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A Comparative Analysis on the Range of Various Geometrically Moment Maps

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Abstract – We begin by identifying a moment map in a rather general setting, and then see how the ideas work in some more specific situations. We hope to show that the moment map point of view is useful, both in understanding certain established results and also in suggesting new problems in geometry and analysis. While these analytical questions are the main motivation for the work, we will concentrate here on the formal aspects and will not make any serious inroads on the analysis.

More than twenty years back, Atiyah watched that the bend could be seen as a minute guide for the activity of the measure gathering on the space of connections. Since then, this thought of a minute guide connected to boundless dimensional symmetry gatherings underlying various-geometric issues has turned out to be extremely productive. It yields an unified perspective on numerous distinctive inquiries, and carries with it a bundle of standard hypothesis which can either be connected straight or in any event, in the deeper perspectives, prescribes what one should attempt to demonstrate. In this paper we will first overview briskly a percentage of the overall made requisitions of these thoughts in the written works.

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INTRODUCTION

This paper is a survey of some recent work concerning the generalizations of the moment map construction appropriate for various different quaternionic geometries. The non-singular versions of these constructions are a little under ten years old, but in many examples one considers, the interesting cases are often singular. It is therefore useful to have some sort of general theory covering the singular case.

In symplectic geometry, Sjamaar & Lerman [SL] have provided such a theory, showing how one obtains a stratification of arbitrary symplectic quotients by symplectic manifolds. In the quaternionic cases, such strong results are not yet known. Indeed, for one type of quaternionic geometry such a result is false, as will be described below. However, the ideas of Sjamaar & Lerman do enable one to divide up the quaternionic quotients into well-behaved pieces.

We will start by describing the quaternionic geometries involved and the non-singular versions of the moment map construction. The emphasis will be on the similarities with symplectic geometry.

THE MOMENT MAP

Throughout this paper, we assume (X, ω) to be a symplectic manifold, meaning that *X* is a manifold and ω is a nondegenerate closed 2-form. A group action of a Lie group *G* on the manifold *X* is a smooth map $\Phi : G \times X \to X$, such that $\Phi(g \cdot h, x) = \Phi(g, \Phi(h, x))$ and $\Phi(1, x) = x$. We often denote $\Phi(g, x)$ by $\Phi_g(x)$ or simply by *g* • *x.* The group properties then reads *h* • *(g* • *x) = (h* • *g)* • *x* and 1 • *x* = *x.* A group action Φ of *G* - sometimes called G-action - is called symplectic if the symplectic form is invariant under each pullback:

$$\Phi_g^*\omega = \omega \text{ for all } g \in G$$

Let $\mathfrak{g}$ be the Lie algebra of *G,* that is, the tangent space 011 *G* at the identity element. The dual of the Lie algebra is denoted by $\mathfrak{g}^*$. For $\xi \in \mathfrak{g}$ and, we sometimes use the following notation: $\ell \in \mathfrak{g}^*$

$$\langle \ell, \xi \rangle := \ell(\xi).$$

Every $\xi \in \mathfrak{g}$ induces a vector field V_ξ on *X* usually called the infinitesimal action,

which is defined by:

$$V_\xi(x) := \frac{d}{dt}\bigg|_{t=0} \exp(t\xi) \cdot x.$$

The definition of the exponential map on a Lie algebra, which is a map from g to G, can be found in the Appendix. For $\xi \in \mathfrak{g}$ and $g \in G$ the Ad-operation is defined

as follows:

$$\mathrm{Ad}_g(\xi) = \frac{d}{dt}\bigg|_{t=0} g \cdot \exp(t\xi) \cdot g^{-1},$$

which can be interpreted as a map from g to itself, and it is convenient to denote it by $\mathrm{Ad}_g(\xi) = g\xi g^{-1}$. Its adjoint, $\mathrm{Ad}_g^* : \mathfrak{g}^* \to \mathfrak{g}^*$, is given by

$$\langle \mathrm{Ad}_g^*(\ell), \xi \rangle = \langle \ell, \mathrm{Ad}_g(\xi) \rangle,$$

where $\ell \in \mathfrak{g}^*$ and $\xi \in \mathfrak{g}$

Definition 1.1. *A* moment map *for a symplectic G-action. with Lie group G on X. is a smooth map* $\mu : X \to \mathfrak{g}^*$ *such that the following properties hold.*

i) $\langle d\mu(x)v, \xi \rangle = \omega_x(V_\xi(x), v)$ *for all* $x \in X,\ v \in T_xX$ *and* $\xi \in \mathfrak{g}$, *where* V_ξ *is the infinitesimal action of* ξ.

ii) $\mu(\Phi_g(x)) = Ad^*_{g^{-1}}\mu(x)$ *holds for all* $x \in X$ *and* $g \in G$

We will call i) the moment map property and ii) equivariance. In some cases we talk about non-equivariant moment maps which are maps where only the first condition is required. We sometimes say equivariant moment map to underline that we require the equivariance property. Not every symplectic Lie group action has a moment map, but under some special assumptions we can prove its existence.

