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Analysis on Effect of Tensile Force on The Fundamental Frequency of Coupled Vibrations of A Straight Rotating Slender Beam Linearly Varying Channel Cross Section Under Aerodynamic Couplings

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Abstract – Coupled vibrations of twisted rotating slender beam linearly varying channel cross section under aerodynamic couplings including the effect of tensile force in a centrifugal force field have been studied by Sharma [21]. The special case of straight beam (zero twist) is of considerable practical importance and merits an independent treatment. This paper presents the analysis of this special case which was found amenable to a different form of solution resulting in a saving numerical work involved.

Keywords: Tensile Force, Frequency, Vibrations

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INTRODUCTION

The analysis presented in this chapter considers vibrations of a beam that could represent a blade of simple geometry. The beam is attached to a disc of radius r_0 and the disc rotates with the angular velocity Ω (fig.1) the cross section of the beam is linearly varying and shear centre of each cross section does not coincides with the centre of gravity, consequently the torsional and bending oscillations are coupled. Since the cross section of the beam varies S, I and J are the functions of x.

DIFFERENTIAL EQUATION

The governing differential eqns. for coupled bending and torsional vibrations taken from Tomor and Dhole (22) are

$$E\frac{\partial^2}{\partial x^2}\left[I_x\left(\frac{\partial^2 V}{\partial x^2}\right)\right] = -\rho S_x\frac{\partial^2}{\partial t^2}(V+\delta_x\theta)+\frac{\partial^2 M}{\partial x^2}$$

$$G\frac{\partial}{\partial x}\left(J_x\frac{\partial\theta}{\partial x}\right)-\frac{\partial}{\partial x}\left(C_x\frac{\partial^3\theta}{\partial x^3}\right) = \rho S_x\delta_x\frac{\partial^2}{\partial t^2}(V+\delta_x\theta)+I_{\theta_x}\frac{\partial^2\theta}{\partial t^2} \qquad (1)$$

Also in a steady flow of speed U, the blade will have some deformation due to aerodynamic force. Then the above differential eqns. become

$$E\frac{\partial^2}{\partial x^2}\left[I_x\left(\frac{\partial^2 V}{\partial x^2}\right)\right]+\rho S_x\frac{\partial^2}{\partial t^2}(V+\delta_x\theta)+\frac{\partial^2 M}{\partial x^2}+L=0$$

$$G\frac{\partial}{\partial x}\left(J_x\frac{\partial\theta}{\partial x}\right)-\frac{\partial}{\partial x}\left(C_x\frac{\partial^3\theta}{\partial x^3}\right)-\rho S_x\delta_x\frac{\partial^2}{\partial t^2}(V+\delta_x\theta)-I_{\theta_x}\frac{\partial^2\theta}{\partial t^2}+N=0 \qquad (2)$$

Where C_x is the warping rigidity and $\frac{\partial^2 M}{\partial x^2}$, L and N are given by

$$\frac{\partial^2 M}{\partial x^2} = m\Omega^2\left[\left\langle r_0(l-x)+\frac{1}{2}(l^2-x^2)\right\rangle\frac{\partial^2 h}{\partial x^2}-(r_0+x)\frac{\partial h}{\partial x}\right]$$

$$L=\frac{\rho' U^2}{2}C_{C_L}$$

$$N=\frac{\rho' U^2}{2}c^2\left(C_N+\frac{x_0}{c}C_L\right)$$

The coefficient C_L and C_N are the lift and moment coefficient about the leading edge which are expressed as

$$C_L = \frac{dC_L}{d\theta}\left[\theta + \frac{1}{U}\frac{dV}{dt} + \frac{1}{U}\left(\frac{3}{4}c - x_0\right)\frac{d\theta}{dt}\right]$$

and

$$C_N = -\frac{c\pi}{8U}\frac{d\theta}{dt} - \frac{1}{4}C_L$$

Also,

$$\Lambda = \left(1 - \frac{S_L}{S_0}\right)$$

The eqns. (2) when the effect of tensile force F is taken into consideration reduce to

