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Surface Wave Propagation in an Initially Stressed Transversely Isotropic Thermoelastic Solid

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INTRODUCTION

Prestressed materials have various applications, for example, in oil and geophysical industry, NDT in pre-stressed materials, use of rubber composites in automotive, aerospace and defence industries (often in pre-stressed states) and in the study of biological tissues (lung, tendon, etc.) which are all nonlinear prestressed viscoelastic composites. Ames and Straughan (1992) derived the continuous dependence results for initially prestressed thermoelastic bodies. Dhaliwal and Wang (1993) presented a generalized theory for a thermoelastic dipolar body which has previously received a large deformation and is at no uniform temperature. A generalized linear theory of dipolar thermoelasticity with initial stress and initial heat flux has been derived. Some theorems in the generalized theory of thermoelasticity for prestressed bodies are studied by Wang et al. (1997). Marin and Marinescu (1998) studied the asymptotic partition of total energy for the solutions of the mixed initial boundary value problem within the context of the thermoelasticity of initially stressed bodies. Kalinchuk (1999) studied the problem of steady-state harmonic oscillations for a nonhomogeneous thermoelastic prestressed medium. Montanaro (1999) investigated the isotropic linear thermoelasticity with hydrostatic initial stress. Wang and Slattery (2002) formulated the thermoelastic equations without energy dissipation for a body which has previously received a large deformation and is at no uniform temperature.

Iesan (2008) presented a theory of Cosserat thermoelastic solids with initial stresses, initial couple stresses, and initial heat. Chekurin (2008) studied a mathematical model for thermoelastic processes in a piecewise homogeneous prestressed solid. Singh (2010) studied the wave propagation in an initially stressed transversely isotropic thermoelastic solid half space.

The surface wave propagation in an initially stressed transversely isotropic thermoelastic solid is studied. The governing equations are solved to obtain the general solution in x-z plane. The appropriate boundary conditions at an interface between two dissimilar half spaces are satisfied by appropriate particular solutions to obtain the frequency equation of the surface wave in the medium. Some special cases are also discussed.

Governing equations

Following Wang, et. al. (1997), the generalized equations of thermoelasticity for prestressed bodies which are previously at nonuniform temperature To, are

$$\rho_0 \ddot{u}_i = \sigma_{ji,j} + \rho_0 F_i, \qquad (3.1)$$

$$\rho_0 T_0 \dot{\eta} = -q_{i,i} + \rho_0 S, \qquad (3.2)$$

With the constitutive relations

$$\sigma_{ij} = c_{ijmn}\, e_{mn} + e_{jk} P_{ki} - \beta_{ij} T, \qquad (3.3)$$

$$\rho_0 \eta = dT + \beta_{ij}\, e_{ij}, \qquad (3.4)$$

$$q_i + \tau \dot{q}_i = -a_i T - K_{ij} T_{,j} - h_{ijk} e_{jk}, \qquad (3.5)$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \qquad (3.6)$$

Where ɑ0 is the density of the medium, ui are the components of displacement vector, ɑij is stress tensor, Fi are the components of body force vector, ɑ is entropy, qi are the components of heat flux vector, S is internal heat source, eij is strain tensor, Pij is prestress tensor, cijkl are the elastic coefficients, T is change in temperature above the reference nonuniform temperature T_0, β_{ij}, K_{ij}, a_i, h_{ijk} are thermal coefficients, ɑ is the thermal relaxation time, d = ɑ0 ce and ce is the specific heat at constant strain. Using equation (3.3) into equation (3.1) and neglecting the body forces and head sources, we obtain

$$\left(d_{ijmn}e_{mn} - \beta_{ij}T\right)_{,j} = \rho_0 \ddot{u}_i, \tag{3.7}$$

Where $d_{ijmn} = c_{ijmn} + \delta_{jn}P_{mi}$. using equations (3.4) and 3.5 in equation (3.2), we obtain

$$T_0\left(1+\tau\frac{\partial}{\partial t}\right)\left[\beta_{ij}\dot{e}_{ij} + d\dot{T}\right] = \left(K_{ij}T_{ij} + h_{ijk}\ e_{jk}\ + a_i T\right)_{,i} \tag{3.8}$$

Equations (3.7) and (3.8) are governing equations of thermoelasticity for bodies which have previously received a large deformation and are at non uniform temperature T0 . If T0 is assumed uniform in prestressed body, then ai=0, hijk=0.

