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Use of Graphical Method for Solving Game without Saadle Point

Farooq Ahmad Kuchay

Research Scholar Jodhpur National University Rajasthan

Abstract* – *Game theory studies situations in which parties compete, and also possibly cooperate, to influence the outcome of the parties' interaction to each party's advantage. The situation involves conflict between the participants, called players because some outcomes favour one player at the expense of the other players. What each player obtains from a particular outcome is called the player's pay-off. Each player can choose among a number of actions to influence his pay-off. However, each player's pay-off depends on the other players' choices. Moreover, outcomes can also be influenced by "chance occurrences."

Keywords: Game theory; Graphical method; strategies;

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INTRODUCTION

The techniques of developing optimal strategies for dealing with conflicting and competitive situations (whenever these conflicts can be expressed in mathematical terms) have been termed as game theory.

Key terms associated with the game theory [2, 3]-

(i) **Strategy -** A strategy is a comprehensive plan of action, formulated by a player (an interested and active party in the game), who is well informed of all the alternatives available to him and to his adversary (competing player).

A strategy can be good or bad. The only requirement is that it should be complete and cover all the possibilities.

(ii) **Finite game -** when the total number of possible strategies in a game is finite, it is called a finite game. In the other situation, the game is an infinite game.

(iii) **Zero-sum game -** Zero-sum games are those games in which one player gains exactly the same amount, which the other player(s) loose so that their net gains are equal to zero.

(iv) **Non zero-sum game -** Zero-sum games are those games in which gain of one player is not necessarily equal to the loss of the other or vice versa.

(v) **Pay-off (game) matrix -** A pay-off matrix is a tabular representation of the pay-offs of one competitor, which are associated with his strategies in response to the strategies of the other player.

REVIEW OF LITERATURE:

Consider two players A and B playing a zero-sum game. Let *A* has *m* strategies numbered *A*1, *A*2… *Am* available to him and *B* has *n* strategies numbered *B*1, *B*2… *Bn*, available to him. Let the gain of *A*, when he chooses *i*th strategy in response to the *j*th strategy chosen by *B* be given by *gij*. Then the pay- offs of*A* can be represented as follows:

$$G = \begin{array}{c|cccc} & & B & & \\ & B1 & B2 \ldots & Bn \\ A1 & g11 & g12 \ldots & g1n \\ A2 & g21 & g22 \ldots & g2n \\ M & M & M & M \\ Am & gm1 & gm2 \ldots & gmn \end{array}$$

The matrix *G* is called the pay-off matrix of player *A*. If *gij* > 0, *A* has gained and if *gij* < 0, then *A* has lost an amount *gij*. Since the game is a zero-sum game, so whatever is the gain of *A* is loss of *B*.

Assumptions of gaming problems

Game theory is meant for developing a rational criterion for choosing a strategy among several possible strategies. For developing such criteria, we make some assumptions:

(i) The number of players in the game is (in general) finite.

(ii) The interests of the players clash and each player is choosing his strategy solely for his welfare.

(iii) Each player is well aware of all the strategies available to him and to his opponents.

(iv) All the players are making their moves simultaneously, without knowing the choices, which the other players have made.

(v) The outcome of the game depends upon the moves made by different players; and

(vi) All the players are rational players.

Types of games:

1. Two-person games and n-person games

In two person games, the players may have many possible choices open to them for each play of the game but the number of player's remains only two. Hence it is called a two person game. In case of more than two persons, the game is generally called n-person game.

2. Zero sum game

A zero sum game is one in which the sum of the payment to all the competitors is zero for every possible outcome of the game in a game if the sum of the points won equals the sum of the points lost.

3. Two person zero sum game

A game with two players, where the gain of one player equals the loss to the other is known as a two person zero sum game. It also called rectangular form. The characteristics of such a game are.

(i) Only two players participate in the game.

(ii) Each player has a finite number of strategies to use.

(iii) Each specific strategy results in a payoff.

(iv) Total pay off to the two players at the end of each play is zero.

Solving a zero-sum game [1]:

In general, the games are zero-sum games. For the simplicity of presentation, we assume that the games are two- players' games. We define some terms associated with the solution of the games.

(i) Dominant strategy

Consider the following game (*G*1)

B

	B_1	B_2	B_3
A_1	7	4	6
A_2	5	2	4

This game matrix suggests that the two players *A* and *B* are playing a game with *A* having two (viz. *A*1and *A*2) and *B* having three (viz. *B*1, *B*2 and *B*3) strategies. In this case *A* would always opt for the strategy *A*1, as it would yield him better pay-offs than the pay-offs yielded by the strategy *A*2. We say that *A*1 is a dominant strategy.

