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REVIEW ARTICLE

A NEW APPROACH FOR AVAILABILITY ANALYSIS

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A New Approach for Availability Analysis

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The researchers have discussed the reliability and availability of many stochastic systems and process industries by using very cumbersome and time-consuming techniques. The general formulae in the closed form are not yet developed to determine the key parameters for a semi Markov renewal process like mean time to system failure (MTSF), availability of the system, and busy periods of the servers doing different jobs, the number of server's visits and the number of replacements of the components/sub-systems. Chander, S. & Mukender Singh [14], Goyal, Rashmi & Ashok Kumar [40], Tuteja, R.K. et al. [110] and many others, have analyzed and discussed various systems under steady state conditions, using the following formulae of the regenerative point technique to find the key parameters of a stochastic system:

a) MTSF = *Lim* ,v->0 ***S***

b) Availability = *Lim* ***Ao(t)*** *= Lim s.Ao{s)*

c) Busy period of the server = *Lim* ***Bo(t)*** = *Lim s. Bq(s)* **QO** J-* 0

d) Expected number of server's visits /replacements = *Lim = Lim s. Vn(s)*

Where *(s)* is obtained on solving recursively the state governing equations after taking the Laplace Stieltjes Transformations (L.S.T.) of each of the state renewal equation, written for each un-failed regenerative state using the equation:

$$ti(t) = I.Q_j\{t)\circledR tj(t) + \pounds 0_{/jjfc}(O \quad \text{..........(2.1.1)}$$
$j \qquad k$

Where *'f* is an un-failed regenerative state to which the given regenerative state '/' can transit and is a failed state to which the state '/' can transit directly.

/4**q(-v**) is obtained on solving recursively the state governing equations after taking the Laplace Transformations (L.T.) of the state renewal equations, written for each regenerative state using the renewal equation:

$$MO = {}_{Mi}(t) +\sim Lq\backslash^{n}]\{f) \copyright Aj(t) \quad \text{...........(2.1.2)}$$
j

Where '/' is any successive regenerative state to which the regenerative state T can transit through n> 1 transitions. **£q(.v**) is obtained as stated for^Q^), using renewal equation:

$$Bf(t) = Wi\{t) + Z.q(\text{»})(t) \copyright Bj\{t) \quad \text{..........(2.1.3)}$$
j

Where ';' 'S a successive regenerative state to which state '/' transits through *ri*>\ transitions.

V*q(s)* is obtained on solving recursively the governing equations after taking the Laplace Stieltjes Transformations (L.S.T.) of the state renewal equations, written for each regenerative state using the equation:

$$V_i\{t) = I,Q_ij(t)\circledR[Sj + Vj\{t)\backslash \quad \text{..........(2.1.4)}$$
j

Where ';" is any regenerative state to which the given regenerative state '/' transits; *§ j-* 1, if '/ is the regenerative state where the server enters afresh/replacement begins, otherwise *S j* ⁼ 0.

Further, the researchers analyzed the systems without giving the due consideration to the reduced states where the system is not fully available because of its partial failure(s).

A new approach for the analysis of a stochastic system is introduced in this chapter. In addition, the concept of the fuzziness measure of the states is introduced to deal with the working of the system in reduced states. This new approach is illustrated in Section 2.6. The statistical models/formulae are presented in the closed form by the equations (2.5.1) to 35 (2.5.4). These are quite easy to apply without using any lengthy and cumbersome calculations, to find the key parameters of a semi-Markov process with underlying Markov renewal process represented by a finite, ir-reducible and aperiodic Markov chain.

FIRST PASSAGE TIME

The probability density function (p.d.f.) of the first passage time from a regenerative state / to a regenerative state ***j*** or to a failed state ***j,*** without visiting any other regenerative state in (0, /] is defined as

$$<li,j(^t) = MXn + \ = j\Xn = ^i\}$$

where X_n denotes the state of the system at «-th epoch of time.

The cumulative distribution function (c.d.f.) of the first passage time from a regenerative state / to a regenerative state ***j*** or to a failed state ***j,*** without visiting any other regenerative state in (0, /], given that the system entered regenerative state / at ***t*** **=** 0, is defined as

$$Qi /(0=J?/ \&)<^{it} = P^r\Xn + \ = J>tn + \-tn^t\Xn = ^i\0$$

It is the probability of transition from a regenerative state / to the regenerative state ***j*** without visiting any other regenerative state in the interval (0, /], given that the system entered regenerative state / at / = 0. It is also denoted as***j(t).***

Further, since for a finite, irreducible, aperiodic Markov chain, the probability of transition /?• ,(/) becomes independent of the initial state / and it tends to a finite limit as'/'

tends to infinity. This limiting value is called the steady-state transition probability of the state ***j*** and is given by

$$p_i j = Lim\ Pjj(t) = Lim\ 5/,/(0 = Lim \ q_i jif)^{\wedge}$$

$$= \ q_{i\ j}\{t) dt = Lim\ l_e-^{SJ} qi \&)^{dt} = Lim\ q_i\ As)$$

The steady-state probability of a simple path from the state 'f (initial state) to the state '/' (terminal state) is defined as the probability of transition from the state to the state $^l f$ along a given directed simple path (/' —> j).