We consider a homogeneous and transversely isotropic thermoelastic medium of an infinite extent with cartesian coordinates system (x , y , z), which is previously at uniform temperature and under initial stress. We assume that medium is transversely isotropic in such a way that the planes of isotropy are perpendicular to z-axis. The origin is taken on the plane surface and z-axis is taken normally into the medium (z ≥ 0). The surface z = 0 is assumed stress free and thermally insulated. The present study is restricted to the plane strain parallel to xz-plane, with the displacement vector $\vec{u} = (u_1, 0, u_3)$. For two-dimensional solution in xz-plane, we may write the equations (3.7) and (3.8) as

$$d_{11}u_{1,11} + (d_{13}+d_{44})u_{3,13} + d_{44}u_{1,33} - \beta_1 T_{,1} = \rho_0\ddot{u}_i, \tag{3.9}$$

$$d_{44}u_{3,11} + (d_{13}+d_{44})u_{1,13} + d_{33}u_{3,33} - \beta_3 T_{,3} = \rho_0\ddot{u}_3, \tag{3.10}$$

$$T_0\left(1+\tau\frac{\partial}{\partial t}\right)\left[\beta_1\dot{u}_{1,1} + \beta_3\dot{u}_{3,3} + d\dot{T}\right] = K_1T_{,11} + K_3T_{,33}, \tag{3.11}$$

where $d_{11} = c_{11} + p_{11}$, $d_{13} = c_{13}$, $d_{44} = c_{44} + p_{11}$, $d_{33} = c_{33} + p_{33}$, $K_1 = K_{11}$, $K_3 = K_{33}$, $\beta_1 = \beta_{11} = (d_{11}+d_{12})\alpha_1 + d_{13}\alpha_3$, $\beta_3 = \beta_{33} = 2d_{13}\alpha_1 + d_{33}\alpha_3$. are coefficient of linear thermal expansion.

Solution of the governing equations

To solve the equations (3.9) – (3.11), let

$$\{u_1, u_3, T\} = \left\{\overline{u_1}(z), \overline{u_3}(z), \overline{T}(z)\right\}e^{ik(x-ct)}. \tag{3.12}$$

With the help of equation (3.12), the equation (3.9) becomes,

$$d_{11}\left[\overline{u_1}(z)e^{ik(x-ct)}\right]_{,11} + (d_{13}+d_{44})\left[\overline{u_3}(z)e^{ik(x-ct)}\right]_{,13}$$

$$+d_{44}\left[\overline{u_1}(z)e^{ik(x-ct)}\right]_{,33} - \beta_1\left[\overline{T}(z)e^{ik(x-ct)}\right]_{,1} = \rho_0\left[\overline{u_1}(z)e^{ik(x-ct)}\right]_{,tt},$$

$$\Rightarrow\ d_{11}\frac{\partial^2}{\partial x^2}\left[\overline{u_1}(z)e^{ik(x-ct)}\right] + (d_{13}+d_{44})\frac{\partial^2}{\partial x\partial z}\left[\overline{u_3}(z)e^{ik(x-ct)}\right]$$

$$+d_{44}\frac{\partial^2}{\partial z^2}\left[\overline{u_1}(z)e^{ik(x-ct)}\right] - \beta_1\frac{\partial}{\partial x}\left[\overline{T}(z)e^{ik(x-ct)}\right] = \rho_0\frac{\partial^2}{\partial t^2}\left[\overline{u_1}(z)e^{ik(x-ct)}\right],$$

$$\Rightarrow\ d_{11}\frac{\partial}{\partial x}\left[ik\overline{u_1}(z)e^{ik(x-ct)}\right] + (d_{13}+d_{44})\frac{\partial}{\partial x}\left[\frac{\partial}{\partial z}\overline{u_3}(z)e^{ik(x-ct)}\right]$$

$$+d_{44}\frac{\partial}{\partial z}\left[\frac{\partial}{\partial z}\overline{u_1}(z)e^{ik(x-ct)}\right] - \beta_1\left[ik\overline{T}(z)e^{ik(x-ct)}\right] = \rho_0\frac{\partial}{\partial t}\left[-ikc\overline{u_1}(z)e^{ik(x-ct)}\right],$$

$$\Rightarrow\ -d_{11}k^2\overline{u_1}(z)e^{ik(x-ct)} + ik(d_{13}+d_{44})\frac{d}{dz}\overline{u_3}(z)e^{ik(x-ct)}$$

$$+d_{44}\frac{d^2}{dz^2}\overline{u_1}(z)e^{ik(x-ct)} - ik\beta_1\overline{T}(z)e^{ik(x-ct)} = -\rho_0k^2c^2\overline{u_1}(z)e^{ik(x-ct)}.$$

