

New Stronger Forms of Fuzzy Continuous Mappings in Fuzzy Topological Spaces

Kiran G. Potadar^{1*}, Sadanand N. Patil²

¹Assistant Prof., Department of Mathematics, Angadi Institute of Technology and Management, Belagavi, Karnataka (India)

²Research Supervisor, VTU RRC, Belagavi, Karnataka (India)

Abstract – The aim of this paper is to introduce and study some stronger forms of fuzzy $g^{\#\#}$ -continuous functions namely, strongly fuzzy $g^{\#\#}$ -continuous, perfectly fuzzy $g^{\#\#}$ -continuous and completely fuzzy $g^{\#\#}$ -continuous functions and their properties.

Keywords: Strongly $f g^{\#\#}$ -continuous, perfectly $f g^{\#\#}$ -continuous, completely $f g^{\#\#}$ -continuous.

1. INTRODUCTION

The basic concept of fuzzy sets and fuzzy set operations were first introduced in the year 1965 by Prof. L.A. Zadeh's [23], Fuzzy subsets are the class of objects with grades of membership ranging between nil memberships (0) and the full membership (1), in the year 1968, C L. Chang [6] introduced the structure of fuzzy topology as an application of fuzzy sets to general topology. Subsequently many researchers like, C.K. Wong [22], R.H. Warren [21], R. Lowen [11], A.S. Mashhour [13], K.K. Azad [2], M. N. Mukherjee [14, 15], G. Balasubramanian & P. Sundaram [3] and many others have contributed to the development of fuzzy topological spaces.

The image and the inverse image of fuzzy subsets under Zadeh's functions and their properties proved by C.L.Chang [6] and R.H.Warren [21] are included. Fuzzy topological spaces and some basic concepts and results on fuzzy topological spaces from the works of C.L.Chang [6], R.H.Warren [20], and C.K.Wong[22] are presented. And some basic preliminaries are included. N.Levine [10] introduced generalized closed sets (g -closed sets) in general topology as a generalization of closed sets. Many researchers have worked on this and related problems both in general topology and fuzzy topology.

Sadanand Patil [16,17&18], S.P.Arya and R.Gupta[1],R.N.Bhaounik and Anjan Mukharjee[3], M.N.Mukharjee and B.Ghosh[15] and so many researchers have introduced and studied some stronger forms of fuzzy continuous functions like, Strongly fuzzy continuous, Perfectly fuzzy continuous and Completely fuzzy continuous functions and their mappings.

2. PRELIMINARIES:

Throughout this paper (X, T) , (Y, σ) & (Z, η) or (simply $X, Y, & Z$) represents non-empty fuzzy topological spaces on which no separation axiom is assumed unless explicitly stated. For a subset A of a space (X, T) , $cl(A)$, $int(A)$ & $C(A)$ denotes the closure, interior and the compliment of A respectively.

Definition 2.01: A fuzzy set A of a fts (X, T) is called:

- 1) A semi-open fuzzy set, if $A \leq cl(int(A))$ and a semi-closed fuzzy set, if $int(cl(A)) \leq A$ [16]
- 2) A pre-open fuzzy set, if $A \leq int(cl(A))$ and a pre-closed fuzzy set, if $cl(int(A)) \leq A$ [16]
- 3) A α -open fuzzy set, if $A \leq int(cl(int(A)))$ and a α -closed fuzzy set, if $cl(int(cl(A))) \leq A$ [17]

The semi closure (respectively pre-closure, α -closure) of a fuzzy set A in a fts (X, T) is the intersection of all semi closed (respectively pre closed fuzzy set, α -closed fuzzy set) fuzzy sets containing A and is denoted by $scl(A)$ (respectively $pcl(A)$, $\alpha cl(A)$).

Definition 2.02: A fuzzy set A of a fts (X, T) is called:

- 1) A generalized closed (g -closed) fuzzy set, if $cl(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [3]
- 2) A generalized pre-closed (gp -closed) fuzzy set, if $pcl(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [16]

