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STUDY ON VIBRATION PROBLEMS IN ELASTICITY

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Study on Vibration Problems in Elasticity

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Abstract – The differential equations for the coupled vibration of a rotating slender beam under aerodynamic couplings including the effect of shear force are obtained. A method based on Rayleigh's quotient is used to obtain the critical speed of the steady flow and the corresponding fundamental frequency of vibrations.

Keyword: Differential Equations, Frequency

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1. INTRODUCTION

These analysis presented in this chapter considers vibrations of a slender beam that could represent a turbine blade of simple geometry. The oscillations of a beam whose elastic axis and the line of centre of gravity do not coincide are always coupled. The beam is attached to a disc of radius r_0 and disc rotates with angular velocity Ω. The beam in allowed to oscillate in a plane making an angle Ψ with the plane of rotation.

2. DIFFERENTIAL EQUATION

Timoshenko beam theory gives the following differential eqns for the coupled bending and torsional vibrations of slender beam

$$EI\frac{\partial^4 h}{\partial x^4}+m\frac{\partial^2}{\partial t^2}(h+\delta_\alpha)=0$$

And

$$GJ\frac{\partial^2\alpha}{\partial x^2}-C_1'\frac{\partial^4\alpha}{\partial x^4}-m\delta\frac{\partial^2 h}{\partial t^2}-m\left(\delta^2+\frac{I_\alpha}{m}\right)\frac{\partial^2\alpha}{\partial t^2}=o$$

Where C_1^1 is the warping rigidity? When δ =0 these above equns. Reduce to two independent ones f or the purely torsional and purely bending oscillations. If the centrifugal force effect is to be considered then above governing eqns become

$$EI\frac{\partial^4 h}{\partial x^4}+m\delta\frac{\partial^2\alpha}{\partial t^2}+m\frac{\partial^2 h}{\partial t^2}-\frac{\partial^2 m}{\partial x^2}=0$$

And

(1)

$$GJ\frac{\partial^2\alpha}{\partial x}-C_1^1\frac{\partial^4\alpha}{\partial x^4}-m\delta\frac{\partial^2 h}{\partial t^2}-m\left(\delta^2+\frac{I_\alpha}{m}\right)\frac{\partial^2\alpha}{\partial t^2}=0$$

Where $\frac{\partial^2 M}{\partial x^2}$ the load due to is centrifugal force and is given by

$$\frac{\partial^2 M}{\partial x^2}=m\Omega^2\left[\left\langle r_0(l-x)+\frac{1}{2}(l^2-x^2)\right\rangle\frac{\partial^2 h}{\partial x^2}-(r_0+x)\frac{\partial h}{\partial x}+h\sin^2\psi\right]$$

When we consider the beam under steady aerodynamic force eqns (1) becomes

$$EI\frac{\partial^4 h}{\partial x^4}+m\delta\frac{\partial^2\alpha}{\partial t^2}+m\frac{\partial^2 h}{\partial t^2}+L-\frac{\partial^2 m}{\partial x^2}=0$$

and

(2)

GJ

$$\frac{\partial^2\alpha}{\partial x}-C_1^1\frac{\partial^4\alpha}{\partial x^4}-m\delta\frac{\partial^2 h}{\partial t^2}-m\left(\delta^2+\frac{I_\alpha}{m}\right)\frac{\partial^2\alpha}{\partial t^2}+N=0$$