THEORETICAL BACKGROUND

Symplectic geometry was designed by Hamilton in the early nineteenth century, as a scientific system for both traditional mechanics and geometrical optics. Physical states in both settings are portrayed by focuses in a fitting stage space (the space of directions and momenta). Hamilton's comparisons cohort to any vigor capacity ("Hamiltonian") on the stage space a dynamical framework.

New systems have converted symplectic geometry into a profound and compelling subject of perfect science. One notion of symplectic geometry that has demonstrated especially suitable in numerous zones of science is the idea of a minute guide. To review the definitive setting for this idea, let M be a symplectic complex, and G a Lie aggregation following up on M by symplectomorphisms. A minute guide for this activity is an equivariant guide $\Phi : M \to \mathfrak{g}^*$ with qualities in the double of the Lie variable based math, with the property that the minute generators of the movement, relating to Lie polynomial math components $\xi \in \mathfrak{g}$, are the Hamiltonian vector fields $X_{\langle \Phi, \xi \rangle}$. The direct energy and calculated energy from traditional mechanics may be seen as minute maps, relating to translational and rotational symmetries, separately.

MINUTE MAPS AND POISSON GEOMETRY

Poisson manifolds are manifolds M furnished with a Poisson section on the variable based math of smooth capacities on M. Symplectic manifolds are exceptional instances of Poisson manifolds, where the section is given as $\{f, g\} \quad X_f(g)$.

A Poisson structure verifies a peculiar foliation (in the feeling of Sussmann) whose leaves are symplectic manifolds.

Rui Fernandes (Instituto Superior Tecnico, Lisbon) (joint work with Crainic). The Poisson section plunges to an authoritative Lie section on the space of 1-structures on any Poisson complex. Along these lines, the cotangent bunch T*m obtains the structure of a Lie algebroid. A worldwide item "combining" this Lie algebroid is a symplectic groupoid, i.e., a groupoid $S \rightrightarrows M$, where S conveys a symplectic structure such that both groupoid maps are Poisson maps, and such that the symplectic structure is perfect with the groupoid increase. Not all Poisson manifolds concede such a symplectic acknowledgement. The exact deterrents were discovered a couple of years prior by Fernandes-Crainic. In his BIRS address, Fernandes ex¬plained how this hypothesis grows to the vicinity of Poisson aggregation activities. He indicated that if M concedes a symplectic acknowledgement S, then the affected movement on S is Hamiltonian with an authoritative minute guide. (This minute guide fulfills a cocycle condition, and is a coboundary if and just if the movement on M concedes a minute guide.) Finally, Fernandez demonstrated in which sense 'symplectic realization' drives with 'reduction'.

Anton Alekseev (University of Geneva). A Poisson Lie assembly is a Lie aggregation K with a Poisson structure for which the product guide is Poisson. This condition demarcates a Lie section on the double of the Lie polynomial math $\mathfrak{k}^*$, which joins to the purported double Poisson Lie bunch K*. Assuming that K conveys the zero Poisson structure, then the double Poisson Lie assembly is $\mathfrak{k}^*$ with the Kirillov Poisson structure. A development of Lu-Weinstein demonstrates that any smaller Lie gathering K concedes an authoritative Poisson Lie bunch structure. Later, Ginzburg-Weinstein demonstrated that, thus, the double Poisson Lie bunch K* is Poisson diffeomorphic to $\mathfrak{k}^*$. Notwithstanding, no express type of such a diffeomorphism was known. Alekseev demonstrated that for the assembly K = U(n), there is a recognized and extremely cement Ginzburg-Weinstein diffeomorphism $\mathfrak{u}(n)^* \to U(n)^*$.

