Journal of Advances in Science and Technology

Vol. X, Issue No. XX, November-2015, ISSN 2230-9659

BAIRE MEASURES ON HOMOGENEOUS COMPACT HYPER GROUPS

AN INTERNATIONALLY INDEXED PEER REVIEWED & REFEREED JOURNAL

Baire Measures on Homogeneous Compact Hyper Groups

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Abstract – An infinite compact group is necessarily uncountable, by the Baire category theorem. A compact "hyper group, in which the product of two points is a probability measure, is much like a compact group, having an everywhere supported invariant measure, an orthogonal system of characters which span the continuous functions in the uniform topology, and a multiplicative semigroup of positive-definite functions. It is remarkable that a compact hyper group can be count ably infinite. In this paper hyper groups, which include the algebra of measures on the p-adic integers which are invariant under the action of the units (for p = 2, 3, 5, • • •) is presented and investigate the question of whether the spectrum or some subset of it has a hyper group structure.

Keywords: Baire, Measures, Homogeneous, Compact, Hyper Groups

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INTRODUCTION

The basic theory of hyper groups has been developed. A hyper group is a compact space on which the space M (H) of (finite) regular Borel measures is a commutative Banach algebra under its natural norm, possessing a multiplication (denoted by *), and such that the space Mp (H) of probability measures is a compact commutative topological (jointly continuous multiplication) semigroup with unit under the weak-topology- a compact commutative topological semigroup with unit. A hyper group is a locally compact space on which the space of finite regular Borel measures has a commutative convolution structure preserving the probability measures (Dunkl, 1973. Dunkl, Ramirez, 1971. Received May 10, 1972. Hasse, Vorlesungen, 1950). The spectrum of the measure algebra of a locally compact abelian group is the semigroup of all continuous semi characters of a commutative compact topological semi group. In this paper we consider the spectrum of abstract measure algebra and investigate the question of whether the spectrum or some subset of it has a hyper group structure (René Spector, 1970. Dunkl, Ramirez, 1971).

REVIEW OF LITERATURE:

For a hyper group H there exists a continuous map $\lambda\colon H \times H \longrightarrow M(H)$ defined by $\lambda(x, y) = \delta_x * \delta_y \in M_p(H)$. For $f \in C(H)$ the space of continuous functions on H, and $x \in H$ define $R(x)f \in C(H)$, by $R(x)f(y) = \int_H f\,d\lambda(y, x) \quad (y \in H)$ if a hyper group H possesses an invariant measure $m \in M_p(H)$, (that is, $\int_H R(x)f\,dm = \int_H f\,dm,\ f \in C(H),\ x \in H)$ and a continuous involution $x \longmapsto x' \quad (x \in H)$ such that,

$$\int_H (R(x)f)\bar{g}\,dm = \int_H f(R(x')g)^{-}\,dm \qquad (f, g \in C(H),\ x \in H),$$

And, $e \in \operatorname{spt} \lambda(x, x') \qquad (x \in H),$

Then H is called a -hyper group (spt μ denotes the minimum closed subset of H carrying the measure μ.).

$$\phi(x)\phi(y) = \int_H \phi\,d\lambda(x, y) \qquad (x, y \in H)$$

Consequences of these definitions are **(l)** the space H of characters is an orthogonal basis for L^2 (H, *dm*), and **(2)** spt m = H.

1. **Symmetrization of hyper groups:** The method of Symmetrization of a hyper group was introduced by the authors in (Pym, 1968). We will in this paper use this construction to produce a denumerable compact F*-hyper group-a striking contrast to infinite compact groups.

Given a homeomorphism t on a compact F*-hyper group //, define T_j: $C(H) \to C(H)$ by $\tau_1 f(x) = f(\tau x),\ f \in C(H),\ x \in H.$ Let τ_1^* be the (weak-* continuous) ad joint of T_1that is, $\int_H f\,d\tau_1^*\mu = \int_H f \circ \tau_1\,d\mu \quad (f \in C(H), \mu \in M(H)).$ the homeomorphism T is called an auto morphism if $\tau_1^*\lambda(x, y) = \lambda(\tau x, \tau y)$

$(x, y \in H)$ this implies that, for $\phi \in \hat{H}$, $\phi \circ \tau \in \hat{H}$, and that $\tau(x)' = \tau(x')$ $(x \in H)$.

2. **A countable compact hyper groups:** Motivated by the results of §3, we will in this section show how to construct for any compact countable F*-hyper group. For p prime and a = 1/p the example agrees with the hyper group H_W constructed in §3 (Glicksberg, 1959. Ragozin, 1972. Taylor, 1965). Let a be such that $0 < a \leqslant ½$ and define H_a to be the compact space Z_+^*. Define the measure *m* on H_a by

$$m(k) = \begin{cases} (1-a)a^k, & k \neq \infty, \\ 0, & k = \infty. \end{cases}$$

For each $n \in Z_+$ define,

$$\chi_n(k) = \begin{cases} 0, & k < n-1, \\ a/(a-1), & k = n-1, \\ 1, & k \geqslant n \text{ or } k = \infty. \end{cases}$$

For $n, m \in Z_+$ with $n \neq m$, define $\lambda(n, m) = \delta(\min(n, m))$, and for $n = m$ let

$$\lambda(n, n)(t) = \begin{cases} 0, & t < n, \\ \frac{1-2a}{1-a}, & t = n, \\ a^k, & t = n + k > n. \end{cases}$$

CONCLUSION:

The theory is then applied to the measure algebra of a compact P*-hyper group, the algebra of central measures on a compact group, or the algebra of measures on certain homogeneous spaces. A further hypothesis, which is satisfied by the algebra of measures given by ultra-spherical series, is given and it is used to give a complete description of the spectrum and the idempotent in this case. In this paper we consider the spectrum of theoretical measure algebra and investigate the question of whether the spectrum or some subset of it has a hyper group structure

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Received May 10, (1972). This research was partly supported by NSF Grant GP-31483X.

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