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NUCLEAR SHELL MODEL AND SPIN-ORBIT INTERACTION

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Nuclear Shell Model and Spin-Orbit Interaction

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INTRODUCTION

In the liquid drop model we have emphasized the properties of nuclear matter have said nothing about single nucleons. This is a great departure from the atomic model where the emphasis is on the motion of the electron in the field provided by the nucleus. Now questions are: Can nucleons be existed in well-ordered controlled nuclear shell? Is there any evidence for the grouping of nucleons into shells? Can quantum number similar to n, l, s, j be applied to the nucleus? For certain numbers of neutrons or protons, *called magic numbers*, nuclei exhibit special characteristics of stability reminiscent of the properties shown by noble gases among the atoms. Nuclei in which either *N or Z* is equal to one of these magic numbers (2, 8, 20, 28, 50, 82, 126, 184) show certain particulars that are not understandable in terms or the liquid drop model. We will see that the magic numbers of the nucleons can be explained with a shell model of the nucleus. Evidently protons and neutrons in the nucleus are not all equivalent as had been assumed in introducing the liquid drop model.

The shell model is partly analogous to the atomic shall model which describes the arrangement of electrons in an atom, in that a filled shall result in greater stability. When adding nucleons (protons or neutrons) to a nucleus, there are certain point where t5he binding energy of the next nucleon is significantly less than the last one. This observation, that there are certain magic numbers of nucleons: 2, 8, 20, 28, 50, 82, 126 which are more tightly bound than the next higher number, is the origin of the shell model.

Note that the shells exist for both protons and neutrons individually, so that we can speak of “magic nuclei” where one nucleon type is at a magic number, and “double magic nuclei”, where both are. Due to some variation in orbital filling, the upper magic are 126 and, speculatively, 184 for neutrons but only 114 for protons, playing a role in the search or the so-called island or stability. There have been found some semi magic numbers, notably Z=40 giving nuclear shell filling for the various elements; 16 may also be a magic number. In order to get these numbers, the nuclear shell model starts from an average potential with a shape something between the square well and the harmonic oscillator. To this potential a spin orbit term is added. Even so, the total perturbation does not coincide with experiment and an empirical spin orbit coupling, named the Nilsson Term and must be added with at least two or three different values of its coupling constant, depending on the nuclei being studied.

Nevertheless, the magic numbers of nucleons, as well as other properties, can be arrived at by approximating the model with a three- dimensional harmonic oscillator plus a spin-orbit interaction. A more realistic but also complicate potential is known as Woods Saxon potential.

IGAL TALMI developed a method to obtain the information from experimental data and use it to calculate and predict energies which have not been measured. This method has been successfully used by many nuclear physicists and led to deeper understanding of nuclear structure. The theory which gives a good description of these properties was developed. This description turned out to furnish the shell model basis of the elegant and successful Interaction boson model.

0	0	0	½
			-½
1	1	1	½
			-½
		0	½
			-½
		1	½
			-½

We can imagine ourselves building a nucleus by adding protons and neutrons. These will always fill the lowest available level. Thus the first two protons fill level zero, the next six protons fill level one, and so on. As with electrons in the periodic table, protons in the outermost shell will be relatively loosely bound to the nucleus if there are only few protons in that shell, because they are farthest from the center of the nucleus. Therefore nuclei which have a full outer proton shell will have a higher binding energy then other nuclei with a similar total no. of protons. All this is true for neutrons as well.

This means that the magic no. are expected to be those in which all the occupied shell are full. We see that for the first two no. we get (level 0 full) and 8 (level 0 and 1 full), in accord with experiment. However the full set of magic no. does not turn out correctly.

These can be computed as follows:

In three dimensional harmonic oscillator the total degeneracy at level n is $\frac{(n+1)(n+2)}{2}$

Thus the magic numbers would be

$$\sum_{n=0}^{k} (n+1)(n+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

for all integer k. this gives the following magic no. 2, 8, 20, 40, 70, 112…., which agree with experiment only in the first three entries. Note that these no. are twice the tetrahedral no. (1, 4, 10, 20, 35, 56…,) from the Pascal Triangle.

