Journal of Advances in Science and Technology

Vol. X, Issue No. XX, November-2015, ISSN 2230-9659

A RESEARCH ON VARIOUS SYSTEMS AND CONTROL THEORY OF MATHEMATICAL MODELING

AN INTERNATIONALLY INDEXED PEER REVIEWED & REFEREED JOURNAL

www.ignited.in

A Research on Various Systems and Control Theory of Mathematical Modeling

Amit Kumar*

Research Scholar

Abstract – A mathematical model is an exclusion law. A mathematical model expresses the opinion that some things can happen, are possible, while others cannot, are declared impossible. The problem of regulation is to design mechanisms that keep convinced to be forbidden variables at stable values against outside fighting that act on the plant that is being regulated, or changes in its properties.

---------------------------♦-----------------------------

INTRODUCTION

In studying control systems the reader must be able to model dynamic systems in mathematical terms and analyze their dynamic characteristics. A mathematical model of a dynamic system is defined as a set of equations that represents the dynamics of the system accurately, or at least fairly well. Note that a mathematical model is not unique to a given system. A system may be represented in many different ways and, therefore, may have many mathematical models, depending on one's perspective.

The dynamics of many systems, whether they are mechanical, electrical, thermal, economic, biological, and so on, may be described in terms of differential equations. Such differential equations may be obtained by using physical laws governing a particular system—for example, Newton's laws for mechanical systems and Kirchhoff's laws for electrical systems.We must always keep in mind that deriving reasonable mathematical models is the most important part of the entire analysis of control systems.

Throughout this book we assume that the principle of causality applies to the systems considered.This means that the current output of the system (the output at time t=0) depends on the past input (the input for t<0) but does not depend on the future input (the input for t>0).

Mathematical models may assume many different forms. Depending on the particular system and the particular circumstances, one mathematical model may be better suited than other models. For example, in optimal control problems, it is advantageous to use state-space representations. On the other hand, for the transient-response or frequency-response analysis of single-input, single-output, linear, time-invariant systems, the transfer-function representation may be more convenient than any other. Once a mathematical model of a system is obtained, various analytical and computer tools can be used for analysis and synthesis purposes.

In obtaining a mathematical model, we must make a compromise between the simplicity of the model and the accuracy of the results of the analysis. In deriving a reasonably simplified mathematical model, we frequently find it necessary to ignore certain inherent physical properties of the system. In particular, if a linear lumped-parameter mathematical model (that is, one employing ordinary differential equations) is desired, it is always necessary to ignore certain nonlinearities and distributed parameters that may be present in the physical system. If the effects that these ignored properties have on the response are small, good agreement will be obtained between the results of the analysis of a mathematical model and the results of the experimental study of the physical system.

In general, in solving a new problem, it is desirable to build a simplified model so that we can get a general feeling for the solution.A more complete mathematical model may then be built and used for a more accurate analysis.

We must be well aware that a linear lumped-parameter model, which may be valid in low-frequency operations, may not be valid at sufficiently high frequencies, since the neglected property of distributed parameters may become an important factor in the dynamic behavior of the system. For example, the mass of a spring may be neglected in lowfrequency operations, but it becomes an important property of the system at high frequencies. (For the case where a mathematical model involves considerable errors, robust control theory may be applied.

Control theory has two main roots: regulation and trajectory optimization. The first, regulation, is the more important and engineering oriented one. The second, trajectory optimization, is mathematics based. However, as we shall see, these roots have to a large extent merged in the second half of the twentieth century. The problem of regulation is to design mechanisms that keep certain to be controlled variables at constant values against external disturbances that act on the plant that is being regulated, or changes in its properties. The system that is being controlled is usually referred to as the plant, a passé partout term that can mean a physical or a chemical system, for example. It could also be an economic or a biological system, but one would not use the engineering term “plant” in that case. Examples of regulation problems from our immediate environment abound.

