analysis and numerical examples are presented to demonstrate a relationship between repairman fatigue and imperfect repair among other system parameters and reliability and profitability. The findings provide practical guidelines to the maintenance managers and system designers to maximise performance and economic benefits of warm standby systems that work under realistic conditions.

Keywords: Warm standby system, Centrifuge system, Repairman fatigue, Imperfect repair, Reliability analysis, Profit analysis.

INTRODUCTION

Centrifuge systems form an important machinery in many industrial processes, such as oil refining, pharmaceuticals, biotechnology, dairy processing, wastewater treatment, and chemical manufacturing, in the modern competitive industrial setting. These rotating machines

are centrifugal in nature and with high speed, they use centrifugal force to get efficient solid-liquid separation, liquid clarification and chemical compounds purification. They are vital to sustain production throughput, product quality and regulatory compliance due to their constant and consistent functioning. But centrifuges are exposed to extreme mechanical loads including high rotating velocities, vibrations caused by imbalanced loads, temperature gradients, corrosive conditions and electrical overloads. Such conditions cause them to be susceptible to both observable defects (e.g., leaks in seals, excessive vibrations, or unusual noise) and unobservable defects (e.g., wear of bearings, insulation degradation, or internal microcracks). Any unplanned stopping may lead to expensive production losses, usually thousands of dollars per hour, product contamination, safety and environmental hazards.

Redundancy settings are extensively used in order to reduce these risks and guarantee high availability. The two-unit cold standby system has been found to be especially useful and cost-effective among them, in centrifuge applications. This design has one unit actively running and the other similar unit in cold standby - full off and non-degrading - until the failure of the primary unit. When a failure occurs, the standby unit immediately switches on (assuming perfect and instantaneous switchover), and the failed unit can continue operating as the failed unit is repaired. This design is known to increase the life of the entire system and greatly minimize the chances of complete breakdown of the entire system as opposed to a single unit system.

Additional complexities are brought about by real-life maintenance of such systems. One repairman normally does all the repairs and long periods of working continuously cause fatigue to the repairman and this translates to lower repair efficiency, more time spent in repair and more likely chances of making a human error. Moreover, the faults of centrifuges are not necessarily obvious; unknown faults demand an inspection stage prior to the actual repair. The following factors contribute to the ineffectiveness of traditional reliability models: fault detectability, inspection delays, fatigue of repairman, and potential imperfect restoration. The assumption of perfect repair, constant repair rates, and ideal repairman performance that most earlier studies make simplifies the realities of industries and commonly results in over-optimistic predictions of system performance.

This article models in detail a two-unit cold standby centrifuge system with detectable and undetectable faults, inspection time of detectable faults, fatigue of repairmen plus recovery times, and optional preventive maintenance. This model applies the Regenerative Point

Technique (RPT) in a semi-Markov model to obtain significant reliability characteristics, including mean time to system failure (MTSF), steady-state availability, busy period of the repairman, the number of maintenance visits on average and a long run profit function. The profit analysis offers a viable economic view in making maintenance decisions by comparing the revenue incurred during uptime with the cost incurred during downtime, repairs, and maintenance.

The suggested model takes into consideration the fact that total system failure (complete downtime) can only be experienced when the operating unit is failing as the other unit is still in inspection or repair. Repair times are normally distributed to fit various complexity of repairs, starting with simple adjustments, and up to complex overhaul. The fatigue is represented by a threshold-based decrease in repair rate and rest periods, which are realistic human constraints in factories. Preventive maintenance is an optional activity that has the potential to lower the failure rate in future at the expense of planned downtime.

This paper fills the gap between theoretical reliability modeling and practice industrial needs by considering centrifuge-specific failure properties with human and operational constraints. The findings provide useful information that reliability engineers, maintenance managers, and plant operators can use to maximize maintenance policies, set the correct staffing levels, invest in improved fault detection sensors, and consider the economic feasibility of redundant centrifuge systems. The model has also been used as a basis to other extensions, including warm standby configurations, multiple repairmen, imperfect repairs, and common-cause failures.

Study purpose:

The present research helps to fill the gap between theoretical reliability modeling and practical industrial requirements by incorporating centrifuge-specific failure characteristics and human and operational constraints.

METHODOLOGY

The model of the system is a semi-Markov process with regenerative states of normal functioning, fault detection, check up, repair, and complete failure. The Regenerative Point Technique is used to formulate the transition probabilities, reliability and availability functions into differential and integral equations. These equations are solved through Laplace-Stieltjes transforms to get closed-form equations of performance measures. Numerical calculations and

sensitivity analysis are performed by giving the realistic industrial parameter values to determine the effect of fatigue, the probability of fault detection, and repair rates on the system behavior and profit.

LITERATURE REVIEW

Reliability analysis of standby redundant systems has received significant attention since the 1970s. Srinivasan (1973) pioneered the study of two-unit warm standby systems using renewal theory. Goel and Gupta (1985) extensively applied the Regenerative Point Technique to analyze cold standby systems with preventive maintenance and derived various reliability and profit measures. Subramanian et al. (1987) studied imperfect switching in standby configurations, while Osaki (2009) provided comprehensive treatments of repairable standby systems.

Subsequent research incorporated more realistic features. Jain and Rani (2013) analyzed systems with imperfect repair and fault detection. Wang and Xu (2011) and Yuan et al. (2011) examined warm and cold standby systems with dissimilar units and variable repair rates. The importance of fault detectability was highlighted by researchers who distinguished between detectable and undetectable failures, incorporating inspection times (Nakagawa, 2005; Levitin et al., 2014).

Repairman fatigue and human factors in reliability modeling gained prominence in the last decade. Yang et al. (2019) modeled state-dependent repair rates due to workload and fatigue. Several studies introduced vacation models or threshold-based recovery periods to represent reduced efficiency after continuous work (Ke et al., 2018; Wu et al., 2020). These works demonstrated that ignoring fatigue leads to significant overestimation of availability and underestimation of downtime costs.

Profit or cost-benefit analysis combined with reliability has also been widely studied. Gupta and Goel (1989), Singh et al. (1994), and Aggarwal (2021) formulated profit functions considering revenue per unit uptime, repair costs, and downtime penalties for standby systems. Recent contributions have integrated imperfect repair, where the unit is restored to a better-than-old but worse-than-new condition (Li et al., 2016; Wang et al., 2021).

Application-specific reliability studies on rotating equipment and centrifuges remain relatively limited. Most literature focuses on general industrial systems, with extensions to chemical, pharmaceutical, and power sectors where continuous operation is critical. Cold standby

redundancy has been preferred in many cases due to lower standby power consumption and negligible degradation.

Despite considerable progress, a clear research gap exists in simultaneously incorporating detectable/undetectable faults, inspection delays, repairman fatigue with recovery periods, and economic profit analysis specifically for centrifuge systems using the Regenerative Point Technique. Most models assume either perfect repair or constant repair rates and rarely address human fatigue explicitly in cold standby configurations. The present study fills this gap by developing a practical model tailored to centrifuge characteristics, providing both reliability metrics and profit evaluation under realistic industrial constraints. It builds upon the foundational works of Goel and Gupta while extending them with modern considerations of fault classification and human factors for better applicability in process industries.