In particular, the first six shells are:

- Level 0 :2 states (l = 0) = 2.
- Level 1 :6 states (l = 1) = 6.
- Level 2 :2 states (l = 0) = +10 states (l = 2) = 12
- Level 3 :6 states (l = 1) = +14 states (l = 3) = 20
- Level 4 :2 states (l = 0) = +10 states (l = 2) +18 states (l = 4) = 30
- Level 5 :6 states (l = 1) = +14 states (l = 3) +22 states (l = 5) = 42

Where for every l there are 2l+ 1 different values of m_l and 2 values of m_s, giving a total of 4l+ 2 states for every specific level.

These no. are twice the values of triangular no. from the Pascal Triangle:

1, 3, 6, 10,15,21…,

INCLUDING A SPIN- ORBIT INTERACTION

We next include a spin- orbit interaction. First we have to describe the system by the quantum no. j, m_j and parity instead of l, m_l and m_s, as in the Hydrogen- like atom. Since every even level includes only even values of l, it includes only states of even (positive) parity; similarly every odd level includes only states of odd (negative) parity. Thus we can ignore parity in counting states. The first six shells, described by the new quantum no. are:-

- Level 0 (n= 0): 2 states (j =½) . Even parity.
- Level 1 (n= 1): 2 states (j =½) + 4 states (j =$^3/_2$) = 6. Odd parity.
- Level 2(n= 2): 2 states (j =½) + 4 states (j =$^3/_2$) +6 states (j =$^5/_2$) =12. Even parity
- Level 3 (n= 3): 2 states (j =½) + 4 states (j =$^3/_2$) +6 states (j =$^5/_2$) +8 states (j= $^7/_2$) =20. Odd parity.
- Level 4 (n= 4): 2 states (j =½) + 4 states (j =$^3/_2$) +6 states (j =$^5/_2$) +8 states (j= $^7/_2$)+ 10 states (j=$^9/_2$) = 30. Even parity.
- Level 5 (n= 5): 2 states (j =½) + 4 states (j =$^3/_2$) +6 states (j =$^5/_2$) +8 states (j= $^7/_2$)+ 10 states (j=$^9/_2$) + 12 states (j=$^{11}/_2$) = 42. Odd parity.

Where for every j there are 2j+1 different state from different values of m_j.

Due to the spin orbit interaction the energies of states of the level but with different j will no longer be identical. This is because in the original quantum no., when ŝ is parallel to î, the interaction energy is negative; in this case j = l + s = l + ½. When ŝ is anti-parallel to î (i.e. aligned oppositely), the interaction energy is positive, and in this case j=l-s=l-½. Furthermore, the strength of the interaction is roughly proportional to l.

For example, consider the states at level 4:

- The 10 states with j= $^9/_2$ come from l=4 and s parallel to l. thus they have a negative spin orbit interaction energy.
- The 8 states with j= $^7/_2$ come from l=4 and s anti- parallel to l. thus they have a positive spin orbit interaction energy.
- The 6 states with j= $^5/_2$ come from l=2 and s parallel to l. thus they have a negative spin orbit interaction energy. However its magnitude is half compared to the states with j=$^9/_2$.
- The 4 states with j= $^3/_2$ come from l=2 and s anti- parallel to l. thus they have a positive spin orbit interaction energy. However its magnitude is half compared to the states with j=$^7/_2$.

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- The 4 states with $j= {}^3/_2$ come from l=2 and thus have zero spin orbit interaction energy.