- 3) A α -generalized closed (ag-closed) fuzzy set, if $\text{acl}(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [16, 17&18]
- 4) A generalized α -closed ($g\alpha$ -closed) fuzzy set, if $\text{acl}(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [16,17&18]
- 5) A generalized semi pre closed (gsp-closed) fuzzy set, if $\text{spcl}(A) \leq U$, whenever $A \leq U$ and U is open fuzzy set in (X, T) . [16,17&18]
- 6) A g^* -closed fuzzy set, if $\text{cl}(A) \leq U$, whenever $A \leq U$ and U is g -open fuzzy set in (X, T) . [10]
- 7) A $g^\#$ -closed fuzzy set, if $\text{cl}(A) \leq U$, whenever $A \leq U$ and U is ag -open fuzzy set in (X, T) . [16,17]
- 8) A $g^{\#\#}$ -closed fuzzy set, if $\text{acl}(A) \leq U$, whenever $A \leq U$ and U is ag -open fuzzy set in (X, T) . [9]
- 8) A fuzzy $g^{\#\#}$ -continuous ($fg^{\#\#}$ -continuous) if $f^{-1}(A)$ is $g^{\#\#}$ -closed fuzzy set in X , for every closed fuzzy set A of Y . [9]
- 9) A fuzzy $g^{\#\#}$ -irresolute ($fg^{\#\#}$ -irresolute) if $f^{-1}(A)$ is $g^{\#\#}$ -closed fuzzy set in X , for every closed fuzzy set A of Y . [9]
- 10) A fuzzy strongly continuous (strongly f -continuous) if $f^{-1}(V)$ is closed fuzzy set in X , for every closed set in Y . [1]
- 11) A fuzzy strongly g -continuous (strongly fg -continuous) if $f^{-1}(V)$ is open fuzzy set in X , for every g -open set in Y . [14]

Complement of g -closed fuzzy (respectively gp -closed fuzzy set, ag -closed fuzzy set, $g\alpha$ -closed fuzzy set, gsp -closed fuzzy set, g^* -closed fuzzy set and $g^\#$ -closed fuzzy set) sets are called g -open (respectively gp -open fuzzy set, ag -open fuzzy set, $g\alpha$ -open fuzzy set, gsp -open fuzzy set, g^* -open fuzzy set, $g^\#$ -open fuzzy set and $g^{\#\#}$ -open fuzzy set) sets.

Definition 2.03: Let X and Y be two fuzzy topological Spaces, A function $f: X \rightarrow Y$ is called:

- 1) A fuzzy continuous (f -continuous) if $f^{-1}(A)$ is closed fuzzy set in X , for every closed fuzzy set A of Y . [3]
- 2) A fuzzy α -continuous ($f\alpha$ -continuous) if $f^{-1}(A)$ is α -closed fuzzy set in X , for every closed fuzzy set A of Y . [16]
- 3) A fuzzy generalized-continuous (fg -continuous) if $f^{-1}(A)$ is g -closed fuzzy set in X , for every closed fuzzy set A of Y . [16]
- 4) A fuzzy generalized α -continuous ($fg\alpha$ -continuous) if $f^{-1}(A)$ is $g\alpha$ -closed fuzzy set in X , for every closed fuzzy set A of Y . [3]
- 5) A fuzzy α -generalized continuous (fag -continuous) if $f^{-1}(A)$ is ag -closed fuzzy set in X , for every closed fuzzy set A of Y . [16]
- 6) A fuzzy g^* -continuous (fg^* -continuous) if $f^{-1}(A)$ is g^* -closed fuzzy set in X , for every closed fuzzy set A of Y . [16]
- 7) A fuzzy $g^\#$ -continuous ($fg^\#$ -continuous) if $f^{-1}(A)$ is $g^\#$ -closed fuzzy set in X , for every closed fuzzy set A of Y . [17]

Definition 2.04: A map $f: X \rightarrow Y$ is called:

- 1) fuzzy α -open ($f\alpha$ -open) iff $f(V)$ is open α -fuzzy set in Y for every open fuzzy set in X [16]
- 2) fuzzy g -open (fg -open) iff $f(V)$ is g -open α -fuzzy set in Y for every open fuzzy set in X [16]
- 3) fuzzy g^* -open (fg^* -open) iff $f(V)$ is g^* -open α -fuzzy set in Y for every open fuzzy set in X [17]
- 4) fuzzy $g^\#$ -open ($fg^\#$ -open) iff $f(V)$ is $g^\#$ -open α -fuzzy set in Y for every open fuzzy set in X [17]
- 5) fuzzy $g^{\#\#}$ -open ($fg^{\#\#}$ -open) iff $f(V)$ is $g^{\#\#}$ -open α -fuzzy set in Y for every open fuzzy set in X [9]

3. STRONGLY $g^{\#\#}$ -CONTINUOUS FUNCTION IN FUZZY TOPOLOGICAL SPACES

Definition 3.01: A function $f: X \rightarrow Y$ is said to be strongly fuzzy $g^{\#\#}$ -continuous (briefly strongly $f g^{\#\#}$ -continuous) iff the inverse image of every $g^{\#\#}$ -open fuzzy set in Y is open fuzzy set in X .

Now we introduce the following.

Theorem 3.02: A function $f: X \rightarrow Y$ is strongly $f g^{\#\#}$ -continuous iff the inverse image of every $g^{\#\#}$ -closed fuzzy set in Y is closed fuzzy set in X .

Proof: The proof follows from the definition.

Theorem 3.03: Every strongly $f g^{\#\#}$ -continuous function is a f -continuous function.