Houses are regulated by thermostats so that the inside temperature remains constant, notwithstanding variations in the outside weather conditions or changes in the situation in the house: doors that may be open or closed the number of persons present in a room, activity in the kitchen, etc. Motors in washing machines, in dryers, and in many other household appliances are controlled to run at a fixed speed, independent of the load. Modern automobiles have dozens of devices that regulate various variables. It is, in fact, possible to view also the suspension of an automobile as a regulatory device that absorbs the irregularities of the road so as to improve the comfort and safety of the passengers. Regulation is indeed a very important aspect of modern technology. For many reasons, such as efficiency, quality control, safety, and reliability, industrial production processes require regulation in order to guarantee that certain key variables (temperatures, mixtures, pressures, etc.) are kept at appropriate values. Factors that inhibit these desired values from being achieved are external disturbances, as for example the properties of raw materials and loading levels or changes in the properties of the plant, for example due to aging of the equipment or to failure of some devices. Regulation problems also occur in other areas, such as economics and biology.

One of the central concepts in control is feedback. A good example of a feedback regulator is a thermostat: it senses the room temperature, compares it with the set point (the desired temperature), and feeds back the result to the boiler, which then starts or shuts off depending on whether the temperature is too low or too high.

THE SYSTEMS APPROACH

The systems approach has beginnings far back in history. But as modern systems analysis has broadened, it has already begun to be controversial and misunderstood. The systems approach has quickly attracted overly zealous proponents and, as often, misinformed detractors. Substantial disagreement exists among the professionals as to how useful the approach is for the bigger problems of society, or for smaller ones when they are more "social" than "technological." This confuses the nonprofessional as to what the approach really is. It impedes its appropriate application. Some hail it as magic, a new all-powerful tool that can demolish any tough problem, engineering or human. Of course, there are always the doubters, the mentally lazy or ignorant who are annoyed with the entry of something new. And there are some aerospace engineers who have used the systems approach but only for narrow problems in their specialized field. They often do not realize they must extend their team capabilities considerably to handle complex social-engineering problems. Some experienced systems engineers go to the other extreme, certain the discipline is inappropriate for "people" problems. In this viewpoint, they are sometimes joined by experts schooled in the more unpredictable behavior of man. Some of these more socially trained individuals are concerned that the systems approach's disciplines cannot be applied successfully to the real-life problems of the human aspects of our civilization.

The systems approach will not solve substantial problems overnight, nor will it ever solve all of them. No matter how broadly skillful is the systems team, the approach is no more than a tool. It will never give us something for nothing, or point the way to an ideal organization of all society, or lead to the planning and production of all of the products of society so as to satisfy all. It will not change the nature of man. It will provide, that is, no miracles. All it can do is help to achieve orderly, timely, and rational designs and decisions. But this "minimum" is something very important. So severe are some of our problems today that chaos threatens. The systems approach to the analysis and design of anything— from a traffic management system to a new city, from a regional medical clinic to a full hospital and medical center, from an automated fingerprint identification system to a fully integrated criminal justice system— will provide no facility of infinite capacity. But it will lead us to designs and operations that will at least not be chaotic. The systems approach, if it is used wisely, is, at the least, a cure for chaos.

SYSTEM DESIGN, A NECESSARY STEP TO COMPONENT AVAILABILITY

The systems approach is flattering vital for still another reason. Without a good systems analysis and system design as a first step, or at least as a parallel effort, it is not easy to understand and specify the necessary pieces of the solution. If the parts required are not called out, no one will set out to make them available. These components, which the systems design will bring together into a pleasant-sounding ensemble to meet the problem, include many items: needed equipment and materiel; people trained in specific jobs with spelled-out functions and procedures; the right kind of information, stored and flowing, so that the people and the things know what to do and where to be to make the whole system operate. For example, the need for systems works to tell us what mechanism we need. For educational system we know that we must greatly enhance educational resources and techniques to provide for more and better education for more young people, for retraining of adults for new jobs, and for growth of the abilities of most of us to keep pace with the requirements of the society. We particularly need a massive rise in educational potency in poverty areas

Now, to meet these needs, we have reason to suppose that technological aids can be very important to extend the effort of the human educator much as X - rays and electrocardiographs and blood tests assist the physician. These aids include special films, closed circuit TV, electronic language laboratories, computer-based education and training programs, and other equipment for the presenting of educational material, the handling of data and information, and for assisting the educator and administrator in planning, analysis of results, and research. But what specific technological devices will accomplish exactly what within what educational framework? If computer-based teaching machines are to be installed, how are they to be used so as to yield real advantages instead of perhaps the disadvantage of creating a sort of robot teacher or evolving to a simple source of entertainment? To answer, we must think such things as the psychology and principles of teaching, the choice of what is to be taught, and how the results will be measured. The actual hardware and software design of some new teaching devices may be the easiest part of the system manufacturing, once we really see what we need.