DEFORMING THE POTENTIAL

The harmonic oscillator potential $V(r) = \mu\omega^2 r^2/2$ grows infinitely as the distance from the center r goes to infinity. A more realistic potential, such as Woods Saxon potential, would approach a constant at this limit. One main consequence is that the average radius of nucleons orbits would be larger in a realistic potential; This leads to a reduced term $h^2 l(l+1)/2mr$ in the Laplace operator of Hamiltonian. Another main difference is that orbits with high average radii, such as those with high n or high l, will have a lower energy than in a harmonic oscillator potential. Both effects lead to a reduction in the energy levels of high l orbits.

PREDICTED MAGIC NUMBERS

Low lying energy levels in a single particle shell model with an oscillator potential (with a small negative l^2 terms) without spin orbit (left) and with spin orbit (right) interaction. The no. to the right of a level indicates its degeneracy, (2j+1). The boxed integers indicate the magic no..

Together with the spin orbit interaction, and for appropriate magnitudes of both effects, one is led to the following qualitative picture: At all levels, the highest j states have their energies shifted downwards, especially for high n (where the highest j is high). This is both due to the negative spin orbit interaction energy and to the reduction in energy resulting from deforming the potential to a more realistic one. The second to highest j states, on the contrary, have their energy shifted up by the first effect and down by the second effect, leading to a small overall shift. The shift in the energy highest j states can thus bring the energy of states of one level to be closer to the states of a lower level. The "shells" of the shell model are then no longer identical to the levels denoted by *n*, and the magic no. are changed.

We may then suppose that the highest *j* states for *n*= 3 have an intermediated energy between the average energies of *n*= 2 and *n* =3, and suppose that the highest *j* states for larger *n*(at least up to *n* =7) have an energy closer to the average energy of *n* -1. Then we get the following shells (see the figure)

- 1st shell:02 states ($n=0$, $j={}^1/_2$).
- 2nd shell:06 states($n=1$,$j= {}^1/_2$ or ${}^3/_2$).
- 3rd shell:12 states ($n=2$, $j={}^1/_2$, ${}^3/_2$ or ${}^5/_2$).
- 4th shell:08 states ($n=3$, $j={}^1/_2$, $j= {}^7/_2$).
- 5th shell:22 states ($n=3$, $j={}^1/_2$, ${}^3/_2$ or ${}^5/_2$; $n=4$, $j={}^9/_2$).
- 6th shell:32 states ($n=4$, $j={}^1/_2$, ${}^3/_2$, ${}^5/_2$ or ${}^7/_2$; $n=5$, $j={}^{11}/_2$).
- 7th shell:44 states ($n=5$, $j={}^1/_2$, ${}^3/_2$, ${}^5/_2$,${}^7/_2$ or ${}^9/_2$; $n=5$, $j={}^{13}/_2$).
- 8th shell:58 states ($n=6$, $j={}^1/_2$, ${}^3/_2$, ${}^5/_2$,${}^7/_2$,${}^9/2$ or ${}^{11}/_2$;$n=7$, $j={}^{15}/_2$).

And so on.

The magic numbers are then

- □□2
- □□8= 2+6
- □20= 2+6+12
- □28= 2+6+12+8
- □50= 2+6+12+8+22
- □82= 2+6+12+8+22+32
- 126= 2+6+12+8+22+32+44
- 184= 2+6+12+8+22+32+44+58

And so on. This gives all the observed magic no., and also predicts a new one (the so-called *island of stability*)at the value of 184 (protons, the magic no. 126 has not been observed yet, and more complicated theoretical considerations predict the magic no. to be 114 instead).

Another way to predict magic (and semi-magic)no. is by laying out the idealized filling order (with spin-orbit splitting but energy levels not overlapping). For consistency s is split into $j={}^1/_2$ and $-{}^1/_2$ components with 2 and 0 members respectively. Taking leftmost and rightmost total counts within sequences marked bounded by / here gives the magic no..