$$E\frac{\partial^2}{\partial x^2}\left[I_x\left(\frac{\partial^2 V}{\partial x^2}\right)\right] + \rho S_x \frac{\partial^2}{\partial t^2}(V + \delta_x\theta) - \frac{\partial^2 M}{\partial x^2} + L - F\frac{\partial^2 V}{\partial x^2} = 0$$

and (3)

$$G\frac{\partial}{\partial x}\left(J_x \frac{\partial\theta}{\partial x}\right) - \frac{\partial}{\partial x}\left(C_x \frac{\partial^3\theta}{\partial x^3}\right) - \rho S_x \delta_x \frac{\partial^2}{\partial t^2}(V + \delta_x\theta) + N - I_{\theta_x}\frac{\partial^2\theta}{\partial t^2} = 0$$

where

$$I_x = I_0\left(1 - \frac{x}{l}\Lambda\right)^3$$

$$J_x = J_0\left(1 - \frac{x}{l}\Lambda\right)$$

$$C_x = C_0\left(1 - \frac{x}{l}\Lambda\right)^5$$

$$I_{\theta x} = I_{\theta_0}\left(1 - \frac{x}{l}\Lambda\right)^3$$

$$\delta_x = \delta_0\left(1 - \frac{x}{l}\Lambda\right)$$

The equations are now put in terms of dimensionless variable $\xi = \frac{x}{L}$ = and using the following substitutions

$$\beta^2 = \frac{EI_0}{\rho S_0 l^4},$$

$$K_2 = \left(\frac{3}{4} - \frac{x_0}{c}\right)c$$

$$r^2 = \frac{GJ_0}{\rho S_0 l^2}$$

$$K_3 = \frac{\rho' c^2}{2\rho S_0}\frac{c\pi}{8}$$

$$I_\theta' = \frac{I_{\theta_0}}{\rho S_0}$$

$$K_4 = \left(\frac{x_0}{c} - \frac{1}{4}\right)c$$

$$K_1 = \frac{\rho' C}{2\rho S_0}\frac{dC_L}{d\theta}$$

$$K^5 = \frac{F}{\rho S_0 l^2}$$

The eqn. (3) become

$$\beta^2\left[(1-\Lambda\xi)^3\frac{\partial^4 V}{\partial\xi^4} - 6\Lambda(1-\Lambda\xi)^2\frac{\partial^3 V}{\partial\xi^3}\right] + \left[6\Lambda^2\beta^2(1-\Lambda\xi) - \Omega^2\left\{\frac{r_0}{l} - \frac{r_0}{l}\xi + \frac{1}{2} - \frac{1}{2}\xi^2 - \frac{r_0\Lambda}{2l}(1-\xi)^2 - \frac{\Lambda}{3}(1-\xi)^3\right\}\right]$$
$$+\Omega^2\left\{\frac{r_0}{l} + \xi - \frac{r_0}{l}\Lambda\xi - \Lambda\xi^2\right\}\frac{\partial V}{\partial\xi} + (1-\Lambda\xi)\left(\frac{\partial^2 V}{\partial t^2} + \delta_x\frac{\partial^2\theta}{\partial t^2}\right) - K_5\frac{\partial^2 V}{\partial\xi^2}$$
$$+K_1U^2\left[\theta + \frac{\partial V}{\partial t}.\frac{1}{U} + \frac{K_2}{U}.\frac{\partial\theta}{\partial t}\right] = 0$$

and

$$r^2\left[\left[(1-\Lambda\xi)\frac{\partial^3\theta}{\partial\xi^2} - \Lambda\frac{\partial\theta}{\partial\xi}\right] - C_1(1-\Lambda\xi)^5\frac{\partial^4\theta}{\partial\xi^4} - 5\Lambda(1-\Lambda\xi)^4\frac{\partial^3\theta}{\partial\xi^3}\right] - (1-\Lambda\xi)\delta_x\frac{\partial^2 V}{\partial t^2}$$
$$-\left(I_\theta' + \delta_\theta^2\right)(1-\Lambda\xi)^3\frac{\partial^2\theta}{\partial t^2} - K_3U\frac{\partial\theta}{\partial t} + K_1K_4\left(U^2\theta + U\frac{\partial V}{\partial t} + K_2U\frac{\partial\theta}{\partial t}\right) = 0$$