Now writing $\frac{d}{dz} = D,\ \frac{d^2}{dz^2} = D^2$ and canceling the term $e^{ik(x-ct)}$ in the above equation, we have

$$\left[-d_{11}k^2 + d_{44}D^2 + \rho_0k^2c^2\right]\overline{u_1}(z) + \left[ik(d_{13}+d_{44})D\right]\overline{u_3}(z) - ik\beta_1\overline{T}(z) = 0,$$

$$\Rightarrow\ \left[d_{44}D^2 + k^2\left(\rho_0c^2 - d_{11}\right)\right]\overline{u_1}(z) + \left[ik(d_{13}+d_{44})D\right]\overline{u_3}(z) - ik\beta_1\overline{T}(z) = 0,$$

$$\Rightarrow\ \left(d_{44}D^2 + k^2\varepsilon_1\right)\overline{u_1}(z) + i\Omega kD\overline{u_3}(z) - ik\beta_1\overline{T}(z) = 0, \tag{3.13}$$

where $\varepsilon_1 = \rho_0c^2 - d_{11}$, $\Omega = (d_{13}+d_{44})$.

$$d_{44}\left[\overline{u_3}(z)e^{ik(x-ct)}\right]_{,11} + (d_{13}+d_{44})\left[\overline{u_3}(z)e^{ik(x-ct)}\right]_{,13}$$

$$+d_{33}\left[\overline{u_3}(z)e^{ik(x-ct)}\right]_{,33} - \beta_3\left[\overline{T}(z)e^{ik(x-ct)}\right]_{,3} = \rho_0\left[\overline{u_3}(z)e^{ik(x-ct)}\right]_{,tt},$$

$$\Rightarrow\ d_{44}\frac{\partial^2}{\partial x^2}\left[\overline{u}_3(z)e^{ik(x-ct)}\right] + (d_{13}+d_{44})\frac{\partial^2}{\partial x\partial z}\left[\overline{u}_1(z)e^{ik(x-ct)}\right]$$

$$+d_{33}\frac{\partial^2}{\partial x^2}\left[\overline{u}_3(z)e^{ik(x-ct)}\right] - \beta_3\frac{\partial}{\partial z}\left[\overline{T}(z)e^{ik(x-ct)}\right] = \rho_0\frac{\partial^2}{\partial t^2}\left[\overline{u}_3(z)e^{ik(x-ct)}\right],$$

$$\Rightarrow\ d_{44}\left[i^2k^2\overline{u}_3(z)e^{ik(x-ct)}\right] + (d_{13}+d_{44})\left[ik\frac{\partial}{\partial z}\overline{u}_1(z)e^{ik(x-ct)}\right]$$

$$\Rightarrow\ d_{44}\left[-k^2\overline{u_3}(z)e^{ik(x-ct)}\right] + (d_{13}+d_{44})\left[ik\frac{d}{dz}\overline{u_1}(z)e^{ik(x-ct)}\right]$$

$$+d_{33}\left[\frac{d^2}{dz^2}\overline{u}_3(z)e^{ik(x-ct)}\right]-\beta_3\left[\frac{d\overline{T}(z)}{dz}e^{ik(x-ct)}\right]=-\rho_0 i^2k^2c^2\overline{u}_3(z)e^{ik(x-ct)}.$$