MATHEMATICAL MODEL

A mathematical model is an exclusion law. A mathematical model expresses the opinion that some things can happen, are possible, while others cannot, are declared impossible. Thus Kepler claims that planetary orbits that do not satisfy his three famous laws are impossible. In particular, he judges no elliptical orbits as unphysical. The second law of thermodynamics limits the transformation of heat into mechanical work. Certain combinations of heat, work, and temperature histories are declared to be impossible. Economic production functions tell us that certain amounts of raw materials, capital, and labor are needed in order to manufacture a finished product: it prohibits the creation of finished products unless the required resources are available. We formalize these ideas by stating that a mathematical model selects a certain subset from a universe of possibilities. This subset consists of the occurrences that the model allows, that it declares possible. We call the subset in question the behavior of the mathematical model.

Definition A mathematical model is a pair (U,B) with U a set, called the universe—its elements are called outcomes—and B a subset of U, called the behavior.

Example 1

Economists believe that there exists a relation between the amount P produced of a particular economic resource, the capital K invested in the necessary infrastructure, and the labor L expended towards its production. A typical model looks like $U = R^3_+$ and $B = \{(P,K,L) \in R^3_+ \mid P = F(K,L)\}$, where $F : R^2_+ \to R+$ is the production function.

Typically, $F : (K,L) 7\to \alpha K^\beta L^\gamma$, with $\alpha, \beta, \gamma \in R+$, $0 \le \beta \le 1$, $0 \le \gamma \le 1$, constant parameters depending on the production process, for example the type of technology used. Before we modeled the situation, we were ready to believe that every triple $(P,K,L) \in R^3_+$ could occur. After introduction of the production function, we limit these possibilities to the triples satisfying $P = \alpha K^\beta L^\gamma$. The subset of R^3_+ obtained this way is the behavior in the example under consideration.

Example 2

During the ice age, shortly after Prometheus stole fire from the gods, man realized that H2O could appear, depending on the temperature, as liquid water, steam, or ice. It took a while longer before this situation was captured in a mathematical model. The generally accepted model, with the temperature in degrees Celsius, is U ={ice, water, steam}× [−273,∞) and B = (({ice}×[−273, 0])∪({water }×[0, 100])∪({steam}× [100,∞)).

SYSTEMS DEFINED BY LINEAR DIFFERENTIAL EQUATIONS

A very common class of dynamical systems consists of the systems that are:

- Linear
- Time-invariant
- Described by differential (or, in discrete time, difference) equations.

The importance of such dynamical systems stems from at least two aspects. First, their prevalence in applications, indeed, many models used in science and (electrical, mechanical, chemical) engineering are by their very nature linear and time-invariant. Secondly, the small signal behavior of a nonlinear time-invariant dynamical system in the neighborhood of an equilibrium point is time-invariant and approximately linear. The process of substituting the nonlinear model by the linear one is called linearization.

Linear systems lend themselves much better to analysis and synthesis techniques than nonlinear systems do. Much more is known about them. As such, the theory of linear systems not only plays an exemplary role for the nonlinear case, but has also reached a much higher degree of perfection. The systems under consideration are those described by linear constant-coefficient differential equations. Dynamical system is determined by its behavior. The systems of differential equations that can be transformed into each other by premultiplication by a unimodular matrix represent the same behavior. Conversely, we will investigate the relation between representations that define the same behavior. It turns out that under a certain condition such differential equation representations can be transformed into each other by means of premultiplication by a suitable unimodular matrix.

MODELING IN STATE SPACE

In this section we shall present introductory material on state-space analysis of control systems.

