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To Define the Hybrid Generalized Multi-valued Contraction Mapping

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Abstract – It is the object in the present research to survey, systematize, and extend a number of recent results concerning the existence of fixed points of noncompact mappings of a subset C of a Hilbert space H into H. The purpose of this research is to define the hybrid generalized multi-valued contraction mapping which is more general than various mappings in literature and to give some properties of this mapping. We also establish the common fixed point theorem. been largely motivated and dominated by questions from nonlinear problems in practice, such as problems of geometric group theory, and others. However, so far, we have seen not many results for the approximation iteration of multi-valued non-expansive mappings in terms of Hausdorff metrics for fixed points in the existing literature. The purpose of this research is to extend the iteration scheme of multi-valued non-expansive mappings from a Banach space to a hyperbolic space by proving Δ-convergence theorems for two multi-valued nonexpansive mappings in terms of mixed type iteration processes to approximate a common fixed point of two multi-valued non-expansive mappings in hyperbolic spaces.

Key Words : Multi-Valued, Approximation Iteration, Hyperbolic, Convergence

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INTRODUCTION

The study of fixed points for multi-valued contractions and non-expansive mappings using the Hausdorff metric was initiated by Markin. Later, various iterative processes have been used to approximate the fixed points of multi-valued non-expansive mappings in Banach space, for example, the authors of and have made extensive research in this direction, which has led to many new results in the study of fixed point theory with applications in control theory, convex optimization, differential inclusion, economics,and related topics.

Many of the most important nonlinear problems of applied mathematics reduce to finding solutions of nonlinear functional equations (e.g., nonlinear integral equations, boundary value problems for nonlinear ordinary or partial differential equations, the existence of periodic solutions of nonlinear partial differential equations) which can be formulated in terms of finding the fixed points of a given nonlinear mapping of an infinite dimensional function space X into itself. For mappings satisfying compactness conditions, a general existence theory of fixed points based upon topological arguments has been constructed over a number of decades (associated with the names of Brouwer, Poincare, Lefschetz, Schauder, Leray, and others). More recently, there has begun the systematic study of fixed points of various classes of noncompact mapping.

REVIEW OF LITERATURE

The structure of the fixed point sets of nonexpansive mappings in Banach spaces with FPP is well understood.

Theorem : Let X be a reflexive space, or a separable space, which has FPP, and let $K \subseteq X$ be nonempty bounded closed and convex. Then the set of common fixed points of any commutative family or nonexpansive self-mappings of K is a nonexpansive retract of K.

This raises the obvious question of whether FPP implies the conclusion of Bruck's theorem in general (as it does in the sparable case). Of course a positive answer to "FPP $\Rightarrow$ reflexive" would settle this affirmatively as well.

Remarks. Under the assumptions of Bruck's theorem, the collection od subsets of K which have f.p.p. includes all the nonexpansive retracts of K.

Proof. Suppose $R : K \to F \subseteq K$ is a nonexpansive retraction, and let $G : F \to F$ be nonexpansive. Then G o R : $K \to F$ is nonexpansive, so by FPP there exists $x \in K$ such that G o $R(x) = x$. But this implies

$x \in F$ and *G(x)=x.* Therefore, *R*(*x*) = *x* is a fixed point of *G*.

Bruck's proof of the above theorem in the single – mapping case is somewhat involved, relying on a clever use of Tychonoff's theorem, and the general case is quite difficult. However:

Corollary . *Bruck's theorem for finite commutative families follows easily from its validity for a single mapping.*

Proof. Suppose *X* and *K* are as in Theorem above, and suppose *T* and *G* are commutative nonexpansive mappings of *K*→*K.* Let fix(*T*) (*etc.*) denote the fixed point set of *T* in *K.* Then since *T* o *G* = *G o T,* it follows that *G* : Fix(*T*) → Fix(*T*). Since Fix(*T*) is by assumption a nonexpansive retract of *K*, by the above Remark Fix(*T*) ∩ Fix(*G*) ≠ Ø. Let *R* be a nonexpansive retraction of *K* onto Fix(*T*). Then

Fix(*T*)∩ Fix(*G*) = Fix(*G o R*),

And the latter set is also a nonexpansive retract of *K*. This shows that the common fixed point set of two commuting mappings of K → *K* is a nonexpansive retract of *K*. The general case for a finite family of commuting nonexpansive mapping follows by induction.