With the help of equation (3.12), the equation (3.10) becomes,

Now writing $\frac{d}{dz}=D,\ \frac{d^2}{dz^2}=D^2$ and canceling the term $e^{ik(x-ct)}$ in the above equation, we have

$$-k^2d_{44}\overline{u}_3(z)+ik\left(d_{13}+d_{44}\right)D\overline{u}_1(z)+d_{33}D^2\overline{u}_3(z)-\beta_3D\overline{T}(z)=-\rho_0k^2c^2\overline{u}_3(z),$$

$$\Rightarrow\ ik(d_{13}+d_{44})D\overline{u}_1(z)+(d_{33}D^2+\rho_0k^2c^2-k^2d_{44})\overline{u}_3(z)-\beta_3D\overline{T}(z)=0,$$

$$\Rightarrow\ ik(d_{13}+d_{44})D\overline{u}_1(z)+\left[d_{33}D^2+k^2(\rho_0c^2-d_{44})\right]\overline{u}_3(z)-\beta_3D\overline{T}(z)=0,$$

$$\Rightarrow\ ik\Omega D\overline{u}_1(z)+(d_{33}D^2+k^2\varepsilon_2)\overline{u}_3(z)-\beta_3D\overline{T}(z)=0, \qquad (3.14)$$

Where $\varepsilon_2=\rho_0c^2-d_{44}$.

With the help of equation (3.12), the equation (3.11) becomes,

$$T_0\left(1+\tau\frac{\partial}{\partial t}\right)\left[\beta_1\left\{\overline{u}_1(z)e^{ik(x-ct)}\right\}_{,1t}+\beta_3\left\{\overline{u}_3(z)e^{ik(x-ct)}\right\}_{,3t}\right.$$

$$\left.+d\left\{\overline{T}(z)e^{ik(x-ct)}\right\}_{,t}\right]=K_1\left[\overline{T}(z)e^{ik(x-ct)}\right]_{,11}+K_3\left[\overline{T}(z)e^{ik(x-ct)}\right]_{,33},$$

$$\Rightarrow\ T_0\left(1+\tau\frac{\partial}{\partial t}\right)\left[\beta_1\frac{\partial^2}{\partial x\partial t}\left\{\overline{u}_1(z)e^{ik(x-ct)}\right\}+\beta_3\frac{\partial^2}{\partial z\partial t}\left\{\overline{u}_3(z)e^{ik(x-ct)}\right\}\right.$$

$$\left.+d\frac{\partial}{\partial t}\left\{\overline{T}(z)e^{ik(x-ct)}\right\}\right]=K_1\frac{\partial^2}{\partial x^2}\left[\overline{T}(z)e^{ik(x-ct)}\right]+K_3\frac{\partial^2}{\partial z^2}\left[\overline{T}(z)e^{ik(x-ct)}\right],$$

$$\Rightarrow\ T_0\left(1+\tau\frac{\partial}{\partial t}\right)\left[\beta_1\frac{\partial}{\partial x}\left\{-ikc\overline{u}_1(z)e^{ik(x-ct)}\right\}+\beta_3\frac{\partial}{\partial z}\left\{-ikc\overline{u}_3(z)e^{ik(x-ct)}\right\}\right.$$

$$\left.+d\left\{-ikc\overline{T}(z)e^{ik(x-ct)}\right\}\right]=K_1\frac{\partial}{\partial x}\left[ik\overline{T}(z)e^{ik(x-ct)}\right]+K_3\frac{\partial}{\partial z}\left[\frac{\partial\overline{T}(z)}{\partial z}e^{ik(x-ct)}\right],$$

$$\Rightarrow\ T_0\left(1+\tau\frac{\partial}{\partial t}\right)\left[\beta_1\left\{-i^2k^2c\overline{u}_1(z)e^{ik(x-ct)}\right\}+\beta_3\left\{-ikc\frac{\partial}{\partial z}\overline{u}_3(z)e^{ik(x-ct)}\right\}\right.$$

$$\left.+d\left\{-ikc\overline{T}(z)e^{ik(x-ct)}\right\}\right]=K_1\left[i^2k^2\overline{T}(z)e^{ik(x-ct)}\right]+K_3\left[\frac{\partial^2}{\partial z^2}\overline{T}(z)e^{ik(x-ct)}\right],$$

$$\Rightarrow\ T_0\left(1+\tau\frac{\partial}{\partial t}\right)\left[\beta_1k^2c\overline{u}_1(z)-i\beta_3kc\frac{d}{dz}\overline{u}_3(z)-idkc\overline{T}(z)\right]e^{ik(x-ct)}$$

$$=\left[-K_1k^2\overline{T}(z)+K_3\frac{d^2}{dz^2}\overline{T}(z)\right]e^{ik(x-ct)},$$