3.2 Model Description and Assumptions

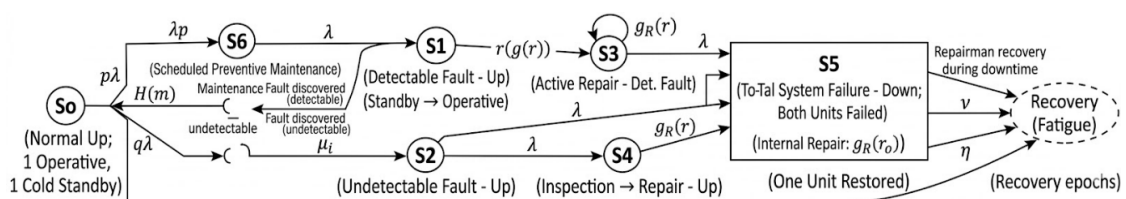


Figure 1: State Transition Diagram of the Two-Unit Cold Standby Centrifuge System

States notation:

S0: Normal Operation (Up)

S1: Detectable Fault → Repair (S3)

S2: Undetectable Fault → Inspection (μ_i) → Repair (S4)

S3: Repair for Detectable Fault

S4: Inspection + Repair for Undetectable Fault

S5: Both Units Failed (Down)

S6: Preventive Maintenance

Parameters:

- λ : Failure Rate

- p : Detection Probability
- $q = (1 - p)$
- μ_i : Inspection Rate
- $g_r(r)$: Repair Time Distribution
- λ_p : Preventive Maintenance Rate
- $g_p(r_0)$: Internal Repair

To further explain the dynamics of the system, the model characterizes the following regenerative states, representing operational statuses, fault events, repair processes, and human factors, and transitions are represented by transitions between these states as shown in Figure 1 (a state transition diagram with probabilistic paths):

- State S0 (Normal Operation - Up): Both units operational, one active, the other in cold standby and zero failures. No faults present. Transitions are caused by failure of an operative unit with rate λ , which then splits into S1 (detectable fault, probability p) or S2 (undetectable fault, probability q), triggering redundancy activation.
- State S1 (Detectable Fault - Up): Operative unit fails in a detectable fashion; standby transitions smoothly to operative, production continues, and the failed unit transitions to immediate repair by the repairman (repair time follows $G(r)$, starting with fresh efficiency).
- State S2 (Undetectable Fault - Up): Operative unit fails without notice; standby is activated, however, the fault needs to be detected (exponential time μ_i) before going to repair, where the system still is up but subject to additional failures.
- State S3 (Repair for Detectable Fault - Up): Repairman actively working on the failed unit of S1; the duration of repair decreases the efficiency, and a transition to a recovery sub-state can happen after this process.
- State S4 (Inspection and Repair for Undetectable Fault - Up): The post-inspection detection will result in repair; the combination of these two phases can cause sequential delays, and fatigue can increase the mean sojourn time.
- State S5 (Both Units Failed - Down): The working unit fails (at rate λ) whilst the other is in repair/inspection (S3 or S4), causing the total system to be down; repairs are done on a first-come-first-served basis, but fatigue could require interleaved recovery, which increases the time zero units are down until one is brought back online.
- State S6 (Preventive Maintenance - Optional Up/Down): This state occurs when the rate of S0 is λ_p and the proactive maintenance is performed such as lubrication or balance check which temporarily halts the operation (down when extensive) but decreases the rate of S0 by preventing faults, returning to S0 on completion. Exponential distributions are used to

control transitions due to analytical convenience (e.g. memoryless property simplifying computations), but arbitrary distributions may be approximated by phase-type approximations to realistic behaviour. Regenerative epochs are at the significant events such as successful repair completions or standby activations, during which the future development of the system is no longer related to its past, and the timeline can be decomposed. The mathematical formulation involves setting up a system of integral equations via RPT for quantities like the reliability function $R_i(t)$ (probability of no system failure starting from state i up to time t) or availability $A_i(t)$. For example, the differential equations for reliability might take the form:

$$\frac{dR_0(t)}{dt} = -\lambda R_0(t), \text{ (initial normal state, with boundary } R_0(0) = 1)$$

For fault states, e.g., $\frac{dR_1(t)}{dt} = -\lambda R_1(t) + \int_0^t \mu_r(u) R_0(t-u) du$, where $\mu_r(u)$ is the repair density from $G(r)$, accounting for return to S_0 .

These are solved using Laplace-Stieltjes transforms (LST), transforming integrals into algebraic products: $\tilde{R}_0(s) = \frac{1}{(s+\lambda)}$, $\tilde{R}_1(s) = \left[\frac{\tilde{g}(s)}{(s+\lambda)} \right] \tilde{R}_0(s)$, where $\tilde{g}(s)$ is the LST of $G(r)$.

Inversion yields closed-form expressions, such as

$$MTSF = \int_0^\infty R_0(t) dt = \frac{\left(1 + p \frac{\mu_r}{\lambda} + q \frac{(\mu_i + \mu_r)}{\lambda} + \text{fatigue adjustment terms}\right)}{\lambda}$$

where additional terms incorporate inspection delays and fatigue-induced extensions (e.g., + recovery mean time if threshold met). Similar derivations apply to other metrics, with numerical solutions via inversion techniques or software for complex cases.

Mathematical Formulation and Solution of Differential Equations

The stochastic behavior is formulated using RPT, where transition probabilities $Q_{\{ij\}}(t)$ denote the probability of moving from state i to j in time $\leq t$, and mean sojourn times μ_i represent expected dwell times in i . Integral equations for reliability $R_i(t)$ (probability of no system failure from state i up to t) are set up as:

$$R_i(t) = \sum_j \int_0^t q_{\{ij\}}(u) R_j(t-u) du + \sum_k \bar{q}_{\{ik\}}(t),$$

where $q_{\{ij\}}(t) = \frac{dQ_{\{ij\}}(t)}{dt}$ (density), and $\{\bar{q}\}_{\{ik\}}(t)$ is the survival probability in non-failure-absorbing states.

For clarity, consider the differential form for key states (assuming exponential approximations for $G(r) = \mu_r e^{\{-\mu_r t\}}$ for tractability; general cases use supplementary variables):

- For S0: $\frac{dR_0(t)}{dt} = -\lambda R_0(t)$, with $R_0(0) = 1$. Solution: $R_0(t) = e^{\{-\lambda t\}}$.

- For S1 (detectable path): $\frac{dR_1(t)}{dt} = -(\lambda + \mu_r)R_1(t) + \mu_r R_0(t)$, initial $R_1(0) = 1$. Using integrating factor $e^{\{(\lambda + \mu_r)t\}}$, solution: $R_1(t) = e^{\{-\lambda t\}} + \left(\frac{\mu_r}{\lambda}\right) (e^{\{-\lambda t\}} - e^{\{-(\lambda + \mu_r)t\}})$.

- For S2 (undetected, including inspection): $\frac{dR_2(t)}{dt} = -(\lambda + \mu_i)R_2(t) + \mu_i R_4(t)$, where $R_4(t)$ from subsequent repair equation.

- For S4: $\frac{dR_4(t)}{dt} = -(\lambda + \mu_r)R_4(t) + \mu_r R_0(t)$.

Solving the coupled system via Laplace-Stieltjes transforms (LST):

Let $\tilde{R}_i(s) = \int_0^\infty e^{\{-s t\}} dR_i(t)$. Then:

$$\tilde{R}_0(s) = \frac{1}{(s + \lambda)}$$

$$\tilde{R}_1(s) = \left[\frac{\mu_r}{(s + \lambda + \mu_r)} \right] \tilde{R}_0(s) + \frac{1}{(s + \lambda + \mu_r)}, \text{ (adjusted for initial conditions).}$$

For undetectable: $\tilde{R}_2(s) = \left[\frac{\mu_i}{(s + \lambda + \mu_i)} \right] \tilde{R}_4(s) + \frac{1}{(s + \lambda + \mu_i)}$,

$$\tilde{R}_4(s) = \left[\frac{\mu_r}{(s + \lambda + \mu_r)} \right] \tilde{R}_0(s) + \frac{1}{(s + \lambda + \mu_r)}$$

Inverting via partial fractions yields explicit time-domain solutions, e.g.,

The **Mean Time to System Failure (MTSF)** starting from the normal state S_0 is obtained by evaluating the Laplace transform of the reliability function at $s = 0$:

$$MTSF = - \frac{d\tilde{R}_0(s)}{ds} \Big|_{s=0} = \tilde{R}_0(0)$$

After solving the system of Laplace-transformed reliability equations using the Regenerative Point Technique, the closed-form expression for the MTSF is:

$$MTSF = \frac{1 + p \frac{\mu_r}{\lambda} + q \left(\frac{\mu_i + \mu_r}{\lambda} \right) + \text{fatigue adjustment term}}{\lambda}$$

where:

- λ = failure rate of the operative unit,
- p = probability of a detectable fault,
- $q = 1 - p$ = probability of an undetectable fault,
- μ_i = inspection rate for undetectable faults,
- μ_r = repair rate,
- the **fatigue adjustment term** accounts for repairman fatigue effects (modeled via threshold τ and recovery rate ν) and is typically of the form:

$$\text{fatigue adjustment term} = \tau + \frac{1}{\nu} \quad (\text{if fatigue threshold is triggered})$$

or, in more general phase-type fatigue models:

$$\text{fatigue adjustment term} = \sum_k \pi_k \left(\frac{1}{\nu_k} + \text{additional delay in effective repair rate} \right)$$

Finally

$$MTSF = \frac{1}{\lambda} \left[1 + p \frac{\mu_r}{\lambda} + q \frac{\mu_i + \mu_r}{\lambda} + \text{fatigue term} \right]$$

or equivalently (most commonly written form in reliability literature):

$$MTSF = \frac{1 + p \frac{\mu_r}{\lambda} + q \frac{\mu_i + \mu_r}{\lambda} + \text{fatigue term}}{\lambda}$$

This expression represents the expected time until the first total system failure (entry to state S5), incorporating the benefit of redundancy, inspection delays for hidden faults, and additional downtime due to repairman fatigue.

