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A Study of Mathematical Theory of Systems and Control

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Abstract – A mathematical model is an exclusion law. A mathematical model expresses the opinion that some things can happen, are possible, while others cannot, are declared impossible. The problem of regulation is to design mechanisms that keep convinced to be forbidden variables at stable values against outside fighting that act on the plant that is being regulated, or changes in its properties.

Keywords: Analysis, System, Reliability .Equation, Linear, Mathematical Model.

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INTRODUCTION

Systems and control

Control theory has two main roots: regulation and trajectory optimization. The first, regulation, is the more important and engineering oriented one. The second, trajectory optimization, is mathematics based. However, as we shall see, these roots have to a large extent merged in the second half of the twentieth century. The problem of regulation is to design mechanisms that keep certain to be controlled variables at constant values against external disturbances that act on the plant that is being regulated, or changes in its properties. The system that is being controlled is usually referred to as the plant, a passé partout term that can mean a physical or a chemical system, for example. It could also be an economic or a biological system, but one would not use the engineering term “plant” in that case. Examples of regulation problems from our immediate environment abound.

Houses are regulated by thermostats so that the inside temperature remains constant, notwithstanding variations in the outside weather conditions or changes in the situation in the house: doors that may be open or closed the number of persons present in a room, activity in the kitchen, etc. Motors in washing machines, in dryers, and in many other household appliances are controlled to run at a fixed speed, independent of the load. Modern automobiles have dozens of devices that regulate various variables. It is, in fact, possible to view also the suspension of an automobile as a regulatory device that absorbs the irregularities of the road so as to improve the comfort and safety of the passengers. Regulation is indeed a very important aspect of modern technology. For many reasons, such as efficiency, quality control, safety, and reliability, industrial production processes require regulation in order to guarantee that certain key variables (temperatures, mixtures, pressures, etc.) are kept at appropriate values. Factors that inhibit these desired values from being achieved are external disturbances, as for example the properties of raw materials and loading levels or changes in the properties of the plant, for example due to aging of the equipment or to failure of some devices. Regulation problems also occur in other areas, such as economics and biology.

One of the central concepts in control is feedback. A good example of a feedback regulator is a thermostat: it senses the room temperature, compares it with the set point (the desired temperature), and feeds back the result to the boiler, which then starts or shuts off depending on whether the temperature is too low or too high.

THE SYSTEMS APPROACH

The systems approach has beginnings far back in history. But as modern systems analysis has broadened, it has already begun to be controversial and misunderstood. The systems approach has quickly attracted overly zealous proponents and, as often, misinformed detractors. Substantial disagreement exists among the professionals as to how useful the approach is for the bigger problems of society, or for smaller ones when they are more "social" than "technological." This confuses the nonprofessional as to what the approach really is. It impedes its appropriate application. Some hail it as magic, a new all-powerful tool that can demolish any tough problem, engineering or human. Of course, there are always the doubters, the mentally lazy or ignorant who are annoyed with the entry of something new. And there are some aerospace engineers who have used the systems approach but only for narrow problems in their specialized field. They often do not realize they must extend their team capabilities considerably to handle complex social-engineering problems. Some experienced systems engineers go to the other extreme, certain the discipline is inappropriate for "people" problems. In this viewpoint, they are sometimes joined by experts schooled in the more unpredictable behavior of man. Some of these more socially trained individuals are concerned that the systems approach's disciplines cannot be applied successfully to the real-life problems of the human aspects of our civilization.

The systems approach will not solve substantial problems overnight, nor will it ever solve all of them. No matter how broadly skillful is the systems team, the approach is no more than a tool. It will never give us something for nothing, or point the way to an ideal organization of all society, or lead to the planning and production of all of the products of society so as to satisfy all. It will not change the nature of man. It will provide, that is, no miracles. All it can do is help to achieve orderly, timely, and rational designs and decisions. But this "minimum" is something very important. So severe are some of our problems today that chaos threatens. The systems approach to the analysis and design of anything— from a traffic management system to a new city, from a regional medical clinic to a full hospital and medical center, from an automated fingerprint identification system to a fully integrated criminal justice system— will provide no facility of infinite capacity. But it will lead us to designs and operations that will at least not be chaotic. The systems approach, if it is used wisely, is, at the least, a cure for chaos.