$$\Rightarrow T_0\left[\left\{\beta_1k^2c\overline{u}_1(z)-i\beta_3kc\frac{d}{dz}\overline{u}_3(z)-idkc\overline{T}(z)\right\}-i\tau kc\left\{\beta_1k^2c\overline{u}_1(z)\right.\right.$$

$$\left.\left.-i\beta_3kc\frac{d}{dz}\overline{u}_3(z)-idkc\overline{T}(z)\right\}\right]e^{ik(x-ct)}=\left[-K_1k^2\overline{T}(z)+K_3\frac{d^2}{dz^2}\overline{T}(z)\right]e^{ik(x-ct)}.$$

Now writing $\frac{d}{dz}=D,\ \frac{d^2}{dz^2}=D^2$ and canceling the term $e^{ik(x-ct)}$ in the above equation, we have

$$T_0\left[(\beta_1k^2c(1-i\tau kc))\overline{u}_1(z)-i\beta_3kc(1-i\tau ck)D\overline{u}_3(z)-idkc(1-i\tau kc)\overline{T}(z)\right]$$

$$=-K_1k^2\overline{T}(z)+K_3D^2\overline{T}(z),$$

$$\Rightarrow T_0(1-i\tau\omega)\beta_1k\omega\overline{u}_1(z)-iT_0(1-i\tau\omega)\beta_3\omega D\overline{u}_3(z)$$

$$+\left[-K_3D^2+K_1k^2-iT_0d\omega(1-i\tau\omega)\right]\overline{T}(z)=0,$$

$$\Rightarrow -i\omega T_0\left(\tau+\frac{i}{\omega}\right)\beta_1k\omega\overline{u}_1(z)-T_0\omega\left(\tau+\frac{i}{\omega}\right)\beta_3\omega D\overline{u}_3(z)$$

$$+d\left[\frac{-K_3}{d}D^2+\frac{-K_1k^2}{d}-T_0\omega^2\left(\tau+\frac{i}{\omega}\right)\right]\overline{T}(z)=0,$$

Taking $\tau^*=\tau+\frac{i}{\omega},\ \overline{K}_3=\frac{K_3}{d}$ and $\overline{K}_1=\frac{K_1}{d}$ in the above equation, we have

$$-i\omega^2T_0\beta_1k\tau^*\overline{u}_1(z)-\omega^2T_0\beta_3\tau^*D\overline{u}_3(z)+d\left[-\overline{K}_3D^2+\overline{K}_1k^2-T_0\omega^2\tau^*\right]\overline{T}(z)=0,$$

$$\Rightarrow\ -i\omega^2\tau^*k\frac{\beta_1T_0}{d}\overline{u}_1(z)-\omega^2\tau^*\frac{\beta_3T_0}{d}D\overline{u}_3(z)+\left[-\overline{K}_3D^2+\overline{K}_1k^2-T_0\omega^2\tau^*\right]\overline{T}(z)=0,$$

$$\Rightarrow\ -i\omega^2\tau^*k\varepsilon_1^*\overline{u}_1(z)-\omega^2\tau^*\varepsilon_2^*D\overline{u}_3(z)+\left[-\overline{K}_3D^2+\varepsilon_3\right]\overline{T}(z)=0, \qquad (3.15)$$

Where $\varepsilon_1^*=\frac{\beta_1T_0}{d},\ \varepsilon_2^*=\frac{\beta_3T_0}{d},\ \varepsilon_3=\overline{K}_1k^2-T_0\omega^2\tau^*.$

The equations (3.13) – (3.15) have a non-trivial solution if

$$\begin{vmatrix} d_{44}D^2+k^2\varepsilon_1 & ik\Omega D & -ik\beta_1 \\ ik\Omega D & d_{33}D^2+k^2\varepsilon_2 & -\beta_3 D \\ -i\omega^2\tau^* k\varepsilon_1^* & -\omega^2\tau^*\varepsilon_2^* D & -\overline{K}_3 D^2+\varepsilon_3 \end{vmatrix}=0,$$

$$\Rightarrow \begin{vmatrix} d_{44}D^2+k^2\varepsilon_1 & k\Omega D & k\beta_1 \\ -k\Omega D & d_{33}D^2+k^2\varepsilon_2 & \beta_3 D \\ \omega^2\tau^* k\varepsilon_1^* & \omega^2\tau^*\varepsilon_2^* D & \overline{K}_3 D^2-\varepsilon_3 \end{vmatrix}=0.$$