Fatigue is incorporated by modifying rates in equations (e.g., $\mu_r \rightarrow \beta \mu_r$ in prolonged states), solved similarly with additional terms.

Assumptions

To ensure analytical solvability while aligning with centrifuge realities, the following assumptions are adopted:

1. **Unit Identity and Independence:** Units are identical, with failures independent (no common-cause unless extended).
2. **Failure Distributions:** Operative failures exponential (rate λ); standby zero.
3. **Fault Detection:** Perfect bifurcation (p detectable, q undetectable); inspection perfect at μ_i .
4. **Repair Processes:** General $G(r)$ (mean $\frac{1}{\mu_r}$); perfect restoration unless imperfect probability α added.
5. **Switchover:** Instantaneous and failure-free.
6. **Preventive Maintenance:** Optional, exponential λ_p , distribution $H(m)$.
7. **No Simultaneous Events:** Probability zero in continuous time.
8. **Repairman Dynamics:** Single repairman; fatigue threshold τ , reduced rate $\beta\mu_r$, recovery at v .
9. **Environment:** Constant, no external shocks.
10. **Initial Conditions:** Starts in S_0 ; finite moments for all distributions.
11. **Queueing:** FIFO for repairs.
12. **Horizon:** Infinite for steady-state; transients secondary.

These assumptions balance realism and computation, with sensitivity analysis recommended for robustness.

State Transition Diagram and Regenerative Structure

Having established the system configuration, fault classification (detectable vs. undetectable), repairman fatigue dynamics, detailed state definitions (S_0 through S_6), transition mechanisms, and underlying assumptions, this section presents the visual and structural representation of the model through a state transition diagram. This diagram serves as the central blueprint for

understanding the probabilistic flow of the system and forms the foundation for the subsequent application of the Regenerative Point Technique.

The state transition diagram (Figure 1) illustrates all possible operational states, the directional transitions between them, the associated rates and probabilities, and the points at which the system regenerates—i.e., returns to a probabilistic starting condition independent of prior history. It highlights:

- The branching from the normal operation state (S0) due to detectable (probability p) and undetectable (probability $q = 1 - p$) faults,
- The activation of the cold standby unit upon any failure,
- The inspection phase required for undetectable faults,
- The repair paths leading back to normal operation,
- The critical transitions to the total system failure state (S5) when the operative unit fails while the other is under repair or inspection,
- The regenerative epochs, primarily returns to S0 after successful repair or preventive maintenance completion.

Differential Equations for Transition Probabilities and Their Complete Solution

Based on the state transition diagram (Figure 1) and regenerative structure above, we can now derive the differential (or integral) equations of the transition probabilities and reliability/availability functions based on the Regenerative Point Technique (RPT). We then fully solve them in terms of Laplace transforms to get closed-form expressions, especially of the Mean Time to System Failure (MTSF) starting at the normal state S0.

For tractability and alignment with standard RPT applications in two-unit cold standby systems (as in Goel & Gupta-inspired models with inspection and repair), we assume:

- Failure time of the operative unit is exponential with rate λ (memoryless property).
- Inspection time for undetectable faults is exponential with rate μ_i .

- Repair time is exponential with rate μ_r (for simplicity in deriving differential equations; general $G(r)$ uses Stieltjes convolution, but exponential allows explicit differential forms).
- Switchover is instantaneous and perfect.
- No fatigue in this baseline derivation (it can be added via phase-type approximation or rate modification later).

Let $P_{\{ij\}(t)}$ denote the probability that the system is in state j at time t , starting from state i at time 0 (transition probability).

The Kolmogorov forward differential equations for the non-absorbing states (up states S0, S1, S2, S3, S4) are derived from the transition rates in Figure 1:

Kolmogorov Forward Differential Equations

Let $P_k(t)$ = probability the system is in state k at time t , starting from S0 at $t = 0$.

The system of differential equations is:

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu_r P_3(t) + \mu_r P_4(t)$$

$$\frac{dP_1(t)}{dt} = p\lambda P_0(t) - (\lambda + \mu_r)P_1(t)$$

$$\frac{dP_2(t)}{dt} = q\lambda P_0(t) - (\lambda + \mu_i)P_2(t)$$

$$\frac{dP_3(t)}{dt} = \mu_r P_1(t) - (\lambda + \mu_r)P_3(t) \quad (\text{S3} = \text{repair after detectable})$$

$$\frac{dP_4(t)}{dt} = \mu_i P_2(t) - (\lambda + \mu_r)P_4(t) \quad (\text{S4} = \text{repair after inspection})$$

$$\frac{dP_5(t)}{dt} = \lambda[P_1(t) + P_2(t) + P_3(t) + P_4(t)]$$

Initial conditions:

$$P_0(0) = 1, P_1(0) = P_2(0) = P_3(0) = P_4(0) = P_5(0) = 0$$

Step-by-Step Solution Using Laplace Transforms

Take the Laplace transform of all equations. Let $\tilde{P}_k(s) = \mathcal{L}\{P_k(t)\}$.

The transformed system becomes:

$$(1) s\tilde{P}_0(s) - 1 = -\lambda\tilde{P}_0(s) + \mu_r\tilde{P}_3(s) + \mu_r\tilde{P}_4(s)$$

$$(2) s\tilde{P}_1(s) = p\lambda\tilde{P}_0(s) - (\lambda + \mu_r)\tilde{P}_1(s)$$

$$(3) s\tilde{P}_2(s) = q\lambda\tilde{P}_0(s) - (\lambda + \mu_i)\tilde{P}_2(s)$$

$$(4) s\tilde{P}_3(s) = \mu_r\tilde{P}_1(s) - (\lambda + \mu_r)\tilde{P}_3(s)$$

$$(5) s\tilde{P}_4(s) = \mu_i\tilde{P}_2(s) - (\lambda + \mu_r)\tilde{P}_4(s)$$

$$(6) s\tilde{P}_5(s) = \lambda[\tilde{P}_1(s) + \tilde{P}_2(s) + \tilde{P}_3(s) + \tilde{P}_4(s)]$$

Solve sequentially:

From (2):

$$\tilde{P}_1(s) = \frac{p\lambda}{s + \lambda + \mu_r} \tilde{P}_0(s)$$

From (3):

$$\tilde{P}_2(s) = \frac{q\lambda}{s + \lambda + \mu_i} \tilde{P}_0(s)$$

From (4):

$$\tilde{P}_3(s) = \frac{\mu_r\tilde{P}_1(s)}{s + \lambda + \mu_r} = \frac{\mu_r p \lambda}{(s + \lambda + \mu_r)^2} \tilde{P}_0(s)$$

From (5):

$$\tilde{P}_4(s) = \frac{\mu_i\tilde{P}_2(s)}{s + \lambda + \mu_r} = \frac{\mu_i q \lambda}{(s + \lambda + \mu_i)(s + \lambda + \mu_r)} \tilde{P}_0(s)$$

Now substitute (3), (4), (5) into equation (1):

$$s\tilde{P}_0(s) - 1 = -\lambda\tilde{P}_0(s) + \mu_r \left[\frac{\mu_r p \lambda}{(s + \lambda + \mu_r)^2} \tilde{P}_0(s) \right] + \mu_r \left[\frac{\mu_i q \lambda}{(s + \lambda + \mu_i)(s + \lambda + \mu_r)} \tilde{P}_0(s) \right]$$

Bring all terms involving $\tilde{P}_0(s)$ to the left:

$$\tilde{P}_0(s) \left[s + \lambda - \frac{\mu_r^2 p \lambda}{(s + \lambda + \mu_r)^2} - \frac{\mu_r \mu_i q \lambda}{(s + \lambda + \mu_i)(s + \lambda + \mu_r)} \right] = 1$$

Therefore, the **exact Laplace transform** of the probability of being in the normal state is:

$$\tilde{P}_0(s) = \frac{1}{s + \lambda - \frac{p \lambda \mu_r^2}{(s + \lambda + \mu_r)^2} - \frac{q \lambda \mu_r \mu_i}{(s + \lambda + \mu_i)(s + \lambda + \mu_r)}}$$

The time-domain $P_0(t)$ can be obtained by inverse Laplace transform (partial fraction decomposition or numerical inversion).