Modern Control Theory - The modern trend in engineering systems is toward greater complexity, due mainly to the requirements of complex tasks and good accuracy. Complex systems may have multiple inputs and multiple outputs and may be time varying. Because of the necessity of meeting increasingly stringent requirements on the performance of control systems, the increase in system complexity, and easy access to large scale computers, modern control theory, which is a new approach to the analysis and design of complex control systems, has been developed since around 1960.This new approach is based on the concept of state. The concept of state by itself is not new, since it has been in existence for a long time in the field of classical dynamics and other fields.

Modern Control Theory Versus Conventional Control Theory - Modern control theory is contrasted with conventional control theory in that the former is applicable to multiple-input, multiple-output systems, which may be linear or nonlinear, time invariant or time varying, while the latter is applicable only to linear timeinvariant single-input, single-output systems. Also, modern control theory is essentially time-domain approach and frequency domain approach (in certain cases such as H-infinity control), while conventional control theory is a complex frequency-domain approach. Before we proceed further, we must define state, state variables, state vector, and state space.

State. The state of a dynamic system is the smallest set of variables (called *state variables*) such that knowledge of these variables at $t=t_0$, together with knowledge of the input for $t \geq t_0$, completely determines the behavior of the system for any time $t \geq t_0$.

Note that the concept of state is by no means limited to physical systems. It is applicable to biological systems, economic systems, social systems, and others.

State Variables. The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. If at least *n* variables $x_1, x_2, \ldots, x_n$ are needed to completely describe the behavior of a dynamic system (so that once the input is given for $t \geq t_0$ and the initial state at $t = to$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables.

Note that state variables need not be physically measurable or observable quantities. Variables that do not represent physical quantities and those that are neither measurable nor observable can be chosen as state variables. Such freedom in choosing state variables is an advantage of the state-space methods. Practically, however, it is convenient to choose easily measurable quantities for the state variables, if this is possible at all, because optimal control laws will require the feedback of all state variables with suitable weighting.

State Vector. If *n* state variables are needed to completely describe the behavior of a given system, then these *n* state variables can be considered the *n* components of a vector x. Such a vector is called a *state vector.* A state vector is thus a vector that determines uniquely the system state \(t) for any time $t \geq t_0$, once the state at $t = t_0$ is given and the input $u(t)$ for $t \geq t_0$ is specified.

State Space. The /t-dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis, $\ldots, x_n$ axis, where $x_1, x_2, \ldots, x_n$ are state variables, is called a *state space.* Any state can be represented by a point in the state space.

State-Space Equations. In state-space analysis we are concerned with three types of variables that are involved in the modeling of dynamic systems: input variables, output variables, and state variables. The state-space representation for a given system is not unique, except that the number of state variables is the same for any of the different state-space representations of the same system.

The dynamic system must involve elements that memorize the values of the input for $t \geq t_1$. Since integrators in a continuous-time control system serve as memory devices, the outputs of such integrators can be considered as the variables that define the internal state of the dynamic system. Thus the outputs of integrators serve as state variables. The number of state variables to completely define the dynamics of the system is equal to the number of integrators involved in the system.

Assume that a multiple-input, multiple-output system involves *n* integrators. Assume also that there are *r* inputs $u_1(t), u_2(t), \ldots, u_r(t)$ and *m* outputs $y_1(t), y_2(t), \ldots, y_m(t)$. Define *n* outputs of the integrators as state variables: $x_1(t), x_2(t), \ldots, x_n(t)$ Then the system may be described by

$$\begin{aligned}
\dot{x}_1(t) &= f_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
\dot{x}_2(t) &= f_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
&\cdot \\
&\cdot \\
&\cdot \\
\dot{x}_n(t) &= f_n(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)
\end{aligned} \quad (1)$$

The outputs $y_1(t), y_2(t), \ldots, y_m(t)$ of the system may be given by

$$\begin{aligned}
y_1(t) &= g_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
y_2(t) &= g_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
&\cdot \\
&\cdot \\
&\cdot \\
y_m(t) &= g_m(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)
\end{aligned} \quad (2)$$

If we define

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n(t) \end{bmatrix}, \quad \mathbf{f}(\mathbf{x}, \mathbf{u}, t) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ f_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ \cdot \\ \cdot \\ \cdot \\ f_n(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \end{bmatrix},$$

$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdot \\ \cdot \\ \cdot \\ y_m(t) \end{bmatrix}, \quad \mathbf{g}(\mathbf{x}, \mathbf{u}, t) = \begin{bmatrix} g_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ g_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ \cdot \\ \cdot \\ \cdot \\ g_m(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \end{bmatrix}, \quad \mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \cdot \\ \cdot \\ \cdot \\ u_r(t) \end{bmatrix}$$

then Equations (1) and (2) become

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \quad (3)$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}, \mathbf{u}, t) \quad (4)$$

where Equation (3) is the state equation and Equation (4) is the output equation. If vector functions f and/or g involve time *t* explicitly, then the system is called a time- varying system.