The verification of this consequence (which confirms a guess of Flaschka-Ratiu) is dependent upon an investigation of Gelfand-Zeitlin frameworks on u(n)* and U(n)*, separately. As a result, one gets the accompanying fascinating consequence: There is an authoritative diffeomorphism $\gamma: \mathrm{Herm}(n) \to \mathrm{Herm}^+(n)$ from hermitian networks to positive categorical Hermitian grids, with the property that the eigenvalues of the kth central submatrix of $\gamma(A)$ are the exponentials of the aforementioned of the kth central submatrix of A.

GEOMETRIC INTERPRETATION OF STABILITY

Lifting the G-action to *L* -

Given a symplectic manifold *X* with an integral symplectic form ω. We assume that a compact Lie group G acts on *M* such that a moment map μ for the action exists. Let n be the order of the torsion subgroup of the fundamental group $\pi_1(G)$, i.e. the smallest integer n such that if γ^r for an $r \in \mathbb{N}$ is contractible for a $\gamma \in \pi_1(G)$ then γ^n is also contractible. Since ω is integral we find a complex line bundle

$$x(t) := g_t x_0, \qquad s(t) := g_t s_0 \in L_{x(t)}$$

$L \to X$

with Chern class $c_1(L) = n[\omega]$. We then choose a connection ∇ such that the curvature of the connection is $-2\pi i n\omega$. In this case we can lift the G-action on *X* to a G-action on *L.*

Theorem 1.18. *Let* (X,ω),G.L *and* ∇ *be as above. Then there exists a moment map* μ *such that the action of G on X lifts to an action on* $G \times L \to L, (g,(x,s)) \mapsto (gx, gs)$ *as follows. For every* $g_t \in G$ *with go = 1. every* $x_0 \in X$ *and every* $s_0 \in L_{x0}$ *the path satisfies*

$$\nabla_t s + 2\pi i n \langle \mu(x(t)), \xi_t \rangle s = 0 \qquad \xi_t := \dot{g}_t g_t^{-1}$$

The equation (1.18) defines the group action on *L* by $gs_0 := s(1)$ where *s(t)* is a solution of the equation and $s(0) = s_0$. We must prove that this group action is well defined, i.e. if *g* = 1, then $s(1) = s(0)$

Lifting the $G^{\mathbb{C}}$-action to *L* -

If we assume that (X, J, ω) is a Kahler manifold with integral Kahler form ω we can choose *L* to be a holomorphic bundle associated with a hermitian metric and ∇ to be a hermitian connection. Then *L* is a complex manifold which can be seen by identifying the tangent space of *L* as follows

$$T_{(x_0,s_0)}L = T_{x_0}X \oplus L_{x_0}.$$

This can be done since *L* can be interpreted as the collection of L_x overall $x \in X$:

$$L = \bigcup_{x \in X} L_x$$

Each L_x is isomorphic to $\mathbb{C}$ such that *L* is locally isomorphic to an open subset of $X \times \mathbb{C}$. The tangent space description of above then follows. We will now introduce a complex structure. Define $\tilde{J}$ by

$$\tilde{J}(v, \hat{s}) = (Jv, is)$$?

which is of course an almost complex structure. That this is the right complex structure can be seen by taking local charts ϕ_α of X and local holomorphic section on L:

$$\begin{aligned}\phi_\alpha &: U_\alpha \subset X \to V_\alpha \subset \mathbb{C}^n \\ \theta_\alpha &: U_\alpha \to L.\end{aligned}$$

This defines the chart on *L*\

$$\begin{aligned}\psi_\alpha : \mathbb{C}^n \times \mathbb{C} &\to L \\ (z, \lambda) &\mapsto (\phi_\alpha^{-1}(z), \lambda\theta(\phi_\alpha^{-1}(z)))\end{aligned}$$

MINUTE MAPS AND PATH INTEGRALS

Jonathan Weitsman (Santa Cruz). Quantum field hypothesis is a hotspot for numerous energizing expectations in science, generally built however with respect to non-thorough 'functional fundamental teachniques'. A model is Witten's equations for crossing point pairings, in view of way vital computations for the Yang-Mills practical (standard square existing apart from everything else guide). In his talk, Weitsman demonstrated that at times, these way essential contentions can indeed be made thorough. The fundamental techique is another development of measures on Banach manifolds cohorted to supersymmetric quantum field speculations. As samples, he examined the Wess-Zumino-Novikov-Witten show for maps of Riemann surfaces into conservative Lie aggregations, and 3-dimensional measure hypothesis.

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