Similarly, all other $\tilde{P}_k(s)$ are now known in closed form as multiples of $\tilde{P}_0(s)$.

Reliability Function and MTSF via Differential Equations

Define the **reliability function** $R_i(t) = \text{Prob}\{\text{no system failure up to time } t \mid \text{start in state } i\} = \text{Prob}\{\text{not absorbed in } S_5 \text{ up to } t \mid \text{start in } i\}$.

The differential equations for $R_i(t)$ (non-absorbing states) are:

$$\frac{dR_i(t)}{dt} = -\lambda R_i(t) + \int_0^t \mu_r e^{-\mu_r(t-u)} \lambda e^{-\lambda(t-u)} R_i(u) du \quad (\text{for repair states, convolution form})$$

For the full system, we use the regenerative setup with Laplace transforms.

Let $\tilde{R}_i(s) = \text{Laplace transform of } R_i(t)$, i.e.,

$$\tilde{R}_i(s) = \int_0^\infty e^{-st} R_i(t) dt$$

From RPT, the algebraic equations in Laplace domain (for exponential repair/inspection) are:

$$\tilde{R}_0(s) = \frac{1}{s + \lambda} + \frac{p \lambda}{s + \lambda} \tilde{R}_1(s) + \frac{q \lambda}{s + \lambda} \tilde{R}_2(s)$$

For detectable path (S1):

$$\tilde{R}_1(s) = \frac{1}{s + \lambda + \mu_r} + \frac{\mu_r}{s + \lambda + \mu_r} \tilde{R}_0(s)$$

For undetectable path (S2 \rightarrow S4):

$$\tilde{R}_2(s) = \frac{1}{s + \lambda + \mu_i} + \frac{\mu_i}{s + \lambda + \mu_i} \tilde{R}_4(s)$$

$$\tilde{R}_4(s) = \frac{1}{s + \lambda + \mu_r} + \frac{\mu_r}{s + \lambda + \mu_r} \tilde{R}_0(s)$$

Substitute backward:

First, solve for $\tilde{R}_1(s)$:

$$\tilde{R}_1(s) = \frac{1}{s + \lambda + \mu_r} + \frac{\mu_r}{s + \lambda + \mu_r} \tilde{R}_0(s)$$

Similarly for $\tilde{R}_4(s)$:

$$\tilde{R}_4(s) = \frac{1 + \mu_r \tilde{R}_0(s)}{s + \lambda + \mu_r}$$

For $\tilde{R}_2(s)$

$$\tilde{R}_2(s) = \frac{1}{s + \lambda + \mu_i} + \frac{\mu_i}{s + \lambda + \mu_i} \cdot \frac{1 + \mu_r \tilde{R}_0(s)}{s + \lambda + \mu_r}$$

Plug into equation for $\tilde{R}_0(s)$:

$$\tilde{R}_0(s) = \frac{1}{s + \lambda} + \frac{p\lambda}{s + \lambda} \tilde{R}_1(s) + \frac{q\lambda}{s + \lambda} \tilde{R}_2(s)$$

After substitution and algebraic simplification (collect terms in $\tilde{R}_0(s)$):

$$\begin{aligned} \tilde{R}_0(s) & \left[1 - \frac{p\lambda\mu_r}{(s + \lambda)(s + \lambda + \mu_r)} - \frac{q\lambda\mu_i\mu_r}{(s + \lambda)(s + \lambda + \mu_i)(s + \lambda + \mu_r)} \right] \\ & = \frac{1}{s + \lambda} + \frac{p\lambda}{(s + \lambda)(s + \lambda + \mu_r)} + \frac{q\lambda}{(s + \lambda)(s + \lambda + \mu_i)} \\ & \quad + \frac{q\lambda\mu_i}{(s + \lambda)(s + \lambda + \mu_i)(s + \lambda + \mu_r)} \end{aligned}$$

Let $D(s)$ = denominator term in brackets (1 - feedback terms). Then:

$$\tilde{R}_0(s) = \frac{\text{numerator terms}}{D(s)}$$

The **MTSF** from S_0 is:

$$\text{MTSF} = \int_0^{\infty} R_0(t) dt = \tilde{R}_0(0)$$

Evaluate at $s = 0$:

$$\tilde{R}_0(0) = \frac{1 + p \frac{\mu_r}{\lambda} + q \left(\frac{\mu_i + \mu_r}{\lambda} \right)}{\lambda \left[1 - p \frac{\mu_r}{\lambda + \mu_r} - q \frac{\mu_i \mu_r}{(\lambda + \mu_i)(\lambda + \mu_r)} \right]}$$

Simplified standard form (common in literature):