If Equations (3) and (4) are linearized about the operating state, then we have the following linearized state equation and output equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \quad (5)$$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t) \quad (6)$$

where A*(t)* is called the state matrix, B(t) the input matrix, C*(t)* the output matrix, and D*(t)* the direct transmission matrix.

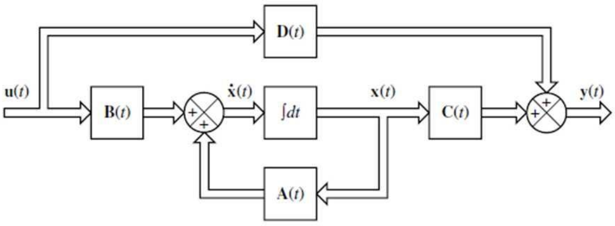

Figure 1: Block diagram of the linear, continuous time control system represented in state space.

A block diagram representation of Equations (5) and (6) is shown in Figure 1.

If vector functions f and g do not involve time *t* explicitly then the system is called a time-invariant system. In this case. Equations (5) and (6) can be simplified to

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (7)$$

$$\dot{\mathbf{y}}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad (8)$$

Equation (7) is the state equation of the linear, time-invariant system and Equation (8) is the output equation for the same system. In this book we shall be concerned mostly with systems described by Equations (7) and (8).

In what follows we shall present an example for deriving a state equation and output equation.

CONCLUSION

The reliability and availability analysis of process industries can benefit in terms of higher production, lower maintenance costs. The Availability of complex systems and continuous process industries can be enhanced by considering maintenance, inspection, repairs and replacements of the parts of the failed units. A mathematical model expresses the opinion that some things can happen, are possible, while others cannot, are declared impossible.

REFERENCES

Ang, K.H., Chong, G.C.Y., and Li, Y. (2005). PID control system analysis, design, and technology, IEEE Trans Control Systems Tech, 13(4), pp. 559-576.

B. Aulbach and T. Wanner (2006). Integral manifolds for Carath´eodory type differential equations in Banach spaces. In B. Aulbach and F. Colonius, editors, Six Lectures on Dynamical Systems, pages 45 119. World Scientific, Singapore. Behaviour and availability analysis of some industrial systems.

Christopher Kilian (2005). Modern Control Technology. Thompson Delmar Learning. ISBN 1- 4018-5806-6.

E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. DuCroz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen (2009). LAPACK Users' Guide, volume 9 of Software - Environments - Tools. SIAM Publications, Philadelphia, PA, 3rd edition.

F. Camilli, L. Gr¨une, and F. Wirth (2000). A regularization of Zubov's equation for robust domains of attraction. In A. Isidori et al., editors, Nonlinear Control in the Year 2000, volume 258 of Lecture Notes in Control and Information Sciences, pages 277– 290. Springer-Verlag, Berlin. Introduction to the mathematical theory of systems and control by Jan Willem Polderman Jan C. Willems

L. Atanassova, D. Hinrichsen, and V. L. Kharitonov (2008). Convex directions for stable polynomials and quasipolynomials: A survey of recent results. In L. Dugard and E. I. Verriest, editors, Stability and Control of Time-Delay Systems, number 228 in Lecture Notes in Control and Information Sciences, pages 72–91. Springer-Verlag, London.

R. N. Clark (2005). Control System Dynamics. Cambridge University Press, Cambridge.

The systems approach fresh solutions to complex problemsthrough combining science and practical common sense by simon ramo, ph.d. and robin k. st.clair, ph.d.

Corresponding Author

Amit Kumar*

Research Scholar

E-Mail – rohitkumarjangra1@gmail.com