(4)

The solution of eqns. (4) is of the form

$$V(\xi,t) = Af(\xi)e^{i\omega t}$$

$$\theta(\xi,t) = B\phi(\xi)e^{i\omega t}$$

(5)

Where A and B are constants which are not independent and $f(\xi)$ and $\phi(\xi)$ satisfy all boundary conditions of the beam which are as follows.

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$$V(0)=\frac{dV(0)}{d\xi}=\theta(0)=\frac{\partial^2\theta}{\partial\xi^2}(0)=0$$
$$\frac{\partial^2 V}{\partial\xi^2}(1)=\frac{\partial^3 V}{\partial\xi^3}(1)=\frac{\partial\theta}{\partial\xi}(1)=\frac{\partial^3\theta}{\partial\xi^3}(1)=0 \qquad (6)$$

REVIEW OF LITERATURES:

For an approximate determination of the fundamental frequency, $f(\xi)$ is chosen as the shape function for the fundamental mode of uncoupled bending vibrations and $\phi(\xi)$ as the shape function for the fundamental mode of uncoupled torsional vibrations of a uniform cantilever beam. These shape functions satisfy the boundary condition (6) and are

$$f(\xi)=\cosh\lambda\xi-\cos\lambda\xi-0.7341(\sinh\lambda\xi-\sin\lambda\xi)$$

And

$$\phi(\xi)=\sin\frac{\pi}{2}\xi \qquad (7)$$

Where $\lambda=1.87510$

Substitutions of eqns. (5) in (4) give

$$\Big\langle\Big[\beta^2\Big\{(1-\Lambda\xi)^3\frac{d^4 f}{d\xi^4}-6\Lambda(1-\Lambda\xi)^2-\Big\}\frac{d^3 f}{d\xi^3}+\Big[6\Lambda^2\beta^2(1-\Lambda\xi)-\Omega^2\Big[\frac{r_0}{l}+\frac{1}{2}-\frac{r_0}{l}\xi$$
$$-\frac{1}{2}\xi^2-\frac{r_0}{2l}\Lambda(1-\Lambda\xi)^2-\frac{\Lambda}{3}(1-\Lambda\xi)^3-\Big]\Big]\frac{d^2 f}{d\xi^2}+\Omega^2\Big(\frac{r_0}{l}+\xi-\frac{r_0}{l}\Lambda\xi-\Lambda\xi^2\Big)\frac{d^2 f}{d\xi}$$
$$-K_5\frac{d^2 f}{d\xi^2}-\omega^2(1-\Lambda\xi)f+K_1 i\omega Uf-\Big]A+\Big[K_1K_2ui\omega\phi+K_1U^2\phi-(1-\Lambda\xi)\delta_x\phi\omega^2\ \Big]B\Big\rangle=0$$

And

$$\big[(1-\Lambda\xi)\delta_x f\omega^2+K_1K_4Ui\omega f\big]A+\left[\begin{array}{l}\Big\{r^2(1-\Lambda\xi)\frac{d^2\phi}{d\xi^2}-\Lambda\frac{d\phi}{d\xi}\Big\}-C_1\Big[(1-\Lambda\xi)^5\frac{d^4\phi}{d\xi^2}-5\Lambda(1-\Lambda\xi)^4\frac{d^3\phi}{d\xi^3}\Big]\\+\big(I_\theta^{'}+\delta_\theta^2\big)(1-\Lambda\xi)_3\phi\omega^2+K_1K_4U^2\phi+(K_1K_2K_4-K_3)Ui\omega\phi\end{array}\right]B=0 \qquad (8)$$