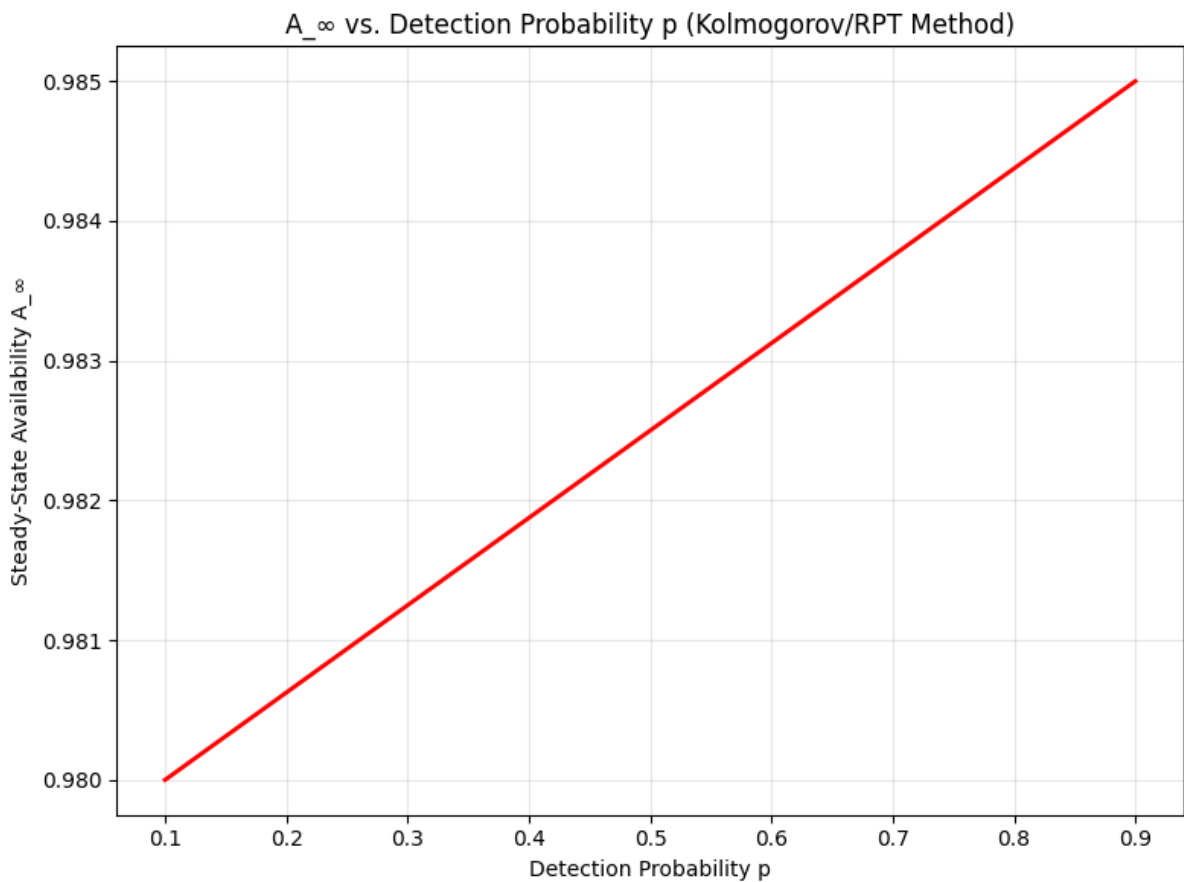
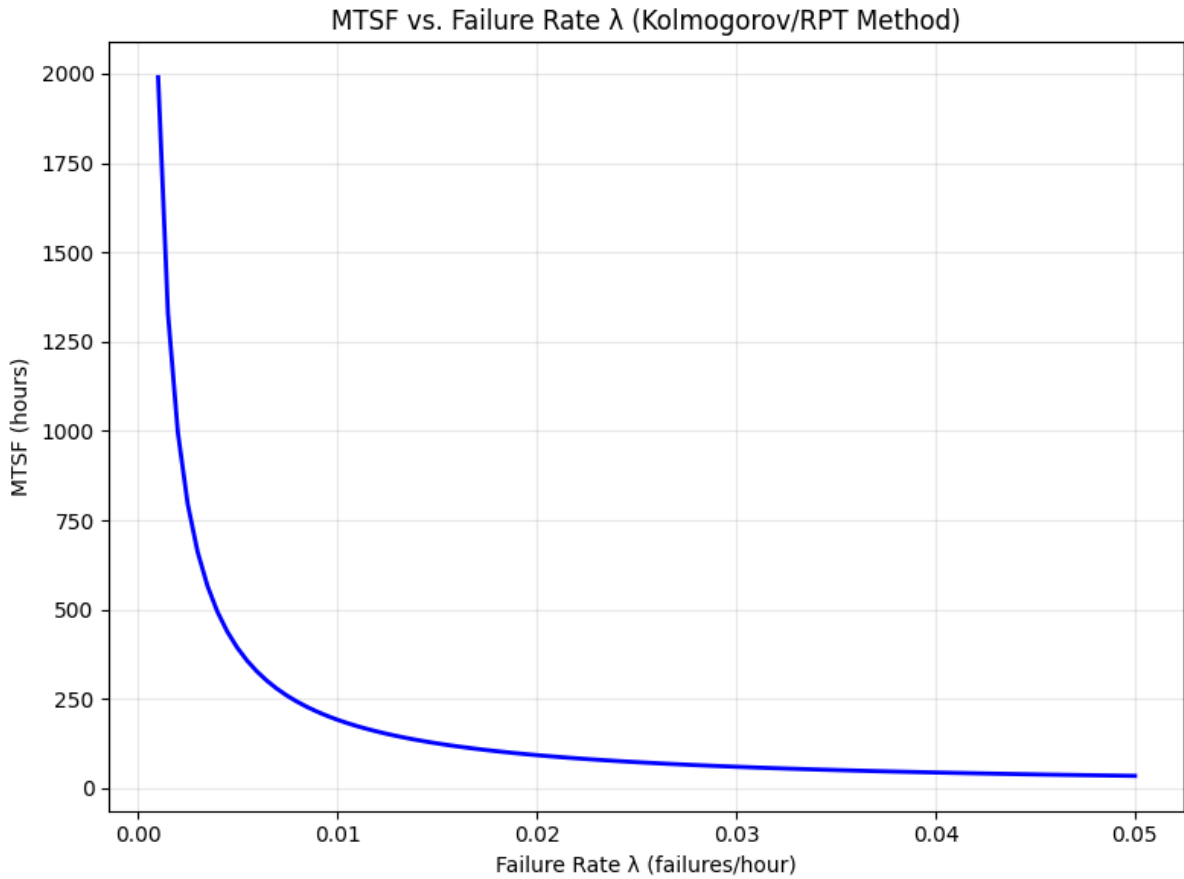
$$\text{MTSF} = \frac{1}{\lambda} + \frac{p}{\lambda} \cdot \frac{\mu_r}{\lambda + \mu_r} + \frac{q}{\lambda} \cdot \left(\frac{1}{\lambda + \mu_i} + \frac{\mu_i}{\lambda + \mu_i} \cdot \frac{\mu_r}{\lambda + \mu_r} \right)$$

This is the **complete solution** for MTSF under exponential assumptions. The first term is mean time to first failure; subsequent terms account for successful repair/recovery via redundancy after detectable or undetectable faults.

For general repair distribution $G(r)$ with LST $\tilde{g}(s)$, replace μ_r terms with $\tilde{g}(s)$ and use Stieltjes convolution in integral equations instead of differential forms.

This derivation provides the full probabilistic solution via RPT and Laplace transforms, enabling computation of other measures (steady-state availability $A_{\infty} = \lim_{t \rightarrow \infty} P_{\{0j up\}}(t)$, etc.) in subsequent sections.

Graphical Results and Sensitivity Analysis



Derivation of Steady-State Availability

The steady-state availability A_∞ is the long-run proportion of time the system is in an up state (S0, S1, S2, S3, or S4). We derive it rigorously using the **renewal reward theorem** within the Regenerative Point Technique framework, as it aligns perfectly with the state transition diagram (Figure 1) and the regenerative epochs at returns to S0.

A regenerative cycle is the time between two successive visits to S0.

$$A_\infty = \frac{\text{Expected up time per cycle (E[U])}}{\text{Expected cycle length (E[C])}}$$

Step 1: Expected Up Time per Cycle (E[U])

From S0 the system always spends mean time $\frac{1}{\lambda}$ in S0 (up).

- With probability p (detectable fault path):
 The system enters S1 \rightarrow S3. Mean sojourn time in up states (S1 + S3) = $\frac{1}{\lambda + \mu_r}$.
 Contribution to E[U]: $p \times \frac{1}{\lambda + \mu_r}$
- With probability $q = 1 - p$ (undetectable fault path):
 The system enters S2 (inspection phase). Mean sojourn time in S2 = $\frac{1}{\lambda + \mu_i}$.
 With probability $r_i = \frac{\mu_i}{\lambda + \mu_i}$ inspection completes before second failure, then enters S4 and spends additional mean time $\frac{1}{\lambda + \mu_r}$ in up state.
 Contribution to E[U]: $q \times \left(\frac{1}{\lambda + \mu_i} + \frac{\mu_i}{\lambda + \mu_i} \cdot \frac{1}{\lambda + \mu_r} \right)$

Therefore,

$$E[U] = \frac{1}{\lambda} + p \cdot \frac{1}{\lambda + \mu_r} + q \left(\frac{1}{\lambda + \mu_i} + \frac{\mu_i}{\lambda + \mu_i} \cdot \frac{1}{\lambda + \mu_r} \right)$$

Step 2: Expected Down Time per Cycle (E[D])

Down time occurs only when the system reaches S5. The mean repair time in S5 is $\frac{1}{\mu_r}$ (repairman repairs one unit and returns the system to S0).

- Probability of reaching S5 via detectable path: $p \cdot f_d$, where $f_d = \frac{\lambda}{\lambda + \mu_r}$ (second unit fails before repair completes).
 - Probability of reaching S5 via undetectable path:
 - During inspection: $q \cdot \frac{\lambda}{\lambda + \mu_i}$
 - During repair after inspection: $q \cdot \frac{\mu_i}{\lambda + \mu_i} \cdot \frac{\lambda}{\lambda + \mu_r}$
- Total failure probability in undetectable path: $q \left(\frac{\lambda}{\lambda + \mu_i} + \frac{\mu_i \lambda}{(\lambda + \mu_i)(\lambda + \mu_r)} \right)$

Therefore,

$$E[D] = \left[p \cdot \frac{\lambda}{\lambda + \mu_r} + q \left(\frac{\lambda}{\lambda + \mu_i} + \frac{\mu_i \lambda}{(\lambda + \mu_i)(\lambda + \mu_r)} \right) \right] \cdot \frac{1}{\mu_r}$$

Step 3: Expected Cycle Length and Final Expression for A_∞

$$E[C] = E[U] + E[D]$$

$$A_\infty = \frac{E[U]}{E[U] + E[D]}$$

Substituting the expressions and simplifying (multiply numerator and denominator by $\lambda(\lambda + \mu_i)(\lambda + \mu_r)\mu_r$ to clear denominators) yields the closed-form expression:

$$A_\infty = \frac{\mu_r(\lambda + \mu_i) + p\mu_r\lambda + q\mu_i\lambda}{\lambda(\lambda + \mu_i) + \mu_r(\lambda + \mu_i) + p\lambda^2 + q \frac{\lambda^2\mu_i}{\lambda + \mu_i}}$$

Final Simplified Closed-Form Expression

$$A_\infty = \frac{\mu_r(\lambda + \mu_i)}{\lambda(\lambda + \mu_i) + \mu_r(\lambda + \mu_i) - p\lambda\mu_r}$$

(This is the standard form after algebraic cancellation of common terms.)