- s(*2,0*) / p(4,*2*)>2,2/6,8,so (semi) magic no. 2,2/6,8
- d(6,4):s(2,0)/ f(8,6):p(4,2)> 14,18:20,20/28,34:38,40, so 14,20/28,40
- g(10,8): d(6,4):s(2,0)/h(12,10): f(8,6):p(4,2)> 50,58,64,68,70,70/82,92,100,106,110,112,so 50,70/82,112

- i(14,12): g(10,8): d(6,4):s(2,0)/j(16,14): h(12,10): f(8,6):p(4,2)> 126,138,148,156,162,166,168,168/184,198,210,220,228,234,238,240,so126,168/184,240

note that the rightmost predicted magic no. of each pair within the quartets bisected by / are double tetrahedral no. from the Pascal Triangle: 2,8,20,40,70, 112, 168,240,are2x 1,4,10,20,35,56,84,120…., and the leftmost no. of the pairs differ from the rightmost by double triangular no. 2-2=0,8-6=2,20-14=6, 40-28=12, 70-50=20,112-82=30,168-126=42,240-184=56, where 0,2,6,12,20,30,42,56….,are2x 0,1,3,6,10,15,21,28…,

OTHER PROPERTIES OF NUCLEI

This model also predicts or explain with some success other properties of nuclei, in particular spin and parity of nuclei ground states, and to some extent their excited states as well.

80as an example – its nucleus has eight protons filling the three first protons 'shell', eight neutrons filling the three first neutrons 'shell', and one extra neutron. All protons in a complete protons shell have total angular momentum zero, since their angular momentum cancel each other; The same is true for neutrons. All protons in the same level (*n*) have the same parity (either +1or -1), and since the parity of a pair of particle is the product of their parities, an even no. of protons from the same level (*n*) will have +1 parity. Thus the total angular momentum of the eight protons and the first eight neutrons is zero, and their total parity is +1. this means that the spin (i.e. angular momentum) of the nucleus, as well as its parity, are fully determined by that of the ninth neutron. This one is the first (i.e. lowest energy) states of the 4^{th} shell, which is a d-shell (*l* = 2), and since p=(-1)^1, this gives the nucleus an overall parity of +1. it should also be noted that 4^{th} d-shell has a $j = {}^5/_2$. thus the nucleus of 17, 80 is expected to have positive parity and spin ${}^5/_2$, which indeed it has.

The rule for the ordering of the nucleus shells are similar to Hund's Rules of the atomic shells, however, unlike its use in atomic physics the completion of a shell is not signified by reaching the next *n*, as such the shell model cannot accurately predict the order of exited nuclei states, through it is very successful in predicting the ground states. The order of the first few terms are listed as follows: 1s, $1p^3/_2$,$1p^1/_2$, $1d^5/_2$,2s, $1d^3/_2$….For further clarification on the notation refer to the article on the Russell- Saunders terms symbol.

For nuclei farther from the magic no. one must add the assumption that to the relation between the strong nuclear force and angular momentum, protons or neutrons with the same *n* tend to form pairs of opposite angular momentum. Therefore a nucleus with an even no. of protons and an even no. of neutrons has 0 spin and positive parity. A nucleus with an even no. of protons and as odd no.of neutrons (or vice versa) has the parity of the last neutron (or protons). By "last" we mean the properties coming from the highest energy level. In the case of a nucleus with an odd no. of protons and an odd no. of neutrons, one must consider the total angular momentum and parity of both the last neutrons and the last proton. The nucleus parity will be a product of theirs, while the nucleus spin will be one of the possible results of the sum of their angular momentum (with other possible result being excited states of the nucleus).

The ordering of angular momentum levels within each shell is according to the principles described above – due to spin orbit interaction, with high angular momentum states having their energies shifted downwards due to the deformation of the potential (i.e. moving from a harmonic oscillator potential to a more realistic one). For nucleon pairs, however, it is often energetically favorable to be at high angular momentum, even if its energy level for a single nucleon would be higher. This is due to the relation between angular momentum and the strong nuclear force.

