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A Study on Nonlinear Dispersive Partial Differential Equations by RDT Method

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Abstract –* *The nonlinear dispersive equations, including a large body of classes, are widely used models for a great number of problems in the fields of physics, chemistry and biology, and have gained a surge of attention from mathematicians ever since they were derived. In addition to mathematical analysis, the numeric of these equations is also a beautiful world and the studies on it have never stopped.

The aim of this paper is to propose and analyze various numerical methods for some representative classes of nonlinear dispersive equations, which mainly arise in the problems of quantum mechanics and nonlinear optics. Extensive numerical results are also reported, which are geared towards demonstrating the efficiency and accuracy of the methods, as well as illustrating the numerical analysis and applications. Although the subjects considered here is merely a small sample of nonlinear dispersive equations, it is believed that the methods and results achieved for these equations can be applied or extended to more general cases.

Keywords: Nonlinear, Dispersive, Partial, Differential, Equations, RDT Method, dispersive, numerical, etc.

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INTRODUCTION

Nonlinear dispersive and wave equations are fundamental models to many areas of physics and engineering like plasma physics, nonlinear optics, Bose-Einstein condensates, water waves, and general relativity. Examples include the nonlinear Schrondinger, wave, Klein-Gordon, water wave, and Einstein's equations of general relativity. This field of PDE has witnessed an explosion in activity in the past twenty, partly because of several successful cross-pollinations with other areas of mathematics; mainly harmonic analysis, dynamical systems, and probability. It also continues to be one of the most active areas of research, rich with problems and open to many interesting directions.

The course is intended as an introduction to nonlinear dispersive PDE, with an objective of exposing some open questions and directions that are fertile areas for future research. Over the past few decades, an extensive body of studies have contributed to the mathematical theories of various classes of dispersive equations; and the analytical results, like local and global well-posedness theory, existence and uniqueness of stationary states and so forth, are rich and vast in the literature (see, e.g., some recent monographs on this topic). In parallel with the analytical studies, a surge of efforts have been devoted to the numeric of these equations, which is a topic of great interests from the point of view of concrete real-world applications to physics and other sciences. Although the numerical approximation of solutions of differential equations is a traditional topic in numerical analysis, has a long history of development and has never stopped, it remains as the beating heart in this field that to propose more sophisticated numerical methods for dispersive equations.

REVIEW OF LITERATURE:

In the early 1990's, Michael Berry, discovered that the time evolution of rough initial data ON periodic domains through the free space linear Schrodinger equation exhibits radically different behavior depending upon whether the elapsed time is a rational or irrational multiple of the length of the space interval. Specifically, given a step function as initial conditions, one finds that, at rational times, the solution is piecewise constant, but discontinuous, whereas at irrational times it is a continuous but nowhere differentiates fractal-like functions.

According to (Olver, P.J. AND Oskolkov, K.I.), it was shown that the same Talbot effect of dispersive quantization and racialization appears in general periodic linear dispersive equations whose dispersion relation is a multiple of a polynomial with integer coefficients (an "integral polynomial"), the prototypical example being the linearized Korteweg-deVries equation. Subsequently, it was numerically observed, that the effect persists for more general dispersion relations which are asymptotically polynomial: $\omega(k) \sim c\,k^n$ for large wave numbers $k \gg 0$, where $c \in \mathbb{R}$ and $2 \leq n \in \mathbb{N}$.

Nonlinear Dispersive (*m*, *n*) Equations by RDT Method:

Searching the solitary solutions of nonlinear equations has decisive role in mathematical physics. There are many nonlinear equations applicable in engineering, fluid mechanics, biology, hydrodynamics and physics (for example plasma physics, solid state physics, fluid mechanics), such as Korteweg-de Wries (KdV) equation, mKdV equation, RLW equation, Sine-Gordon equation, Boussinesq equation, Burgers equation, etc. Firstly Wadati developed KdV solution and the mKdV solution. Here, we mention a simple form of the well-known KdV equation.