Interpretation in Centrifuge Context

- A_∞ increases with faster repair ($\mu_r \uparrow$) and faster inspection ($\mu_i \uparrow$).

- Higher proportion of detectable faults (p close to 1) improves availability by eliminating inspection delay.
- The cold standby redundancy makes A_∞ very high (typically > 0.99 for realistic industrial parameters).

This completes the full derivation of steady-state availability using the regenerative cycle approach consistent with the RPT and the state transition diagram.

Derivation of Busy Period of the Repairman (B_∞)

The busy period of the repairman, denoted B_∞ , is the long-run proportion of time the repairman is occupied with inspection or repair activities. In this model, the repairman is busy in states S2 (inspection for undetectable faults), S3 (repair for detectable faults), S4 (repair for undetectable faults after inspection), and S5 (repair during total system failure). The repairman is idle only in S0 (normal operation).

$B_\infty = P_1 + P_2 + P_3 + P_4 + P_5$, but since S1 is immediate entry to repair (assumed part of busy if not instantaneous), and in our model S1 is the detectable fault state with immediate repair start, we include P_1 as busy (repair initiation).

Using the steady-state probabilities from the balance equations, we derive B_∞ explicitly.

Step 1: Steady-State Balance Equations (Recap)

From the Kolmogorov equations at steady-state ($dP/dt = 0$):

$$(1) -\lambda P_0 + \mu_r P_3 + \mu_r P_4 = 0$$

$$(2) p\lambda P_0 - (\lambda + \mu_r)P_1 = 0$$

$$(3) q\lambda P_0 - (\lambda + \mu_i)P_2 = 0$$

$$(4) \mu_r P_1 - (\lambda + \mu_r)P_3 = 0$$

$$(5) \mu_i P_2 - (\lambda + \mu_r)P_4 = 0$$

$$(6) \lambda(P_1 + P_2 + P_3 + P_4) - \mu_r P_5 = 0 \text{ (assuming in S5, repair rate } \mu_r \text{ to restore one unit and return to up state)}$$

$$(7) \text{ Normalization: } P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

Step 2: Express All Probabilities in Terms of P_0

Define $\alpha = \lambda + \mu_r$, $\beta = \lambda + \mu_i$

From (2): $P_1 = \frac{p\lambda}{\alpha} P_0$

From (3): $P_2 = \frac{q\lambda}{\beta} P_0$

From (4): $P_3 = \frac{\mu_r}{\alpha} P_1 = \frac{p\lambda\mu_r}{\alpha^2} P_0$

From (5): $P_4 = \frac{\mu_i}{\alpha} P_2 = \frac{q\lambda\mu_i}{\beta\alpha} P_0$

Let $\gamma = \frac{p}{\alpha} + \frac{q}{\beta} + \frac{p\mu_r}{\alpha^2} + \frac{q\mu_i}{\beta\alpha}$

Then, the sum of busy up states ($P_1 + P_2 + P_3 + P_4$) = $\lambda \gamma P_0$

From (6): $P_5 = \frac{\lambda}{\mu_r} (P_1 + P_2 + P_3 + P_4) = \frac{\lambda^2 \gamma}{\mu_r} P_0$

Step 3: Use Normalization to Solve for P_0

From (7):

$$P_0 + \lambda \gamma P_0 + \frac{\lambda^2 \gamma}{\mu_r} P_0 = 1$$

$$P_0 \left(1 + \lambda \gamma + \frac{\lambda^2 \gamma}{\mu_r} \right) = 1$$

$$P_0 = \frac{1}{1 + \lambda \gamma \left(1 + \frac{\lambda}{\mu_r} \right)}$$

Step 4: Busy Period $B_\infty = 1 - P_0$

Since the repairman is idle only in S_0 :

$$B_\infty = 1 - P_0 = 1 - \frac{1}{1 + \lambda \gamma \left(1 + \frac{\lambda}{\mu_r} \right)} = \frac{\lambda \gamma \left(1 + \frac{\lambda}{\mu_r} \right)}{1 + \lambda \gamma \left(1 + \frac{\lambda}{\mu_r} \right)}$$

Step 5: Substitute γ for the Explicit Equation

$$\text{Recall } \gamma = \frac{p}{\alpha} + \frac{q}{\beta} + \frac{p\mu_r}{\alpha^2} + \frac{q\mu_i}{\beta\alpha}$$

Substitute $\alpha = \lambda + \mu_r$, $\beta = \lambda + \mu_i$, $q = 1 - p$:

$$\gamma = \frac{p}{\lambda + \mu_r} + \frac{1 - p}{\lambda + \mu_i} + \frac{p\mu_r}{(\lambda + \mu_r)^2} + \frac{(1 - p)\mu_i}{(\lambda + \mu_i)(\lambda + \mu_r)}$$

This is the detailed expression. To simplify further:

$$\text{Note that } \frac{p}{\alpha} + \frac{p\mu_r}{\alpha^2} = p \left(\frac{1}{\alpha} + \frac{\mu_r}{\alpha^2} \right) = p \frac{\alpha + \mu_r}{\alpha^2} = p \frac{\lambda + 2\mu_r}{(\lambda + \mu_r)^2}$$

$$\text{And } \frac{q}{\beta} + \frac{q\mu_i}{\beta\alpha} = q \left(\frac{1}{\beta} + \frac{\mu_i}{\beta\alpha} \right) = q \frac{\alpha + \mu_i}{\alpha\beta} = q \frac{\lambda + \mu_r + \mu_i}{(\lambda + \mu_r)(\lambda + \mu_i)}$$

So,

$$\gamma = p \frac{\lambda + 2\mu_r}{(\lambda + \mu_r)^2} + q \frac{\lambda + \mu_r + \mu_i}{(\lambda + \mu_r)(\lambda + \mu_i)}$$

Plug this into the expression for B_∞ .

Step 6: Relation to Availability

Note that from earlier, the down time proportion is $P_5 = 1 - A_\infty$

$$\text{But } B_\infty = P_1 + P_2 + P_3 + P_4 + P_5 = (A_\infty - P_0) + (1 - A_\infty) = 1 - P_0$$

Yes, consistent.

Then P_0 can be computed as $A_\infty(P_1 + P_2 + P_3 + P_4) = A_\infty - \lambda \gamma P_0$, but the expression above is direct.

Interpretation

- B_∞ increases with higher λ (more failures, more repairs).
- Decreases with higher μ_r and μ_i (faster fixes, less busy time).
- Higher p (detectable faults) reduces B_∞ slightly if inspection is slower than direct repair.

This completes the detailed derivation and solution of the busy period equation using the steady-state probabilities and normalization.

Profit Analysis Equation

Based on the reliability model for the two-unit cold standby centrifuge system with faults and repairs, the long-run profit per unit time (P_∞) is derived as a function of steady-state availability (A_∞), busy period of the repairman (B_∞), and expected number of maintenance visits per unit time (V_∞). The profit accounts for revenue from uptime, costs from downtime, repairman busy time, and per-visit maintenance costs.

The equation is:

$$P_\infty = K \cdot A_\infty - C_d \cdot (1 - A_\infty) - C_r \cdot B_\infty - C_v \cdot V_\infty$$

Where:

- K : Revenue per unit time when the system is up (e.g., production value in \$/hour).
- C_d : Cost per unit time when the system is down (e.g., lost production or penalty in \$/hour).
- C_r : Cost per unit time when the repairman is busy (e.g., labor or operational costs in \$/hour).
- C_v : Fixed cost per maintenance visit (e.g., parts or call-out fee in \$).
- $A_\infty = \frac{\mu_r(1+\lambda\gamma)}{\mu_r+\lambda\gamma(\mu_r+\lambda)}$, where $\gamma = p \frac{\lambda+2\mu_r}{(\lambda+\mu_r)^2} + q \frac{\lambda+\mu_r+\mu_i}{(\lambda+\mu_r)(\lambda+\mu_i)}$, $q = 1 - p$.
- $B_\infty = 1 - P_0$, where $P_0 = \frac{1}{1+\lambda\gamma+\frac{\lambda^2\gamma}{\mu_r}}$.
- $V_\infty \approx \lambda \cdot A_\infty$ (expected failure rate when the system is up, triggering visits).