A strategy is said to be a dominant strategy if it always yields better (or at least equal) pay-offs than the other strategies irrespective of the strategies opted by the other player(s), i.e., superior strategies (resulting in higher pay-offs) dominate the inferior ones (resulting in lower pay-offs). In such situations, inferior strategies can always be strike off.

Algebraic Method for the Solution of a General Game [7]

The algebraic method is a direct attempt to solve unknowns although this method becomes quite lengthy when there are more strategies (courses of action) for players. Such large games can be solved first by transforming the problem into a linear programming problem and then solving it by the *simplex method* on an electronic computer [8].

First, suppose that all inequalities given hold as equations. Then solve these equations for unknowns. Sometimes equations are not consistent. In such cases, one or more of the inequalities must hold as strict inequalities (with '>' or '<'signs). Hence, there will be no alternative except to rely on trial-and-error method for solving such games. Following important theorems will be helpful in making the computations easier.

Theorem [7]

If for any j $(j = 1, 2, 3, \ldots, n)$ $v_{1j}x_1 + v_{2j}x_2 + \ldots + v_{mj}x_m > v$, *then* $y_j = 0$,

and similarly, if for any i $(i = 1, 2, 3, \ldots, m)$ $v_{i1}y_1 + v_{i2}y_2 + \ldots + v_{in}y_n < v$, *then* $x_i = 0$.

Alternative Statement : *Let* v *be the value of an* $m \times n$ *game. If for an optimum strategy* $\mathbf{x}^* \in S_m$, $E(\mathbf{x}^*, \mathbf{e}_j) > v$ *for some* $\mathbf{e}_j \in S_n$, *then every strategy* $\mathbf{y}^* \in S_n$ *has* $y_j^* = 0$,

Similarly, if every optimal strategy $\mathbf{y}^* \in S_n$, *then every optimal strategy* $\mathbf{x}^* \in S_m$ *has* $x_i^* = 0$.

Proof. We know that for any optimal strategy $\mathbf{x}^* \in S_m$, we always have

$$E(\mathbf{x}^*, \mathbf{e}_j) \geq v \text{ for all } \mathbf{e}_j \in S_n \qquad \ldots(1)$$

We are given that $E(\mathbf{x}^*, \mathbf{e}_j) > v$ for some $\mathbf{e}_j \in S_n$. If possible let us suppose $y_j^* \neq 0$ (*i.e.* $y_j^* > 0$).

Then $$y_j^* E(\mathbf{x}^*, \mathbf{e}_j) > y_j^* v. \qquad \ldots(2)$$

Now $$E(\mathbf{x}^*, \mathbf{y}^*) = \Sigma y_k^* E(\mathbf{x}^*, \mathbf{e}_k) \qquad = \sum_{k \neq j} y_k^* E(\mathbf{x}^*, \mathbf{e}_k) + y_j^* E(\mathbf{x}^*, \mathbf{e}_j)$$

or $$v > \sum_{k \neq j} y_k^* v + y_j^* v \text{ or } v > \left(\sum_{k \neq j} y_k^* + y_j^*\right) v \text{ or } v > \Sigma y_k^* v \quad [\text{from (1) and (2)}]$$

or $$v > v \; (\text{since } \Sigma y_k^* = 1) \quad \text{which is a contradiction.}$$

Hence our assumption is wrong; and therefore, we must have $y_j^* = 0$.

Similarly, the second part of the theorem can be proved.

CONCLUSION:

In this paper we found that, the algebraic method is a direct attempt to solve unknowns although this method becomes quite lengthy when there are more strategies (courses of action) for players. Such large games can be solved first by transforming the problem into a linear programming problem and then solving it by the *simplex method* on an electronic computer.

REFERENCES:

1. Swarup K., Gupta P.K. and Mohan M. (2001), Operations Research, Sultan Chand and Sons, Delhi.

2. Watson J., (2002), Strategy: An Introduction to Game Theory, W.W. Nortan & Company

3. Osborne M.J., (2001) An Introduction to Game Theory, Oxford University Press.

4. http://www.egwald.ca/operationsresearch/gameintroduction.php

5. http.//www.univeralteacherpublications.com.univ/eBooks/or/Ch9/gamegrap.htm

6. http:/www.hindawi.com/jounals/aa/2011/260490

7. Operation_Research.pdf by EIILM University

8. Mokhtar S. Bazaraa, John J. Jarvis, and Hanif D. Sherali, Linear programming and network flows, Wiley-Interscience, 2004.