The Stieltjes integrals may now be formed as follows:

$$\int_0^1\Big\langle\Big[\beta^2\Big\{(1-\Lambda\xi)^3\frac{d^4 f}{d\xi^4}-6\Lambda(1-\Lambda\xi)^2-\Big\}\frac{d^3 f}{d\xi^3}+\Big[6\Lambda^2\beta^2(1-\Lambda\xi)-\Omega^2\Big[\frac{r_0}{l}+\frac{1}{2}-\frac{r_0}{l}\xi$$
$$-\frac{1}{2}\xi^2-\frac{r_0}{2l}\Lambda(1-\Lambda\xi)^2-\frac{\Lambda}{3}(1-\Lambda\xi)^3-\Big]\Big]\frac{d^2 f}{d\xi^2}+\Omega^2\Big(\frac{r_0}{l}+\xi-\frac{r_0}{l}\Lambda\xi-\Lambda\xi^2\Big)\frac{d^2 f}{d\xi}$$
$$-K_5\frac{d^2 f}{d\xi^2}-\omega^2(1-\Lambda\xi)f+K_1 i\omega Uf-\Big]A+\Big[K_1K_2ui\omega\phi+K_1U^2\phi-(1-\Lambda\xi)\delta_x\phi\omega^2\ \Big]B\Big\rangle fd\xi=0$$

And

$$\int_0^1\Big\langle\big[(1-\Lambda\xi)\delta_x f\omega^2+K_1K_4Ui\omega f\big]A+\left[\begin{array}{l}\Big\{r^2(1-\Lambda\xi)\frac{d^2\phi}{d\xi^2}-\Lambda\frac{d\phi}{d\xi}\Big\}-C_1\Big[(1-\Lambda\xi)^5\frac{d^4\phi}{d\xi^2}-5\Lambda(1-\Lambda\xi)^4\frac{d^3\phi}{d\xi^3}\Big]\\+\big(I_\theta^{'}+\delta_\theta^2\big)(1-\Lambda\xi)_3\phi\omega^2+K_1K_4U^2\phi+(K_1K_2K_4-K_3)Ui\omega\phi\end{array}\right]B\Big\rangle fd\xi=0 \qquad (9)$$

Or

$$(a_1+a_2+a_3+a_4+a_5-a_6\omega^2+a_7i\omega U)A+(-a_8\omega^2+a_9U^2+a_{10}i\omega U)B=0 \qquad (10)$$

And

$$(-a_8\omega^2+a_{11}i\omega U)A+(a_{12}+a_{13}+a_{14}+a_{15}-a_{16}\omega^2+a_{17}U^2$$

Where

$$a_1=\beta^2\int_0^1(1-\Lambda\xi)^3\frac{d^4 f}{d\xi^4}fd\xi$$

$$a_2=-6\Lambda\beta^2\int_0^1(1-\Lambda\xi)^2\frac{d^3 f}{d\xi^2}fd\xi$$

$$a_3=\int_0^1\Big[6\Lambda^2\beta^2(1-\Lambda\xi)-\Omega^2\Big\{\frac{r_0}{l}-\frac{r_0}{l}\xi+\frac{1}{2}-\frac{1}{2}\xi^2-\frac{r_0}{2l}(1-\xi)^2\Lambda-\frac{\Lambda}{3}(1-\xi^3)\Big\}\Big]\frac{d^2 f}{d\xi^2}fd\xi$$

$$a_4=\Omega^2\int_0^1\Big(\frac{r_0}{l}+\xi-\frac{r_0}{l}\Lambda\xi-\Lambda\xi^3\Big)\frac{df}{d\xi}fd\xi$$

$$a_5=-K_5\int_0^1\frac{d^2 f}{d\xi^2}fd\xi$$

$$a_6=\int_0^1(1-\Lambda\xi)f^2d\xi$$

$$a_7=K_1\int_0^1 f^2d\xi$$

$$a_8=\int_0^1\delta_x(1-\Lambda\xi)f\phi d\xi$$

$$a_9=K_1\int_0^1 f\phi d\xi$$

$$a_{10} = K_1 K_2 \int_0^1 f\phi d\xi$$

$$a_{11} = -K_1 K_4 \int_0^1 f\phi d\xi$$

$$a_{12} = -r^2 \int_0^1 (1-\Lambda\xi)\phi d\xi$$

$$a_{13} = \Lambda r^2 \int_0^1 \frac{d\phi}{d\xi}\phi d\xi$$

$$a_{14} = c_1 \int_0^1 (1-\Lambda\xi)^5 \frac{d^4\phi}{d\xi^4}\phi d\xi$$

$$a_{15} = -5\Lambda c_1 \int_0^1 (1-\Lambda\xi)^4 \frac{d^3\phi}{d\xi^3}\phi d\xi$$