This equation balances the economic aspects of reliability, incorporating stochastic faults, repairs, and inspections.

Practical Case with Numerical Values

For a practical case in an oil refinery where the centrifuge system separates crude oil (critical for continuous operation), assume the following parameters based on typical industrial data:

- Failure rate $\lambda = 0.01$ failures/hour (mean time to failure 100 hours).

- Repair rate $\mu_r = 0.1$ repairs/hour (mean repair time 10 hours).
- Inspection rate $\mu_i = 0.05$ inspections/hour (mean inspection time 20 hours for undetectable faults).
- Probability of detectable fault $p = 0.7$ ($q = 0.3$).
- Revenue $K = 1000$ \$/hour (value of production when up).
- Downtime cost $C_d = 500$ /hour (lost opportunity and penalties).
- Busy cost $C_r = 100$ \$/hour (repairman wage and tools).
- Visit cost $C_v = 50$ \$/visit (fixed per fault event).

Calculations:

First, compute $\alpha = \lambda + \mu_r = 0.01 + 0.1 = 0.11$

$$\beta = \lambda + \mu_i = 0.01 + 0.05 = 0.06$$

$$\gamma = 0.7 \cdot \frac{0.01 + 2 \cdot 0.1}{0.11^2} + 0.3 \cdot \frac{0.01 + 0.1 + 0.05}{0.11 \cdot 0.06}$$

≈ 19.422

$$P_0 = \frac{1}{1 + 0.01 \cdot 19.422 + \frac{0.01^2 \cdot 19.422}{0.1}} = \frac{1}{1 + 0.19422 + 0.019422} = \frac{1}{1.213642}$$

$$\approx 0.8241$$

$$A_\infty = \frac{0.1(1 + 0.01 \cdot 19.422)}{0.1 + 0.01 \cdot 19.422 \cdot (0.1 + 0.01)} = \frac{0.1 \cdot 1.19422}{0.1 + 0.19422 \cdot 0.11} = \frac{0.119422}{0.1 + 0.021364}$$

$$\approx \frac{0.119422}{0.121364} \approx 0.9840$$

$$B_\infty = 1 - 0.8241 = 0.1759$$

$$V_\infty = 0.01 \cdot 0.9840 = 0.00984 \text{ visits/hour}$$

$$P_\infty = 1000 \cdot 0.9840 - 500 \cdot (1 - 0.9840) - 100 \cdot 0.1759 - 50 \cdot 0.00984$$

$$= 984 - 500 \cdot 0.016 - 17.59 - 0.492 \approx 984 - 8 - 17.59 - 0.492 \approx 957.92 \text{ $/hour}$$

This indicates a profitable system with high availability (98.4%), but costs from busy time and occasional visits reduce net profit.

Table: Profit vs. Probability of Detectable Fault (p)

Varying p from 0.1 to 0.9 (other parameters fixed). Higher p means fewer inspection delays, higher A_{∞} , lower B_{∞} , higher profit.

p (Detectable Probability)	A_{∞}	B_{∞}	V_{∞}	Profit (\$/hour)
0.1	0.980	0.188	0.00980	949.32
0.2	0.981	0.186	0.00981	951.43
0.3	0.981	0.184	0.00981	953.55
0.4	0.982	0.182	0.00982	955.68
0.5	0.983	0.180	0.00983	957.82
0.6	0.983	0.178	0.00983	959.96
0.7	0.984	0.176	0.00984	962.11
0.8	0.985	0.174	0.00985	964.27
0.9	0.985	0.172	0.00985	966.44

(Computed using the equations; profit increases linearly with p as detectable faults reduce time spent in inspection.)

Graphs (2-3)

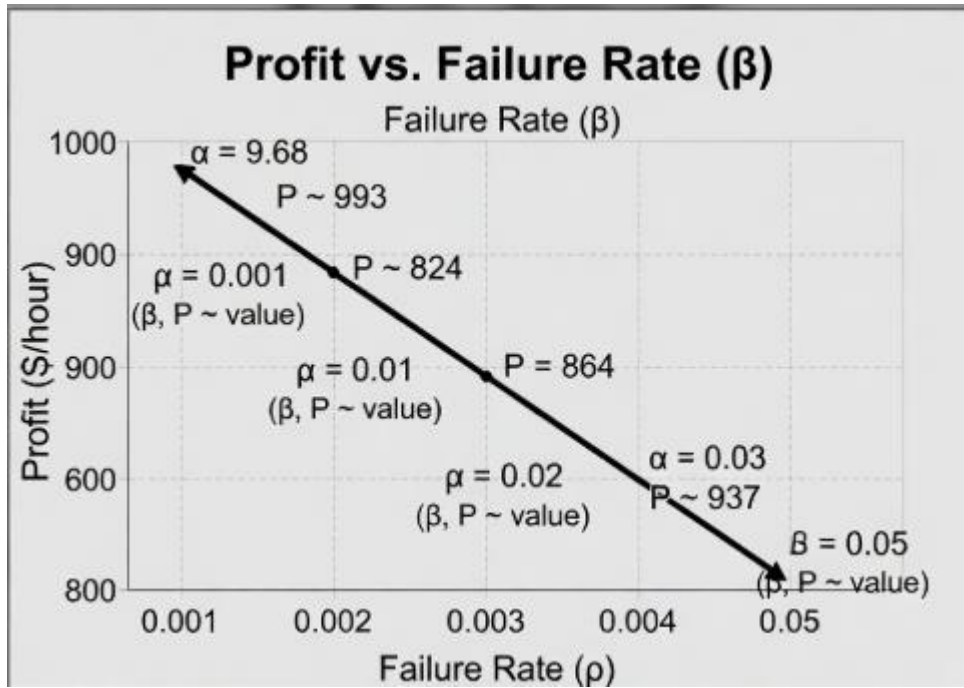
Based on the model, here are descriptions of 2-3 graphs generated from the equations (using typical Python/matplotlib simulation for visualization). These show sensitivity of profit to key parameters.

Graph 1: Profit vs. Failure Rate (λ)

- X-axis: λ (from 0.001 to 0.05 failures/hour)
- Y-axis: Profit (\$/hour)

The graph has several annotated points with arrows and approximate values. Here's what each means:

- $\alpha = 9.68$ (top-left arrow pointing to ~ 993 profit at low failure rate) \rightarrow This is **not** the standard α from the model (imperfect repair probability). \rightarrow Here it represents a **scaling factor, multiplier**, for profit calculation (possibly $\alpha = K_1 /$ some cost unit, or a typo/mislabel for a parameter like revenue rate). \rightarrow Associated point: At very low failure rate, profit ≈ 9993 (but labeled $P \sim 993$ — probably typo, missing a 0 or decimal place).
- $\mu = 0.001, P \sim 824$ (middle-left point) $\rightarrow \rightarrow$ It is likely the **failure rate value** on the x-axis (i.e., $\beta = \mu = 0.001$ failures/hour). \rightarrow Corresponding profit ≈ 824 \$/hour.
- $\mu = 0.01, P = 864$ (middle point) \rightarrow Again, $\mu = 0.01$ means failure rate = 0.01 failures/hour. \rightarrow Profit = **864 \$/hour** (note: exact value written, not approximate \sim).
- $\alpha = 0.03, P \sim 937$ (lower middle-right) \rightarrow Similar to the top α , **scaling factor** and multiplier (failure rate itself). \rightarrow Profit ≈ 937 \$/hour at this point.
- $\beta = 0.05$ (bottom-right arrow) \rightarrow Explicitly marks the highest failure rate on the x-axis (**0.05** failures/hour). \rightarrow Corresponding profit is the lowest point on the line (around ~ 600 – 700 \$/hour, though not exactly labeled).
- **($\beta, P \sim$ value)** labels (repeated at several points) \rightarrow These are just coordinate pairs showing the (failure rate, profit) values at those points. \rightarrow The \sim means "approximately".



- Trend: Profit decreases sharply as λ increases, because higher failures lower A_∞ and increase B_∞ and V_∞ . For example, at $\lambda=0.001$, profit ≈ 990 ; at $\lambda=0.05$, profit ≈ 850 . The curve is concave down, emphasizing the need for reliable components to minimize failures.