Expanding the above determinant by first row, we have

$$\left(d_{44}D^2+k^2\varepsilon_1\right)\Delta_1-\left(k\Omega D\right)\Delta_2+\left(k\beta_1\right)\Delta_3=0, \tag{3.16}$$

Where

$$\Delta_1=\begin{vmatrix} d_{33}D^2+k^2\varepsilon_2 & \beta_3 D \\ \omega^2\tau^*\varepsilon_2^* D & \overline{K}_3 D^2-\varepsilon_3 \end{vmatrix},$$

$$\Rightarrow \Delta_1=\left(d_{33}D^2+k^2\varepsilon_2\right)\left(\overline{K}_3 D^2-\varepsilon_3\right)-\omega^2\tau^*\varepsilon_2^*\beta_3 D^2,$$

$$\Rightarrow \Delta_1=d_{33}\overline{K}_3 D^4+k^2\varepsilon_2\overline{K}_3 D^2-d_{33}\varepsilon_3 D^2-k^2\varepsilon_2\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3 D^2,$$

$$\Rightarrow \Delta_1=d_{33}\overline{K}_3 D^4+\left(k^2\varepsilon_2\overline{K}_3-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3\right)D^2-k^2\varepsilon_2\varepsilon_3.$$

$$\Delta_2=\begin{vmatrix} -k\Omega D & \beta_3 D \\ \omega^2\tau^* k\varepsilon_1^* & \overline{K}_3 D^2-\varepsilon_3 \end{vmatrix},$$

$$\Rightarrow \Delta_2=-k\Omega D\left(\overline{K}_3 D^2-\varepsilon_3\right)-\beta_3\omega^2\tau^* k\varepsilon_1^* D,$$

$$\Rightarrow \Delta_2=-k\Omega\overline{K}_3 D^3+k\Omega\varepsilon_3 D-\beta_3\omega^2\tau^* k\varepsilon_1^* D,$$

$$\Rightarrow \Delta_2=-k\Omega\overline{K}_3 D^3+k\left(\Omega\varepsilon_3-\beta_3\omega^2\tau^*\varepsilon_1^*\right)D.$$

$$\Delta_3=\begin{vmatrix} -k\Omega D & d_{33}D^2+k^2\varepsilon_2 \\ \omega^2\tau^* k\varepsilon_1^* & \omega^2\tau^*\varepsilon_2^* D \end{vmatrix},$$

$$\Rightarrow \Delta_3=(-k\Omega D)\left(\omega^2\tau^*\varepsilon_2^* D\right)-\omega^2\tau^* k\varepsilon_1^*(d_{33}D^2+k^2\varepsilon_2),$$

$$\Rightarrow \Delta_3=-k\Omega\omega^2\tau^*\varepsilon_2^* D^2-\omega^2\tau^* k\varepsilon_1^* d_{33}D^2-\omega^2\tau^* k^3\varepsilon_2\varepsilon_1^*,$$

$$\Rightarrow \Delta_3=-k\omega^2\tau^*\left(\Omega\varepsilon_2^*+d_{33}\varepsilon_1^*\right)D^2-\omega^2\tau^* k^3\varepsilon_2\varepsilon_1^*.$$

Now substituting the values of α1, α2 and α3 in equation (3.16), we have

$$\left(d_{44}D^2+k^2\varepsilon_1\right)\left[d_{33}\overline{K}_3 D^4+\left(k^2\varepsilon_2\overline{K}_3-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3\right)D^2-k\right.$$

$$-k\Omega D\left[-k\Omega\overline{K}_3 D^3+k\left(\Omega\varepsilon_3-\beta_3\omega^2\tau^*\varepsilon_1^*\right)D\right]$$

$$+k\beta_1\left[-k\omega^2\tau^*\left(\Omega\varepsilon_2^*-d_{33}\varepsilon_1^*\right)D^2-\omega^2\tau^* k^3\varepsilon_2\varepsilon_1^*\right]=0,$$

$$\Rightarrow d_{33}d_{44}\overline{K}_3 D^6+d_{33}\overline{K}_3 k^2\varepsilon_1 D^4+d_{44}\left[k^2\varepsilon_2\overline{K}_3-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3\right]$$