$$u_t - a u u_x + u_{xxx} = 0. \tag{1.1}$$

The dispersion term u_{xxx} in the equation (1.1) makes the wave form spread. Solitons has been studied by many numerical and analytical methods such as Adomian decomposition method, homotopy perturbation method, variational methods, exp-function method, generalized auxiliary equation method, Hirota's bilinear method, homogeneous balance method, inverse scattering method, sine-cosine method, differential transform method, Backlund transformation, tanh-coth method and finite difference method . In this study we will apply the semi-functional or reduced differential transform method (RDTM) to solve the nonlinear dispersive equation, which is a compacton, called generalized KDV equation

$$u_t \pm a(u^m)_x + (u^n)_{xxx} = 0, \qquad m, n \geq 1 \tag{1.2}$$

If function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0}, \tag{1.3}$$

Definition-

The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k. \tag{1.4}$$

Then combining equation (1.3) and (1.4) we write

$$u(x,t) = \sum_{k=0}^{n} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0} t^k \tag{1.5}$$

For the purpose of illustration of the methodology to the proposed method, we write the nonlinear dispersive K(m, n) equation in the standard operator form

$$L(u(x,t)) + R(u(x,t)) + N(u(x,t)) = g(x,t) \tag{1.6}$$

With initial condition

$$u(x,0) = f(x) \tag{1.7}$$

Where $L = \frac{\partial}{\partial t}$ is a linear operator, $N(u(x,t)) = a(u^m)_x + (u^n)_{xxx}$ is a nonlinear term, $R(u(x,t))$ is remaining linear term and $g(x,t)$ is an inhomogeneous term.

According to the RDTM and Table 1, we can construct the following iteration formula:

$$(k+1)U_{k+1}(x) = G_k(x) - R(U_k(x)) - N(U_k(x)) \tag{1.8}$$

Where $R(U_k(x))$, $N(U_k(x))$ and $G_k(x)$ are the transformations of the functions $R(u(x,t))$, $N(u(x,t))$ and $g(x,t)$ respectively. We can write first few nonlinear terms as

$$N_0 = a\left(\tfrac{\partial}{\partial x} U_0^m(x)\right) + \left(\tfrac{\partial^3}{\partial x^3} U_0^n(x)\right),$$

$$N_1 = a\left(\tfrac{\partial}{\partial x} m U_0^{m-1}(x) U_1(x)\right) + \left(\tfrac{\partial^3}{\partial x^3} n U_0^{n-1}(x) U_1(x)\right)$$

$$N_2 = a\left(\tfrac{\partial}{\partial x}\left(m(m-1)U_0^{m-2}(x)U_1(x) + mU_0^{m-1}(x)U_2(x)\right)\right) +$$

$$\left(\tfrac{\partial^3}{\partial x^3}\left(n(n-1)U_0^{n-2}(x)U_1(x) + nU_0^{n-1}(x)U_2(x)\right)\right)$$

It is clear that $R(U_k(x)) = 0$ and $G_k(x) = 0$ at this equation. From the initial condition (1.7), we write

$$U_0(x) = f(x), \tag{1.9}$$

Substituting (1.9) into (1.8) and after recursive calculations, we get the following $U_k(x)$ values. Then the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{\infty}$ gives an approximate solution as,

$$\tilde{u}_n(x,t) = \sum_{k=0}^{n} U_k(x) t^k \tag{1.10}$$

CONCLUSION:

The main goal of this study is to construct an approximate analytical solution for nonlinear dispersive equations. We have achieved this goal by applying reduced differential transform method. The main advantage of the RDT is to provide the user an analytical approximation to the solution, in many cases, an exact solution, in a rapidly convergent sequence with elegantly computed terms. RDT needs small size of computation contrary to other numerical methods, converges rapidly and introduces a significant improvement solving nonlinear dispersive equations over existing methods. The solution procedure of the RDT is simpler than classical Differential Transform Method (DTM) and this method needs less computational effort than classical DTM. For initial value problems RDT gets infinite power series which can be easily expressed in a closed form which is usually exact solution of the problem. The results show that the RDT is a powerful mathematical tool for solving nonlinear dispersive equations; it is also a promising method to solve other nonlinear equations. Maple package programmer is used for all calculations.

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