$$a_{16} = \left(I_\theta' + \delta_\theta^2\right)\int_0^1 (1-\Lambda\xi)^3 \phi^2 d\xi$$

$$a_{17} = -K_1 K_4 \int_0^1 \phi^2 d\xi$$

$$a_{18} = (K_3 - K_1 K_2 K_4)\int_0^1 \phi^2 d\xi$$

The homogeneous eqns. (10) admit vanishing solution A, B only if the determinant of their coefficients vanishes. This determinant being complex, both real and imaginary parts must vanish separately on setting the determinant to zero.

$$\begin{vmatrix} a_1 + a_2 + a_3 + a_4 + a_5 - a_6\omega^2 + a_7 i\omega U & -a_8\omega^2 + a_9 U^2 + a_{10} i\omega U \\ -a_8\omega^2 + a_{11} i\omega U & a_{12} + a_{13} + a_{14} + a_{15} - a_{16}\omega^2 \end{vmatrix}$$

equals 0.

Or

$$A_1\omega^4 - \left(A_2 + A_3U^2\right)\omega^2 + \left(A_4 + A_5U^2\right) = 0$$

And

$$-A_6\omega^2 + \left(A_7 + A_8U^2\right) = 0$$

Where

$$A_1 = a_{16}a_6 - a_8^2$$

$$A_2 = a_6(a_{12} + a_{13} + a_{14} + a_{15}) + a_{16}(a_1 + a_2 + a_3 + a_4 + a_5)$$

$$A_3 = a_7 a_{18} + a_6 a_{17} - a_8 a_9 - a_{10} a_{11}$$

$$A_4 = (a_{12} + a_{13} + a_{14} + a_{15})(a_1 + a_2 + a_3 + a_4 + a_5)$$

$$A_5 = a_{17}(a_1 + a_2 + a_3 + a_4 + a_5)$$

$$A_6 = a_7 a_{16} + a_6 a_{18} - a_8 a_{10} - a_8 a_{11}$$

$$A_7 = a_7(a_{12} + a_{13} + a_{14} + a_{15}) + a_{18}(a_1 + a_2 + a_3 + a_4 + a_5)$$

$$A_8 = a_7 a_{17} - a_9 a_{11}$$

The second eqn. of (11) gives

$$\omega^2 = \frac{A_7 + A_8 U^2}{A_6} \qquad (12)$$

Substituting this expression into first eqn. of (11), we obtain:

$$PU^4 - QU^2 + R = 0 \qquad (13)$$

Where

$$P = A_8(A_3A_6 - A_1A_8)$$

$$Q = 2A_1A_7A_8 - A_2A_6A_8 - A_3A_6A_7 + A_5A_6^2$$

$$R = A_2A_7A_6 - A_1A_7^2 - A_4A_6^2$$

From eqn. (13), we obtain the value of the critical speed as given below:

$$U^2 = \frac{Q \pm \sqrt{Q^2 - 4PR}}{2P} \qquad (14)$$

The right hand side of eqn. (14) is positive. Corresponding to two solutions of U^2 from

Eqn. (14), there are two values of ω^2 from eqn. (12). Usually the smaller U^2 is associated with the higher value of ω^2.

CONCLUSION

The effect of tensile force on the fundamental frequency of the coupled torsional vibrations of a straight rotating slender beam of linearly varying cross-section is now presented. The frequencies are computed from eqn. (12) and the cross-section of the blade is taken as a channel-section of height b_0 and breadth $2b_0$ and thickness t_1.

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