Nuclear magnetic momentum is partly predicted by this simple version of the model. The magnetic momentum is calculated through *j*, l and s of the "last" nucleon, but nuclei are not in states of well-defined l and s. furthermore, for odd-odd nuclei, one has to consider the two "last" nucleons, as in deuterium. Therefore one gets several possible answers for the nuclear magnetic moment, one for each possible combined l and s states, and the real states of the nucleus is a superposition of them. Thus the real (measured) nuclear magnetic moment is somewhere in between the possible answers.

The electric dipole of a nucleus is always zero, because its ground state has a definite parity, so its matter density where is the wave function is always invariant under parity. This is usually the situations with the atomic electric dipole as well.

Higher electric and magnetic multipole moments cannot be predicted by this simple version of the shell model, for the reasons similar to those in the case of the deuterium.

The study of nuclear properties shows evidence of nuclear shells analogous to those observed in the atoms. One clear piece of evidence in the nuclear case is the sharp discontinuity in nucleon separation energies for certain numbers of N and Z known as magic numbers. In the case of the electronic shells in atoms the picture is very clear, since there is a central Coulomb potential, due to the charge carried by the nucleons and electrons. In the case of the nucleus there is no such external potential but the nucleons move in the potential created by them. This potential contains many terms: central, spin-orbit, and tensor, spin spin, etc. At long distances it has a Yukawa form while at short distances it shows an extremely repulsive core. The idea of a shell model for the nucleus may seem contradictory with these strong correlations because this rudely breaks the independent particle picture. We shall consider the nucleus as composed of Z protons and N neutrons, that interact via two-body forces and obey the Schrödinger equation, the general time independent form of which is

$$(-h^2/2\,m\,\Delta^2+v)\,|\psi\;\rangle = E\,|\psi\rangle$$

Where V is the potential and $|\psi\rangle$ is the wave function with an associated energy E. The experimental idea of magic numbers led M. Goeppert-Mayer and H. Jensen to the construction of the nuclear mean field, a harmonic oscillator, whose main novelty was the very strong spin-orbit splitting needed to explain the experimental magic numbers. This idea originates from atomic physics in which the magnetic moment of an electron interacts with a magnetic field generated by its motion around the nucleus.

$$\mathbf{V(r) = ½\, m\, \omega^2 r^2 + Dl^2 - Cl.s}$$

Where $1/2\ m\ \omega^2\ r^2$ is the kinetic energy of an harmonic oscillator with frequency ω and mass m, l is the orbital angular momentum operator, s is the spin operator, D and C are constants to fit and where,

$$\mathbf{l.s = -1/2\,(j^2 - l^2 - s^2) = -1/2\,(j(j+1) - l\,(l+1) - 3/4)}$$

$$\mathbf{= 1+1 \text{ for } j = l - ½}$$

$$\mathbf{= -1 \text{ for } j = l + ½}$$

The single-particle levels of the nuclear mean field are represented in Fig. The left-hand side shows the shell structure of the isotropic harmonic oscillator, then the splitting due to the l^2 term and finally the single-particle levels taking into account the spin-orbit splitting. To the right are the predicted magic numbers. Therefore, due to the lst term in the potential, the total degeneracy becomes 2j+l. This means that, for example, a 1p level, with a total degeneracy of 2(2l+1) = 6, will split into two levels according to Equation 1p1/2 and 1p3/2 with degeneracy 2 and 4 respectively.

For a given nucleus (N, Z) the mean field dictates which levels are occupied. However, these states can be close enough in energy or have a structure such that the residual two body interaction can mix them to produce correlated states. Therefore, the infinite set of mean field orbits will be divided in three parts:

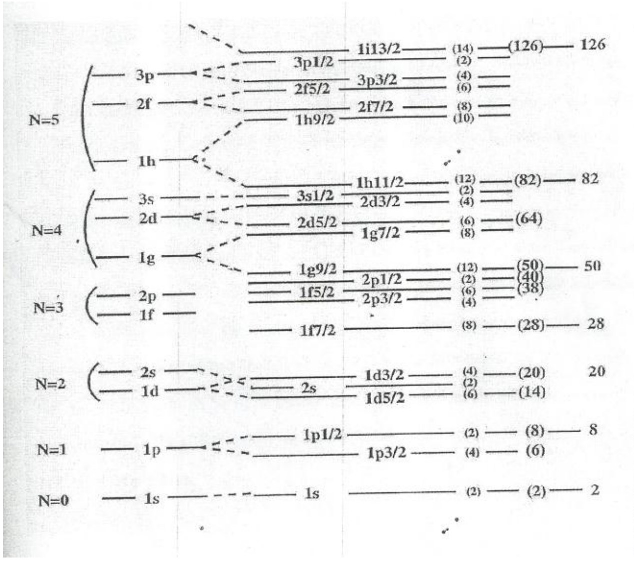

Figure The schematic structure of the single particle orbital's resulting from the simple, spherical nuclear mean field

Inert core: the orbits that are forced to be always full. Imagine that the core consists of Nc neutrons and Zc protons, thus if we are studying a nucleus (N,Z) there will remain nv = N –Nc valence neutrons and zv =Z-Zc valence protons.

Valence space: The orbits available to the valence partials, that will partially occupied by them according to the effective interaction.

External space: The remaining orbits that are always empty.

This section focuses on the description of an effective shell model developed by the Saitama group. In a shell model calculation it is necessary to include all the relevant orbits to describe a nucleus, but since this is not always feasible for medium-heavy nuclei, various truncation schemes are commonly used. To determine which orbitals should be included and make feasible the calculations, physical arguments are considered. The principal case of study of this these n terms of the shell model is $^{136}{}_{56}Ba_{80}$ and the following considerations will be made for this nucleus. To truncate the model space, the proton single-particle orbital's involved in the calculations are restricted to the three orbitals, $^0g_{7/2}$, $^1d_{5/2}$ and $^0h_{11/2}$. The neutron single-particle orbital's include all of the five orbitals between the N = 50 and 82 shell, i.e. the $^1d_{3/2}$, $^0h_{11/2}$, $^2s_{1/2}$, $^1d_{5/2}$ and $^0g_{7/2}$. The single-particle energies are extracted from experiment. The effective shell-model Hamiltonian is written as,

$$\mathbf{H = H\nu + H\pi + H\nu\pi}$$

Where Hν, Hπ, and Hνπ represent the neutron interaction, the proton interaction and the neutron-

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proton interaction respectively. The interaction among like nucleons H(ı = ν or π) consists of spherical single-particle energies, a monopole-pairing interaction, a quadrupole-pairing interaction, a quadrupole-pairing interaction, a quadrupole-quadrupole interaction, a hexa decapole-pairing and a hexa decapole-hexadecapole interaction. The strengths of these interactions are determined so as to reproduce the corresponding experimental energies of single-closed-shell nuclei. A detailed description of these interactions can be found in Ref. The definition of the HP and HH interactions are extensions of the QP and the QQ interactions from angular momentum coupling two to four, but no radial dependence is assumed.

The transition rates between levels are studied using the resultant shell-model wave functions. To study the basic structure of the levels in ^{136}Ba and to keep the basis to a reasonable truncation, the pair-truncated shell model approach has been used. This approach is very similar to the interacting boson model in concept, but the bosons are now replaced by correlated nucleon pair to treat Pauli effects explicitly. In addition to the S: J = 0 pairs, the truncated valence space only allows pair excitations of the following type, D: J = 2, G: J = 4 and H. note that the calculation is limited to a single H pair that can only be formed by the coupling of two $h_{1/2}$. Proton particles or neutron holes to angular momentum J = 0, 2, 4, 6, 8, 10. in contrast, the other pairs in this model space are collective and can be made from linear combinations of other angular momentum couplings between pairs of nucleons in different single-particle orbital's. All these pairs have positive parity so that only positive parity states are predicted. The PTSM model allows the study of the structure of the levels in terms of the expectation number of pairs.

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