Let $(oQ -> a_n) = \{«Q,oJ,«2' ••■>^a{}_n-\i\ ^a{}_n\}^{\wedge e\ a}$ given directed simple path from the state to the state a_n in the transition diagram of the system where *no k- cycle* is formed for A: $a^,a^,a_2$, •• > $^a{}_n-l$ ^{anc*} $^an'$ $^{m\ otner\ wor}$ds, $aQ,a^,a_2$, $^an-\ ^{an(*}\ ^an$ ^{are} not the terminals of any circuit in the transition diagram. Let the first passage transition times from to , to a_2, $^{,a}n-\^{oa}n$ ^{are} $^{ran^om}$ variables *X\, X2, X-****i,...*** , X_n having the probability density functions as *q^^t), $q^2(t)$, #2 3^)" $q_{n\_\ n}$ if)* respectively. The p.d.f. of the stochastic variable: S„ = $X\ + X_2 + X_3 + ...+ X_n$ is defined by

$$q(t) = q_Q^t)©q_{12}(t)©q_{2>3}(0©......®q_n-l_{,n}(t), \quad \text{Cox [15]}$$

$$\cdot_1 ••••!\ JtfOd^l)'\ \#1,2\ ("2\sim«l)\ ....q_n-\_n^< u_n-u_n-\)du\du2-du_n-\$$

On talcing Laplace transformation

Therefore, probability of the given simple path («**q**—> $\#_w$) under steady-state conditions, is given by

$$pr(a_0 ^{\wedge}> a_n) = \int_0^{\infty} q\{t)dt = \lim_{t\to\infty} \int_0^t q(t)dt = \lim_{s\to 0} q^*(s) = q^*(u)$$

$$= q^*0,l(0)-q\{_2(0y .....q^*{}_{n\_h},,(0)$$

$$= {}^{\wedge}0,1-^1,2-^2,3Pn-\,n$$

Symbolically,

$$pr(a_0 -> a_n) = (a_Q,\ 0^.(^,0^ ....(^a{}_n- V^a n)$$

$$= (a_0, a_v a_2,\ldots, a_{n\_y} a_n)$$

STEADY STATE TRANSITION PROBABILITY OF A REACHABLE STATE

The steady-state transition probability of a state ***'j'*** reachable from the state '/' is defined as the sum of the steady-state path probabilities of transition from the state '/' to the state *'j'* along all the directed simple paths *(/—> j)* for different values of V, from the state '/' to the state '/.

A simple path *(/—> j)* for a given value of V, from the state /' to the state *j* in the state-transition diagram may have the regenerative state(s) /cat which /c-cycle(s) are formed. The steady-state transition probability of the state *j* reachable from the state / (denoted by *Vi j)* is the conditional probability defined by

$$\frac{pr(i \xrightarrow{Sr} j)}{\prod_k \{1\text{-Z M*- } cycle)\}} \quad (2.4.1)$$

where *k* is a regenerative state belonging to the simple path *(/—^—> j)* where /c-cycle(s) are formed. And £ P^r*(k - cycle)* = the sum of probabilities of all the different *k-* simple circuit(s) formed at the given regenerative point *k* and for a given value of *r.*

If *k* is a regenerative point belonging to the given simple path *(/—*^{Sr} *> j)* such that no /c-cycle is formed, then ^ *pr(k. - cycle)* = 0. And if for a. fixed V the simple path *(/* ^{Sr} *> j)* has one or more regenerative path point(s) where *k-* cycle(s) are formed in the transition diagram of the system, then all such state(s) ^{i}*k'* are considered for the calculation of *T\{\~T.pr(k-cycle)}, k* The steady-state transition probability *V^j* of the state **'7'** reachable from the initial state / = **0** (at *t* = **0)** is **J/q** *j* and it forms the probability distribution < ***Vo,j>*** *J* > °f^{tne} stochastic variable y.

