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EXISTENCE OF FIXED POINTS OF NONCOMPACT MAPPINGS

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Existence of Fixed Points of Noncompact Mappings

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Abstract – The results presented for various classes of nonlinear mappings (contractive, strictly pseudo contractive, pseudo contractive) may be considered as somewhat sophisticated, sharpened forms of the classical iteration scheme (the method of successive approximations) of Picard-Banach-Cacciopoli et al. that work in contexts in which the classical iteration scheme no longer applies (and in particular, outside the class of strictly contractive mappings). We have restricted the discussion to the case of mappings defined in Hilbert space both to avoid technical complications in the presentation of the results and proofs.

Key Words: Contractive, Strictly Pseudo contractive, Pseudo contractive

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INTRODUCTION

Fixed points of noncompact mappings using different iterative processes on different domains has remained at the heart of fixed point theory. Non-expansive mappings constitute one of the most important classes of nonlinear mappings which have remained a crucial part of such studies. Another important role is that of ambient spaces in this regard. Banach spaces (linear domains) with some geometric structure have been studied extensively: one of such structures is convexity. Since every Banach space is a vector space, it is easier to assign a convex structure to it. However, metric spaces do not enjoy this structure. Takahashi introduced the notion of convex metric spaces and studied the fixed point theory for non-expansive mappings in this setting. Later on, several attempts were made to introduce different convex structures on a metric space. One such convex structure is available in a hyperbolic space introduced by Kohlenbach. Kohlenbach hyperbolic space is more restrictive than the hyperbolic space introduced in and more general than the concept of hyperbolic space in. Spaces like CAT(0) and Banach are special cases of a hyperbolic space. T

REVIEW OF LITERATURE

A fixed point of *T*. However in 1971, Kaniel obtained a rather complicated discrete convergence procedure for appromaximating fixed points of nonexpansive mappings in such spaces. Quite recently, Moloney obtained a refinement of Kaniel's method for constructing such a sequence, a method which in fact applies to asymptotically nonexpansive mappings. We briefly describe this result, beginning with the relevant definations.

The *modulus of convexity* of a Banach space *X* is the function $\delta_x:[0,2]\to[0,1]$ defined as follows:

$$\delta_x(\in)=\inf\{1-\|\frac{x+y}{2}\|:\|x\|\le 1,\|y\|\le 1,\|x-y\|\ge\in\}$$

.

It is known that the function δ_x is nondecreasing, and continuous on [0,2). A Banach space is said to be *uniformly convex* if δ_x > 0 whenever $\in$ > 0.

We assume that *X* is a uniformly convex Banach space and K $\subseteq$ *X* is a given bounded closed and convex subset of *X*. A mapping $T:K\to K$ is said to be *asymptotically nonexpansive* if there exists a sequence $\{k_n\}$ of positive real numbers for which $\lim_{n\to\infty}k_n=1$ and $\|T^n(x)-T^n(y)\|\le k_n\|x-y\|$ for all $x,y\in K$. Goebel and Krik show that such a mapping *T* always has a fixed point. (There is an extensive literature on asymptotically nonexpansive and related classes of mappings which we do not take up here.) Using some technical lemmas, Moloney constructs an auxiliary mapping $S:K\to K$ which has the properties:

(a) $T(p)=p\Leftrightarrow S(p)=p;$

(b) $\| p - S(x) \| \leq \| p - x \|$;

(c) $\lim_{n\to\infty} x_n = x \text{ and } \lim_{n\to\infty} \| S(x_n) - x_n \| = 0$, then $S(x) = x = T(x)$.

Using the mapping *S* he then constracts a sequence $\{y_n\}$ which always converges strongly to a fixed point *T.* However, as in the case of Kaniel's construction, it is not possible to determine how to close the fixed point one is at any step.

MATERIAL AND METHOD

It is the object in the present research to survey, systematize, and extend a number of recent results concerning the existence of fixed points of noncompact mappings of a subset C of a Hilbert space H into H. From the point of view of application, it is essential not only to show the existence of fixed points of such mappings under suitable hypotheses, but also to develop systematic techniques for the construction or calculation of such fixed points.

The results presented for various classes of nonlinear mappings (contractive, strictly pseudocontractive, pseudocontractive) may be considered as somewhat sophisticated, sharpened forms of the classical iteration scheme (the method of successive approximations) of Picard-Banach-Cacciopoli et al. that work in contexts in which the classical iteration scheme no longer applies (and in particular, outside the class of strictly contractive mappings). We have restricted the discussion to the case of mappings defined in Hilbert space both to avoid technical complications in the presentation of the results and proofs.

Thus the study of fixed point theory for hyperbolic spaces has Banach's contraction mapping principle extended to fixed point theorems about multi-valued contraction mappings by Nadler In 1973, the study of fixed points for multi-valued contractions using the Hausdorff metric was initiated by Markin Afterward, an interesting and rich fixed point theory for such mappings was developed in many directions. The theory of multi-valued mapping has applications in optimization problems, control theory, differential equations and economics. Kamran extended the notion of weak contraction mapping which is more general than the contraction mapping and introduced the notion of multi-valued (f , θ, L)-weak contraction mapping and generalized multi-valued (f , α, L)-weak contraction mapping. He established some coincidence and common fixed point theorems. We state the results for convenience as follows:

Let (X, d) be a metric space, f : X → X and T : X → CB(X) be a multi-valued (f , θ, L)-weak contraction such that TX ⊂ fX. Suppose fX is complete. Then the set of coincidence points of f and T , C(f , T), is nonempty.

Further, if f is T -weakly commuting at coincidence point u and ffu = fu, then f and T have a common fixed point.

Theorem: Let (X, d) be a metric space, f : X → X and T : X → CB(X) be a generalized multi-valued (f , α, L)-weak contraction such that TX ⊂ fX. Suppose fX is a complete subspace of X. Then f and T have a coincidence point u ∈ X. Further, if f is T -weakly commuting at u and ffu = fu, then f and T have a common fixed point.

Recently, Sintunavarat and Kumam extended the notion of a generalized multi-valued (f , α, L)-weak contraction mapping to a generalized multi-valued (f , α, β)-weak contraction mapping and also established the common fixed point theorem for this mapping.

CONCLUSION

This is so because of the fact that in general almost all problems in various disciplines of science are nonlinear in nature, and most results of fixed point theory are proposed under the framework of normed linear spaces or Banach spaces as the property of nonlinear mappings may depend on the linear structure of the underlying spaces. Thus it is necessary to study fixed point theory for nonlinear mappings under the space which does not have a linear structure but is embedded with a kind of '*convex structures*'. The class of hyperbolic spaces, being nonlinear in nature, is a general abstract theoretic setting with rich geometrical structures for metric fixed point theory.

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