We look at the structure of the fixed point sets in a more abstract metric space setting in the next section.

Asymptotic regularity and approximate fixed points. At the outset we call attention to the survey of Bruck.

If *K* is a subset of a Banach space *X*, then *f* : *K* → *K* is said to be asymptotically regular (at $x \in K$) if $\| f^n(x) - f^{n+1}(x) \| \to 0$.

In 1976 Ishikawa obtained a surprising results,a special case of which may be stated as follows: Let *K* be an arbitrary bounded closed convex subset of a Banach space *X* , *T* : *K* → *K* nonexpansive, and λ ∈ (0, 1). Set *T λ = (1− λ)I* + λ*T*. Then for each x ∈ *K* :

$$\| T_\lambda^n(x) - T_\lambda^{n+1}(x) \| \to 0.$$

Thus by Iterating the 'averaged' mapping T_λ one can always reach points which are approximately fixed (but on the other hand, these points may not be near fixed points−indeed, it need not be the case that *T* even have a fixed point).

In 1978, Edelstein and O'Brien proved that $\{T_\lambda^n(x) - T_\lambda^{n+1}(x)\}$ converges to 0 uniformly for $x \in K$ and, in 1983, Goebel and Krik proved that this convergence is given uniform for $T \in \Im$, where $\Im$ denotes the collection of all nonexpansive self-mappings of *K.*

MATERIAL AND METHOD

Let (X, d) be a metric space, f : X → X be a single-valued mapping and T : X → CB(X) be ageneralized multi-valued (f , α, β)-weak contraction mapping. If fX is complete subspace of X and Tx ⊂ fX, then f and T have acoincidence point u ∈ X. Moreover, if ffu = fu, then f and T have a common fixed point.Extended, improved, unified and generalized several fixed point theorems.

The purpose of this research is to define the hybrid generalized multi-valued contraction mapping which is more general than various mappings in literature and to give some properties of this mapping. We also establish the common fixed point theorem. been largely motivated and dominated by questions from nonlinear problems in practice, such as problems of geometric group theory, and others. However, The results presented in this research are new and can be regarded as an extension of corresponding results from Banach spaces to hyperbolic spaces in the existing literature given by the authors.

The fixed point theorem states the existence of fixed points under suitable conditions.

Recall that in case f : X→X is a function, then y is a fixed point off if fy = y is satisfied .

The famous Brouwer fixed point theorem was given in 1912 .

2 Brouwer fixed point theorem

The theorem states that if f : B→B is a continuous function and B is a ball in Rn, then f has a fixed point.

This theorem simply guarantees the existence of a solution, but gives no information about the uniqueness and determination of the solution.

For example, if f : [0; 1]→[0; 1] is given by fx = x2, then f0 = 0 and f1 = 1, that is, f has 2fixedpoints.

Several proofs of this theorem are given. Most of them are of topological in nature. A classical proof due to Birkho_ and Kellog was given in 1922, Similar classical proof was given in Linear Operators Volume 1, Dunford and Schwartz 1958.

Brouwer theorem gives no information about the location of fixedpoints. However,effective methods have been developed to approximate the fixed points. Such tools are useful in calculating zeros of functions. A polynomial equation Px = 0 can be written as Fx - x = 0 where Fx - x = Px:

For example, consider x^2 - 7x + 12 = 0; where Px = x^2 - 7x + 12: We can write

Fx - x = Px = x^2 - 7x+12; so x = (x^2+12)=7 = Fx. Here F has twofixedpoints,

F3 = 3 and F4 = 4.

This theorem is not true in in_nite dimensional spaces. For example, if B is a unit ball in an in_nite dimensional Hilbert space and f : B→B is a continuous function, then f need not have afixedpoint. This was given by Kakutani in 1941.

CONCLUSION

So far, we have seen not many results for the approximation iteration of multi-valued non-expansive mappings in terms of Hausdorff metrics for fixed points in the existing literature. The purpose of this research is to extend the iteration scheme of multi-valued non-expansive mappings from a Banach space to a hyperbolic space by proving Δ-convergence theorems for two multi-valued nonexpansive mappings in terms of mixed type iteration processes to approximate a common fixed point of two multi-valued non-expansive mappings in hyperbolic spaces.

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