A Particular Case: In particular, if / and *j* are consecutive regenerative states in the transition diagram of the system and (/ ■> *j) = {i, j}* be the only simple path, then

$$pr\,(/ \text{---}> j) = \text{--}^{P}\text{-}\pm\pm\text{-----} \quad 0\text{-}U)0\text{-}Lj) \qquad \text{...........(2.4.2)}$$

where *L k* = *V k, k* = **Z** *pr(k - cycle)).*

And further, if no cycles are formed at / and *j* states, then *Lf = 0, Lj=0.* And in this case the steady-state transition probability of a state '/ reachable from the state '/' is

$$pr(i\text{-----}>J) = Pi,j \qquad \text{............ (2-4.3)}$$

A NEW APPROACH FOR AVAILABILITY ANALYSIS

On considering the concepts explained in Sections 2.3 and 2.4, the vital and key parameters like mean time to system failure (MTSF), availability of the system, busy periods for inspection/ instructions/ repairs/ replacements and the number of visits by different servers doing different types of jobs such as inspection/ instructions/ repairs/ replacements; the number of different types of replacements and the number of preventive & corrective maintenance actions can be evaluated by using the formulae (2.5.1) to (2.5.4) explained as follows:

Mean Time To System Failure *(MTSF):*

The mean time to failure of a system is the statistical average time for which the system is operative before any failure(s) of the system. The term *MTSF* is used when the system undergoes either preventive or corrective maintenance actions. Mean time to system failure (under steady state conditions) of the system is given by **Z** *Vq* /./^ where ***i*** is an un-failed ***I*** regenerative state in the state-transition diagram of the system. On using (2.4.1), the mean time to system failure is

MTSF **Z** {pr(0- (#) -i)].H/ pr(ki- cycle)} **Ui*0** i-z- {pr(0 ^(*#)-)0)} Tl{l-I.pr(k2-cycle)}\ k2*0 1_ .... (2.5.1)

Explanation:

i: a regenerative un-failed state to which the system can transit before entering any failed state while entering the initial 0-state at time t = 0.

k : a regenerative state along the path 39 *s Mf)* (0 ------------------ >*i*), at which a ***k\- cycle*** is formed through regenerative un-failed states.

k_2 : a regenerative state along the path *s* (0 ---------------- > 0), at which ajt$_2$ - ***cycle*** is formed through regenerative un-failed states.

[In the numerator, the coefficient of ***jUj***, for / = 0, is equal to ***pr****(0* —» 0)= (0, 0) = 1, and in the denominator the expression also contributes the term 1 - (0, 0) =1 ~*Pqq* provided there is a loop at the 0-state].

Steady State Availability Of A System:

It is defined as the proportion of time that the system is operational when the time- interval is very^2 large and the corrective, preventive maintenance down times and the waiting times are included.

$$\frac{MTBM}{MTBM + MDT} \qquad AO$$

Where *MTBM* = mean time between maintenance; *MDT* (mean down time) = statistical mean of the down times caused due to breakdowns, including supply down time, administrative down time.

The stale transition diagram takes into account all the times under consideration of the stochastic system/process (under steady state conditions). Therefore, *HVo^'-^j* *IS me* *J* measure of the numerator and *Y.Vo I-m]* i$^{s\ me}$ *i* measure of the denominator, where '/' is a reachable un-failed and is a regenerative state in the state-transition diagram of the system, *ju]* is the total un-conditional time spent before transiting to any other regenerative state(s), given that the system entered regenerative state at *t* = 0]. Thus, steady state availability of a system is given by

$$AO = [ZVo,j\,.\mathbf{jUj}] + [ZVoj.<u\}] \qquad j \qquad i$$

Tn case the system fails partially and is not fully available for its purpose then the availability of the system is discounted according to the proportions to the fuzziness measure of the states that the system can visit. Accordingly, the steady state availability of a system is modified to

Ao = [HVoj-fj.jUj] * i^Voj-juj]
j i

where / • is the fuzziness measure of the un-failed state

On using (2.4.1), the steady state availability of a system is

{pr(0-^i)}.M\
AO I nV-Zpriki-cycle)} n{i-iM£2-^/e)}|
J,s,r £i*0

Explanation:

The 0- state is the state at time *t* = 0.

j: a reachable state which is an available state (which may be down/ or reduced state). ***i:*** a regenerative state.

ytj-(^O): a regenerative path-point (may be an interior or the terminal point of the path) at which a ***kf*** cycle is formed (may be formed through non-regenerative/failed-states). ***k*** is a regenerative state visited along the path (0—^{Sr} >/) and£i can be equal to ***j***. And &$_2$ is ^{a} regenerative state visited along the path (0—^{Sr} ***>i)*** and ***k2*** can be equal to [In the numerator coefficient of *juj* , for ***j*** **=** 0, is equal to ***pr****(0* -> 0)= (0,0) = 1 and in the denominator coefficient of *ju*, for /' = 0, is ***pr****(0* -> 0)= (0,0) =1].

In case a down state of the system is treated as a failed state then for availability purposes the said state is to be treated as un-available state.

Busy Period of the Server:

Busy period of the server (under steady state conditions) doing a given job is defined by

$$B_o = \frac{MTTR}{MTBM + MDT}$$

Where *MTTR* = mean time to repair; *MTBM* = mean time between maintenance; *MDT* = mean down time. *(MDT* is replaced by *M* or *MTTR* as per the real situation to which the stochastic process is subjected during its operation).

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