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A Study on Multi-Objective Linear Fractional Programming Problem

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Abstract – This paper presents a linear fractional programming approach for solving multi objective linear programming problem. We develop a model to solve multi objective linear programming problem into fractional programming problem and proposed a method for linearization of a fractional objective.

In particular, we consider the optimization of ratio of profit and cash expenditure as fractional objective and used rest of objectives as constraints by obtaining aspiration levels. As the objectives are conflicting in nature, we used the concept of conflict and non-conflict between objectives for computation of appropriate aspiration level. The method is illustrated on a problem of agricultural production system to show its suitability.

Keywords: Fractional Programming, Multi Objective Programming Problem, Fuzzy Goal Programming.

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INTRODUCTION

In several decisions making problems in Economics and Management science, one has to optimize ratios. Such ratios arise in modeling the resource allocation, production, profit, transportation etc. In economic application these ratio like productivity, profit/cash input, return/cost etc. describe efficiency of a system. Such type of problems is inherently fractional programming problem. A variety of applications of fractional programming can be seen in a survey article Schaible.

Consider the modeling of complex system where several objectives are to be optimized at a time. For example, agricultural production system is a real life example of such complex systems comprising of multiple objectives that too of conflicting nature.

In general, modeling of an agricultural management system requires optimizing the profit cutting the cost of cultivation. Thus the core objective of the problem is to maximize the profit subject to minimization of resource requirements. But since situations are not ideal and have limited scope in production system. With changing scenario of capital intensive agricultural with mechanization and proper availability of resources, the problem changes to develop a model giving optimal profit in accordance of fulfilling resource goals.

Thus, such problems of multi objective linear programming can be better dealt with goal programming approach. Here, we deal such problem as fractional programming problem in which fractional function (profit/ cash-input) is to be optimized and other objectives are to be dealt as constraints for getting optimal solution.

REVIEW OF RELATED LITERATURE

Charnes and Cooper (Charnes, Cooper, 2012). developed a method in which linear fractional programming problem can be optimized by solving two linear programs. A comprehensive study of linear fractional programming can be viewed in Craven (Craven, 2008. Craven, Mond., 2012).

Kornbluth and Steuer (Kornbluth, Steuer, 2011. Kornbluth, Steuer, 2011) Discussed the feasibility region for linear fractional programming problems and multi objective linear fractional programming problems and considered its solution. It was Luhandjula (Chakraborty, Sandipan, 2002). who first proposed the fuzzy set theory approach to reach a satisfactory solution, which was later modified by Dutta et.al. (Dutta, *et. al.,* 2012) and further by Stancu-Minasian and Pop (Kornbluth, Steuer, 2011).

Further Nykowski and Zolkiewski gave a compromise procedure for multiple objective linear fractional programming problems.

Ohta and Yamaguchi [6], (Chakraborty, Sandipan, 2002). studied the solution of linear fractional programming problems in fuzzy environment. Many more applications of fractional programming may be obtained in literature (Kao, Liu, 2010).

A goal programming procedure for solving multi objective linear fractional programming problem has been given by Pal et.al..

Stanojevic and Stancu-Minasian studied a fully fuzzified linear fractional programming problem where all parameters and variables are fuzzy numbers.

Mehrjerdi (Charnes, Cooper, 2012) developed a fuzzy goal programming model using a linear approximation technique for solving nonlinear fractional programming problem.

Recently Zeng et.al considered a crop area planning problem and proposed a fuzzy multi objective linear programming problem with fuzzy numbers and transformed the fuzzy multi objective programming problem in an equivalent goal programming problem to crisp ones to find its solution.

The motivation of the present study is to demonstrate that converting a multi objective cropping problem of agricultural production system into a fractional programming problem lead to a better solution and optimal profit.

LINEAR FRACTIONAL PROGRAMMING APPROACH

Let there be two objectives out of k objectives such that their ratio is to be maximized as core of the problem. Let these two linear objectives are and whose ratio forms a new objective function, giving rise to linear fractional programming as

$$\text{Maximize} \quad \frac{Z_p(X)}{Z_q(X)}$$

Assuming that $Z_p(X) > 0$ *and* $Z_q(X) > 0$, therefore it is equivalent to *Maximize* $Z_p(X) - Z_q(X)$ and $Z_p(X) - rZ_q(X) \geq 0$

where r is a positive real number, which is a restriction that the ratio should always be greater than to a level r . The rest of objectives may be converted into fuzzy goals by obtaining the maximum and minimum values for constructing the membership function with a method given by Zimmermann (Chakraborty , Sandipan, 2002).

Consider a problem under situation that aspiration levels for each of the objective with respective tolerances are available with the decision maker. Then the rest of the objectives can be treated as fuzzy goals having fuzzy aspiration level.

Thus, the problem (1) can be equivalently written in a fuzzy goal programming problem as

$$\text{Maximize} \quad Z_p(X) - Z_q(X)$$

$$\text{s.t.} \quad Z_p(X) - rZ_q(X) \geq 0$$

$$\begin{aligned} Z_k(X) &\gtrsim b_k, \\ Z_s(X) &\lesssim b_s, \\ Z_t(X) &\approx b_t, \\ x_j &\geq 0; \ j = 1, 2, \dots, n. \end{aligned} \tag{2}$$

$$where\ k, s, t \in (1,2, \dots k) - \{p, q\}$$

Fuzzy goal programming approach For solution of the above fuzzy goal programming problem (5), we consider the fuzzy goal programming approach given by Zimmermann where membership function for various goal in the solution set is can be obtained as For the goal of type maximization:

$$\mu_{Z_k}(X) = \begin{cases} 1 & Z_k(X) \geq b_k \\ \frac{Z_k(X) - l_k}{(b_k - l_k)} & l_k \leq Z_k(X) \leq b_k \\ 0 & Z_k(X) \leq l_k \end{cases} \tag{3}$$

and for the goal of type minimization:

$$\mu_{Z_k}(X) = \begin{cases} 1 & Z_k(X) \leq b_k \\ \frac{u_k - Z_k(X)}{(u_k - b_k)} & b_k \leq Z_k(X) \leq u_k \\ 0 & Z_k(X) \geq u_k \end{cases} \tag{4}$$

Where is aspiration level for goal and are upper and lower tolerance limits for one of the major problems in solution of multi objective programming problem by fuzzy goal programming approach is to obtain appropriate priorities to various goals.

As a matter of fact in agricultural planning system various goals are conflicting in nature like : production and expenditure, profit and cost of cultivation etc. Thus computation of appropriate aspiration levels corresponding to for the fuzzy goals need to consider the conflict and non-conflict among various goals for realistic modeling of the problem.

CONCLUSION

The result obtained by our proposed method of using fractional programming clearly shows the superiority of achievement of various goals under taken in the study. Thus the proposed method of fractional programming using the simple linearization of fractional function into linear function, we get much

better result. Here using the proposed method we obtain a profit/ expenditure ratio of 7.05 against a ratio of 2.78 as obtained by Biswas and Pal (Biswas, Pal , 2005). Hence the developed method is suitable method for dealing such cropping problem of agricultural system for optimal profits.

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