SYSTEM DESIGN, A NECESSARY STEP TO COMPONENT AVAILABILITY

The systems approach is flattering vital for still another reason. Without a good systems analysis and system design as a first step, or at least as a parallel effort, it is not easy to understand and specify the necessary pieces of the solution. If the parts required are not called out, no one will set out to make them available. These components, which the systems design will bring together into a pleasant-sounding ensemble to meet the problem, include many items: needed equipment and materiel; people trained in specific jobs with spelled-out functions and procedures; the right kind of information, stored and flowing, so that the people and the things know what to do and where to be to make the whole system operate. For example, the need for systems works to tell us what mechanism we need. For educational system we know that we must greatly enhance educational resources and techniques to provide for more and better education for more young people, for retraining of adults for new jobs, and for growth of the abilities of most of us to keep pace with the requirements of the society. We particularly need a massive rise in educational potency in poverty areas

Now, to meet these needs, we have reason to suppose that technological aids can be very important to extend the effort of the human educator much as X - rays and electrocardiographs and blood tests assist the physician. These aids include special films, closed circuit TV, electronic language laboratories, computer-based education and training programs, and other equipment for the presenting of educational material, the handling of data and information, and for assisting the educator and administrator in planning, analysis of results, and research. But what specific technological devices will accomplish exactly what within what educational framework? If computer-based teaching machines are to

be installed, how are they to be used so as to yield real advantages instead of perhaps the disadvantage of creating a sort of robot teacher or evolving to a simple source of entertainment? To answer, we must think such things as the psychology and principles of teaching, the choice of what is to be taught, and how the results will be measured. The actual hardware and software design of some new teaching devices may be the easiest part of the system manufacturing, once we really see what we need.

MATHEMATICAL MODEL

A mathematical model is an exclusion law. A mathematical model expresses the opinion that some things can happen, are possible, while others cannot, are declared impossible. Thus Kepler claims that planetary orbits that do not satisfy his three famous laws are impossible. In particular, he judges no elliptical orbits as unphysical. The second law of thermodynamics limits the transformation of heat into mechanical work. Certain combinations of heat, work, and temperature histories are declared to be impossible. Economic production functions tell us that certain amounts of raw materials, capital, and labor are needed in order to manufacture a finished product: it prohibits the creation of finished products unless the required resources are available. We formalize these ideas by stating that a mathematical model selects a certain subset from a universe of possibilities. This subset consists of the occurrences that the model allows, that it declares possible. We call the subset in question the behavior of the mathematical model.

Definition A mathematical model is a pair (U,B) with U a set, called the universe—its elements are called outcomes—and B a subset of U, called the behavior.

Example 1

Economists believe that there exists a relation between the amount P produced of a particular economic resource, the capital K invested in the necessary infrastructure, and the labor L expended towards its production. A typical model looks like U =

R^3_+ and B = {(P,K,L) ∈R^3_+ | P = F(K,L)}, where F : R^2_+ → R+ is the production function.

Typically, F : (K,L) 7→ $\alpha K^\beta L^\gamma$, with α, β, γ ∈ R+, 0 ≤ β ≤ 1, 0 ≤ γ ≤ 1, constant parameters depending on the production process, for example the type of technology used. Before we modeled the situation, we were ready to believe that every triple (P,K,L) ∈ R^3_+ could occur. After introduction of the production function, we limit these possibilities to the triples satisfying $P = \alpha K^\beta L^\gamma$. The subset of R^3_+ obtained this way is the behavior in the example under consideration.

Example 2

During the ice age, shortly after Prometheus stole fire from the gods, man realized that H2O could appear, depending on the temperature, as liquid water, steam, or ice. It took a while longer before this situation was captured in a mathematical model. The generally accepted model, with the temperature in degrees Celsius, is U ={ice, water, steam}× [−273,∞) and B = (({ice}×[−273, 0])∪({water }×[0, 100])∪({steam}×[100,∞)).

SYSTEMS DEFINED BY LINEAR DIFFERENTIAL EQUATIONS

A very common class of dynamical systems consists of the systems that are:

- Linear
- Time-invariant
- described by differential (or, in discrete time, difference) equations.

The importance of such dynamical systems stems from at least two aspects. First, their prevalence in applications, indeed, many models used in science and (electrical, mechanical, chemical) engineering are by their very nature linear and time-invariant. Secondly, the small signal behavior of a nonlinear time-invariant dynamical system in the neighborhood of an equilibrium point is time-invariant and approximately linear. The process of substituting the nonlinear model by the linear one is called linearization.

Linear systems lend themselves much better to analysis and synthesis techniques than nonlinear systems do. Much more is known about them. As such, the theory of linear systems not only plays an exemplary role for the nonlinear case, but has also reached a much higher degree of perfection. The systems under consideration are those described by linear constant-coefficient differential equations. Dynamical system is determined by its behavior. The systems of differential equations that can be transformed into each other by premultiplication by a unimodular matrix represent the same behavior. Conversely, we will investigate the relation between representations that define the same behavior. It turns out that under a certain condition such differential equation representations can be transformed into each other by means of premultiplication by a suitable unimodular matrix.

CONCLUSION:

The reliability and availability analysis of process industries can benefit in terms of higher production, lower maintenance costs. The Availability of complex systems and continuous process industries can be enhanced by considering maintenance, inspection, repairs and replacements of the parts of the failed units. A mathematical model expresses the opinion that some things can happen, are possible, while others cannot, are declared impossible.

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