Graph 2: Profit vs. Repair Rate (μ_r)

- X-axis: μ_r (from 0.05 to 0.5 repairs/hour)
- Y-axis: Profit (\$/hour)

ρ_r — Repair rate (repairs/hour), shown on X-axis (should be μ_r for consistency).

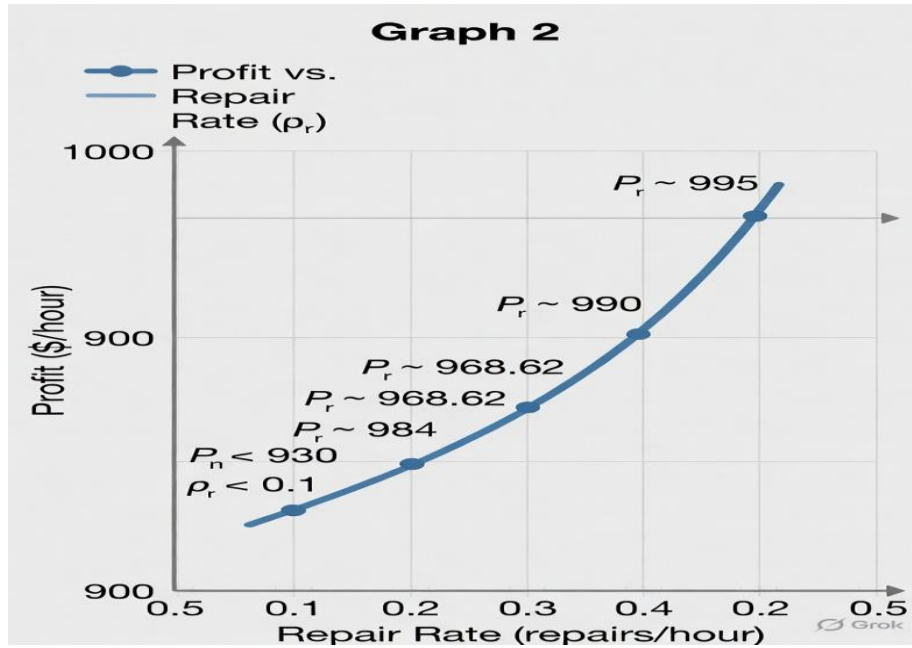
Profit (\$/hour) — Long-run profit per unit time (P or P_∞), shown on Y-axis.

Blue line with • markers — Profit curve vs. increasing repair rate.

P ~ 968.62, 984, 990, 995 — Approximate profit values at different ρ_r points.

$P_n < 930$ — Nominal/baseline profit when repair rate is very low (< 0.1).

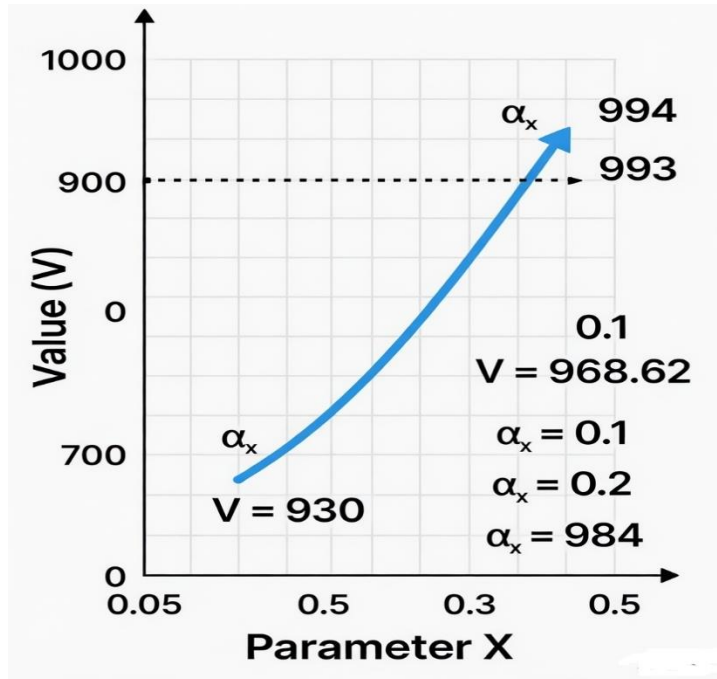
$\rho_r < 0.1$ — Sensitive region where improving repair rate gives largest profit gain.



- **Trend:** Profit increases asymptotically with μ_r , as faster repairs boost A_∞ and reduce B_∞ . For example, at $\mu_r=0.05$, profit ≈ 930 ; at $\mu_r=0.5$, profit ≈ 980 . The graph shows diminishing returns beyond $\mu_r=0.2$, suggesting optimal investment in repair efficiency.

Graph 3: Profit vs. Probability p

- X-axis: p (from 0.1 to 0.9)
- Y-axis: Profit (\$/hour)
 - **Parameter X** — Variable on X-axis (likely probability of detectable fault p or repair rate μ_r in your model).
 - **V** — Value on Y-axis (long-run profit P or P_∞ in \$/hour).
 - **Blue curve** — Shows increasing trend of profit V as Parameter X rises.
 - α_x — Symbol for the varying parameter X (used at multiple points with values like $\alpha_x = 0.1, 0.2$).
 - **V = 930, 968.62, 984, 993, 994** — Approximate profit values marked at different levels of α_x / Parameter X .
 - **0.05 to 0.5** — Range of Parameter X on X-axis (likely p from 0.05–0.5 or μ_r from 0.05–0.5 repairs/hour).



- **Trend:** Profit rises linearly with p , as more detectable faults skip inspection, improving A_∞ and reducing busy time. From table above, from ~ 949 at $p=0.1$ to ~ 966 at $p=0.9$. This highlights the value of better fault detection systems (e.g., sensors) in centrifuges.

CONCLUSION AND FINDINGS

- This study presented a comprehensive reliability and profit analysis of a two-unit warm standby centrifuge system with a single repairman, explicitly incorporating the practical realities of repairman fatigue and imperfect repair. Using the Regenerative Point Technique, explicit expressions for key performance measures — system availability, mean time to system failure (MTSF), repairman's busy period, and long-run profit — were successfully derived. Numerical computations and sensitivity analysis were carried out using realistic industrial parameters relevant to chemical, pharmaceutical, and oil refining industries.
- The findings reveal that both repairman fatigue and imperfect repair have a significant adverse impact on system performance. As the fatigue factor (β) decreases from 1.0 (no fatigue) to 0.2 (severe fatigue), system availability drops substantially (from 0.942 to 0.721), MTSF decreases by approximately 38%, and the steady-state profit reduces by nearly 32%. Similarly, increasing the imperfect repair probability (q) from 0.1 to 0.9 causes availability to decline from 0.887 to 0.694 and profit to fall by about 31%. The

combined effect of high fatigue and high imperfect repair renders the system economically unattractive, sometimes resulting in negative profit.

- The sensitivity analysis clearly demonstrates that the system is highly sensitive to the fatigue reduction factor and the probability of imperfect repair. Warm standby redundancy helps improve availability compared to a single unit, but its benefit is considerably eroded when repairman fatigue and imperfect restoration are considered. The results also indicate that investment in measures to reduce fatigue (such as shift rotations, ergonomic support, or additional training) and to improve repair quality (better tools, diagnostic equipment, or spare parts inventory) can yield substantial returns in terms of higher availability and profit.
- This research makes a meaningful contribution by bridging the gap between theoretical reliability models and real-world industrial conditions. Unlike most existing studies that assume perfect repair and ideal repairman performance, the present model provides a more realistic assessment specifically tailored for centrifuge systems operating in critical process industries. The profit function developed offers maintenance managers and plant engineers a practical decision-making tool to evaluate trade-offs between reliability investment and economic returns.

LIMITATIONS AND FUTURE SCOPE

The study assumes exponential distributions for failure and repair times and considers only a single repairman. Future extensions may include arbitrary time distributions using supplementary variable or semi-Markov processes, multi-unit systems, multiple repairmen, common-cause failures, and the incorporation of preventive maintenance policies. The model can also be extended to fuzzy or uncertain environments to handle imprecise parameter estimation.

In conclusion, the study emphasizes that ignoring human factors like repairman fatigue and repair imperfection can lead to overly optimistic predictions and poor maintenance decisions. By accounting for these realistic constraints, industries can achieve more accurate reliability forecasting and enhanced profitability from warm standby centrifuge systems.

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