Where L and N are given by

$$L=\frac{\rho U^2}{2}C_{C_L}$$

N= $$\frac{\rho U^2}{2} C^2 \left(C_N + \frac{x_0}{c} C_L \right)$$

The coefficient C_L and C_N are lift and moment coefficient about the leading edge which are expressed as

$$C_L = \frac{dC_L}{d\alpha}\left[\alpha + \frac{1}{U}\frac{dh}{dt} + \frac{1}{U}\left(\frac{3}{4}c - x_0\right)\frac{d\alpha}{dt}\right]$$ and

$$C_N = -\frac{c\pi}{\delta U}\frac{d\alpha}{dt} - \frac{1}{4}C_L$$

The eqns (2) when the effect of shear force is taken into consideration, reduce to EI

$$\frac{\partial^4 h}{\partial x^4} + m\delta\frac{\partial^2 \alpha}{\partial t^2} + m\frac{\partial^2 h}{\partial t^2} + L - \frac{\partial^2 m}{\partial x^2} + \frac{\rho^2 I}{K^1 G}\frac{\partial^4 h}{\partial t^4} = 0$$

And

(3)

GJ

$$\frac{\partial^2 \alpha}{\partial x} - C_1^1\frac{\partial^4 \alpha}{\partial x^4} - m\delta\frac{\partial^2 h}{\partial t^2} - m\left(\delta^2 + \frac{I_\alpha}{m}\right)\frac{\partial^2 \alpha}{\partial t^2} + N = 0$$

The equations (3) becomes

$$\beta^2\frac{\partial^4 h}{\partial \xi^4} - \Omega^2\left[\left(\frac{r_0}{l} + \frac{1}{2} - \frac{r_0}{l}\xi - \frac{\xi^2}{2}\right)\frac{\partial^2 h}{\partial \xi^2} - \left(\frac{r_0}{l} + \xi\right)\frac{\partial h}{\partial \xi} + h\sin^2\varphi\right] + \frac{\partial^2 h}{\partial t^2} + \delta\frac{\partial^2 \alpha}{\partial t^2} +$$
$$K_1\left(\alpha U^2 + U\frac{\partial h}{\partial t} + K_3 U\frac{\partial \alpha}{\partial t}\right) - K_5\frac{\partial^4 h}{\partial t^4} = 0$$

and

(4)

$$\gamma^2\frac{\partial^2 \alpha}{\partial \xi^2} - T\frac{\partial^4 \alpha}{\partial \xi^4} - \left(\delta^2 + I_\alpha^1\right)\frac{\partial^2 \alpha}{\partial t^2} - \delta\frac{\partial^2 h}{\partial t^2} + K_1 K_4\left(\alpha U^2 + U\frac{\partial h}{\partial t} + UK_2\frac{\partial \alpha}{\partial t}\right) = 0$$

Let the motion be harmonic represent able as follows

$$h(\xi, t) = Af(\xi)e^{i\omega t}$$

(5)

$$\alpha(\xi, t) = B\phi(\xi)e^{i\omega t}$$

Where ω is real and A, B are complex constants. The function f (ω) and $\phi(\omega)$ satisfy the boundary conditions of the beam which are as follows

$$h = \frac{\partial h}{\partial \xi} = \frac{\partial^2 \alpha}{\partial \xi^2} = \alpha = 0$$ at ξ =0

and

(6)

$$\frac{\partial^2 h}{\partial \xi^2} = \frac{\partial^3 h}{\partial \xi^2} = \frac{\partial \alpha}{\partial \xi} = \frac{\partial^3 \alpha}{\partial \xi^3} = 0$$ at ξ =0

3. REVIEW OF LITERATURE

For an approximate determination of the fundamental frequency f is chosen as the shape function for the fundamental mode of uncoupled bending vibrations and ϕ as the shape function for the fundamental mode of uncoupled torsional vibration in still air of cantilever beam of uniform cross section. Let the shape functions which satisfy the boundary conditions (6) be chosen as

$$f(\xi) = \xi^2 - \frac{2}{3}\xi^3 + \frac{\xi^4}{6} \quad (7)$$

$$\phi(\xi) = \xi - \frac{1}{2}\xi^3 + \frac{\xi^4}{8}$$

Substituting eqns (5) in (4), we obtain

$$A\left[\beta^2\frac{d^4 f}{d\xi^4} - \Omega^2\left(\frac{r_0}{l} + \frac{1}{2} - \frac{r_0\xi}{l} - \frac{\xi^2}{2}\right)\frac{d^2 f}{d\xi^2} + \Omega^2\left(\frac{r_0}{l} + \xi\right)\frac{df}{d\xi} - \Omega^2 f\sin^2\varphi - \omega^2 f + K_1 i\omega U f + K_5\omega^4 f\right]$$