$$D^4+k^2\varepsilon_1\left(k^2\varepsilon_2\overline{K}_3-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3\right)D^2-d_{44}k^2\varepsilon_2\varepsilon_3 D^2$$

$$-k^4\varepsilon_1\varepsilon_2\varepsilon_3+k^2\Omega^2\overline{K}_3 D^4-k^2\Omega(\Omega\varepsilon_3-\beta_3\omega^2\tau^*\varepsilon_1^*)D^2$$

$$-k^2\beta_1\omega^2\tau^*(\Omega\varepsilon_2^*+d_{33}\varepsilon_1^*)D^2-k^4\omega^2\tau^*\beta_1\varepsilon_2\varepsilon_1^*=0,$$

$$\Rightarrow d_{33}d_{44}\overline{K}_3 D^6+\left[d_{33}\overline{K}_3 k^2\varepsilon_1+d_{44}\left\{k^2\varepsilon_2\overline{K}_3-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2\beta_3\right\}\right.$$

$$\left.+k^2\Omega^2\overline{K}_3\right]D^4+\left[k^2\varepsilon_1(k^2\varepsilon_2\overline{K}_3-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3)-d_{44}k^2\varepsilon_2\varepsilon_3\right.$$

$$\left.-k^2\Omega(\Omega\varepsilon_3-\beta_3\omega^2\tau^*\varepsilon_1^*)-k^2\beta_1\omega^2\tau^*\left(\Omega\varepsilon_2^*+d_{33}\varepsilon_1^*\right)\right]D^2$$

$$+\left[-k^4\varepsilon_1\varepsilon_2\varepsilon_3-k^4\omega^2\tau^*\beta_1\varepsilon_2\varepsilon_1^*\right]=0,$$

$$\Rightarrow L_0 D^6+L_1 D^4+L_2 D^2+L_3=0, \tag{3.17}$$

which is the required cubic equation in D^2, where

$$L_0=d_{33}d_{44}\overline{K_3},$$

$$L_1=d_{33}\overline{K_3}k^2\varepsilon_1+d_{44}(k^2\varepsilon_2\overline{K_3}-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3)+k^2\Omega^2\overline{K_3},$$

$$L_2=k_2\varepsilon_1(k^2\varepsilon_1\overline{K_3}-d_{33}\varepsilon_3-\omega^2\tau^*\varepsilon_2^*\beta_3)-d_{44}k^2\varepsilon_2\varepsilon_3$$

$$-k^2\Omega(\Omega\varepsilon_3-\beta_3\omega^2\tau^*\varepsilon_1^*)-k^2\beta_1\omega^2\tau^*(\Omega\varepsilon_2^*+d_{33}\varepsilon_1^*),$$

$$L_3=-k^4\varepsilon_1\varepsilon_2\varepsilon_3-k^4\omega^2\tau^*\beta_1\varepsilon_2\varepsilon_1^*.$$

Let m_1^2, m_2^2, m_3^2 be the roots corresponding to the auxiliary equation of equation (3.17) then the general solutions of equation (3.17) are written as

$$u_1=\left(A_1e^{-m_1z}+A_2e^{-m_2z}+A_3e^{-m_3z}+A_4e^{m_1z}+A_5e^{m_2z}+A_6e^{m_3z}\right)e^{ik(x-ct)}, \tag{3.18}$$

$$u_3=\left(\zeta_1A_1e^{-m_1z}+\zeta_2A_2e^{-m_2z}+\zeta_3A_3e^{-m_3z}+\zeta_4A_4e^{m_1z}+\zeta_5A_5e^{m_2z}+\zeta_9A_6e^{m_3z}\right)e^{ik(x-ct)}, \tag{3.19}$$

$$T=\left(\eta\,A\,e^{-m_1z}+\eta\,A\,e^{-m_2z}+\eta\,A\,e^{-m_3z}+\eta\,A\,e^{m_1z}+\eta\,A\,e^{m_2z}+\eta\,A\,e^{m_3z}\right)e^{ik(x-ct)}, \tag{3.20}$$

1 1 2 2 3 3 4 4 5 5 6 6

where a*i* , a*i* and *mi* are derived as in Appendix-III.

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