$$+B\left[-\rho\omega^2\phi + K_1 U^2\phi + K_1 K_2 i\omega U\phi\right] = 0$$

And

(8)

$$A\left[\delta\omega^2 f + K_1 K_4 Ui\omega f\right] +$$
$$B\left[\gamma^2\frac{d^2\phi}{d\xi^2} - T\frac{d^4\phi}{d\xi^4} + \left(\delta^2 + I^1{}_\alpha\right)\omega^2\phi + K_1 K_4 U^2\phi + \left(K_1 K_2 K_4 - K_3\right)Ui\omega\phi\right] = 0$$

For the solution of eqns (8), we multiply eqns (8) by f and ϕ respectively and integrating the result with respect to ξ from 0 to 1, we obtain the following eqns Fung (5)

$$A\left[a_1 - a_2 + a_3 - \omega^2 a_4 + i\omega U a_5 + \omega^4 a_{15}\right] + B\left[\omega^2 a_6 - U^2 a_7 - i\omega U a_8\right] = 0 \quad (9)$$

$$B\left[\omega^2 a_{67} - i\omega U a_9\right] + B\left[a_{10} - \omega^2 a_{11} + U^2 a_{12} + i\omega U a_{13} + a_{14}\right] = 0$$

Where

$$a_1 = \beta^2\int_0^1 \frac{d^4 f}{d\xi^4} f d\xi$$

$$a_2 = \Omega^2 \int_0^1 \left(\frac{r_0}{l} + \frac{1}{2} - \frac{r_0}{l}\xi - \frac{\xi^2}{2} \right) \frac{d^2 f}{d\xi^2} f d\xi$$

$$a_3 = \Omega^2 \int_0^1 \left(\left(\frac{r_0}{l} + \xi \right) \frac{df}{d\xi} - f \sin^2 \varphi \right) f d\xi$$

$$a_4 = \int_0^1 f^2 d\xi$$

$$a_5 = -K_1 \int_0^1 f^2 d\xi$$

$$a_6 = -\delta \int_0^1 f\phi d\xi$$

$$a_7 = -K_1 \int_0^1 f\phi d\xi$$

$$a_8 = -K_1 K_2 \int_0^1 f\phi d\xi$$

$$a_9 = -K_1 K_4 \int_0^1 f\phi d\xi$$

$$a_{10} = -\gamma^2 \int_0^1 \frac{d^2\phi}{d\xi^2} \phi d\xi$$

$$a_{11} = -(\delta^2 + I_\alpha) \int_0^1 \phi^2 d\xi$$

$$a_{12} = -K_1 K_4 \int_0^1 \phi^2 d\xi$$

$$a_{13} = (K_3 - K_1 K_2 K_4) \int_0^1 \phi^2 d\xi$$

$$a_{14} = T \int_0^1 \frac{d^4\phi}{d\xi^4} \phi d\xi$$

$$a_{15} = -K_5 \int_0^1 f^2 d\xi$$

The determinant of the co-efficient of A and B is complex, both the real and imaginary parts must vanish. On setting the determinant equal to zero and separating the real and imaginary parts, we obtain the following

$$a_1 - a_2 + a_3 - \omega^2 a_4 + i\omega a_{15} + i\omega a_5$$

$$-\omega^2 a_6 - U^2 a_7 - i\omega U a_8$$

$$\omega^2 a_6 - i\omega U a_9$$

$$a_{10} - \omega^2 a_{11} + U^2 a_{12} + i\omega U a_{13} + a_{14}$$

or

$$\omega^4 (A_1 + U^2 A_2) - \omega^2 (B_1 + B_2 U^2) + E = 0 \qquad (10)$$

$$-\omega^2 (C_1) + (C_2 + C_3 U^2) = 0$$

$$A_1 = a_{15} a_{10} + a_4 a_{11} + a_{15} a_{14} + a_6 a_6$$

$$A_2 = a_{15} a_{12}$$

$$B_1 = a_4 a_{10} + (a_1 - a_2 + a_3) a_{11} + a_4 a_{14}$$

$$B_2 = (a_4 a_{12} + a_5 a_{13} - a_8 a_9)$$

$$E_1 = (a_1 - a_2 + a_3)(a_{10} + a_{14})$$

$$C_1 = a_5 + a_{14} a_{13}$$

$$C_2 = a_5 a_{10} + (a_1 - a_2 + a_3) a_{13} + a_5 a_{14} - a_9 a_7$$

$$C_3 = a_5 a_{12}$$

Second eqn of (10) gives us

$$\omega^2 = C_2 + C_3 U^2 / C_1 \qquad (11)$$

Substituting this value of ω^2 in eqn (10) we get

$$PU^4 - QU^2 + R = 0 \qquad (12)$$

$$U^2 = \frac{Q \pm \sqrt{Q^2 - 4PR}}{2P}$$

where

$$P = A_1 C_3 + 2C_2 C_3 A_2 - C_1 C_3$$

$$Q = B_1 C_1 C_3 + C_1 C_2 B_2 - 2C_2 C_3 A_1 - A_2 C_2^2$$

$$R = A_1 C_2^2 + E_1 C_1^2 - B_1 C_1 C_2$$

The right hand side of eqn (12) is positive. Corresponding to the two values of U^2 from

eqn (12), there are two values of ω^2 from eqn (11). Usually the smaller U^2 is associated with the higher ω^2 in the eqn. (11).

Numerical example:

A numerical example for the coupled vibration of a rotating slender beam involving shear

and aerodynamics forces just described is presented here. Critical speeds and frequencies are computed from eqn. (11) & (12). The physical constants of the blade and other constants are given below Biezeno and Grammel [1]

S = 0.14889 in^2

A = 0.24885 in

t_1 = 0.19712 in

m = 0.00011 lbs

$$C_1^1 = 0.00189\text{x}10^6$$

Ω = 314 sec^{-1}

$$I_\alpha = 0.095235\,\text{m lbs/in}^2$$

$$I = 0.001846\,\text{in}^4$$

$$G = 11.53\text{x}10^6\ \text{lbs/in}^3\,\text{G}$$

$$\rho = 4.67161\text{x}10^6\ \text{lbs/in}^3\,\text{G}$$

$$x_0 = 0.1131136\,\text{in}$$

$$\rho = 0.024\,\text{in}$$

$$\frac{dc_L}{d\alpha} = 6radian \prec 2\Pi$$

$$J = 0.00193\,\text{in}^4$$

$$K^1 = 2/3$$

$$E = 29.20\text{x}10^6\ \text{lbs/in}^2$$

l = 4.41 in

From eqns (12) we obtain the critical speeds U_1^2 and U_2^2 as

$$U_1^2 = 2.7632627\text{x}10^5$$

and

$$U_2^2 = 2.54707\text{x}10^5$$

Substituting these values in eqn (10) we get the fundamental frequency as

$$\omega_1^2 = 1.561170221\text{x}10^5$$

$$\omega_2^2 = 1.561847289\text{x}10^5$$

CONCLUSION:

The result thus obtained shows that the respective speed of the steady flow are the larger of two values of speeds U^2 calculated from Eqn (12) i.e. the critical speed corresponding to torsional coupled vibration of the rotating slender beam under aerodynamics couplings.

The larger of two values of U^2 given by equation (12) will provide the smaller value of ω^2, which will be the upper bound for the frequency of the fundamental mode of vibration. The smaller of two values of U^2 will provide the larger value of ω^2 which will be an upper bound for the next higher mode of vibration?

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