Proof: Let $f: X \rightarrow Y$ be strongly $f g^{\#\#}$ -continuous function. Let V be open fuzzy set in Y , and V is $g^{\#\#}$ -open set in Y . Then $f^{-1}(V)$ is open fuzzy set in X . Hence f is f -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.04: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows. $A = \{(a,0.7), (b,0.5), (c,0.8)\}$, $B = \{(a,0.3), (b,0.5), (c,0.2)\}$, $C = \{(a,0.8), (b,0.5), (c,0.9)\}$. Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. then (X, T) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is f -continuous as B is open fuzzy set in Y and $f^{-1}(B) = B$ is open fuzzy set in X . But f is not strongly f $g^{##}$ -continuous as the fuzzy set C is $g^{##}$ -closed fuzzy set in Y and $f^{-1}(C) = C$ is not closed fuzzy set in X .

Theorem 3.05: Every f -strongly continuous function is a strongly f $g^{##}$ -continuous function.

Proof: Let $f: X \rightarrow Y$ be f -strongly continuous function. Let V be $g^{##}$ -open fuzzy set in Y . And then $f^{-1}(V)$ is both open and closed fuzzy set in X as f is f -strongly continuous function. Hence f is strongly f $g^{##}$ -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.06: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A_1, A_2, A_3, A_4, A_5 and A_6 be defined as follows.

$A_1 = \{(a,1), (b,0), (c,0)\}$, $A_2 = \{(a,0), (b,1), (c,0)\}$, $A_3 = \{(a,0), (b,0), (c,1)\}$, $A_4 = \{(a,1), (b,1), (c,0)\}$, $A_5 = \{(a,1), (b,0), (c,1)\}$ and $A_6 = \{(a,0), (b,1), (c,1)\}$.

Consider $T = \{0, 1, A_1, A_2, A_4\}$ and $\sigma = \{0, 1, A_4\}$. Then (X, T) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is strongly f $g^{##}$ -continuous but not f -strongly continuous as A_1 in Y is such that $f^{-1}(A_1) = A_2$ is open fuzzy set in X not closed fuzzy set in X .

Theorem 3.07: Let $f: X \rightarrow Y$ be strongly f $g^{##}$ -continuous and $g: Y \rightarrow Z$ is strongly f $g^{##}$ -continuous. Then the composition map $g \circ f: X \rightarrow Z$ is strongly f $g^{##}$ -continuous function.

Proof: Let V be $g^{##}$ -open fuzzy set in Z . Then $g^{-1}(V)$ is open fuzzy set in Y . since g is strongly f $g^{##}$ -continuous. Therefore $g^{-1}(V)$ is $g^{##}$ -open fuzzy set in Y . Also since f is strongly f $g^{##}$ -continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is open fuzzy set in X . Hence $g \circ f$ is strongly f $g^{##}$ -continuous function.

Theorem 3.08: Let $f: X \rightarrow Y$, $g: Y \rightarrow Z$ be maps such that f is strongly f $g^{##}$ -continuous and g is f $g^{##}$ -continuous then $g \circ f: X \rightarrow Z$ is f -continuous.

Proof: Let F be a closed fuzzy set in Z . Then $g^{-1}(F)$ is $g^{##}$ -closed fuzzy set in Y . Since g is f $g^{##}$ -continuous. And since f is strongly f $g^{##}$ -continuous, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is closed fuzzy set in X . Hence $g \circ f$ is f -continuous.

Theorem 3.09: If $f: X \rightarrow Y$ be strongly f $g^{##}$ -continuous and $g: Y \rightarrow Z$ is f $g^{##}$ -irresolutive, then the composition map $g \circ f: X \rightarrow Z$ is strongly f $g^{##}$ -continuous.

Proof: Omitted.

4. PERFECTLY $g^{##}$ -CONTINUOUS FUNCTION IN FUZZY TOPLOGICAL SPACES

Definition 4.01: A function $f: X \rightarrow Y$ called perfectly fuzzy $g^{##}$ -continuous (briefly perfectly f $g^{##}$ -continuous) if the inverse image of every $g^{##}$ -open fuzzy set in Y is both open and closed fuzzy set in X .

Theorem 4.02: A map $f: X \rightarrow Y$ is perfectly f $g^{##}$ -continuous iff the inverse image of every $g^{##}$ -closed fuzzy set in Y is both open and closed fuzzy set in X .

Proof: The proof follows from the definition.

Theorem 4.03: Every perfectly f $g^{##}$ -continuous function is f -continuous function.

Proof: Let $f: X \rightarrow Y$ be perfectly f $g^{##}$ -continuous. Let V be open fuzzy set in Y , and V is $g^{##}$ -open fuzzy set in Y . Since f is perfectly f $g^{##}$ -continuous, then $f^{-1}(V)$ is both open and closed fuzzy set in X . That is $f^{-1}(V)$ is open fuzzy set in X . Hence f is f -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 4.04: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows. $A = \{(a,0.7), (b,0.5), (c,0.8)\}$, $B = \{(a,0.3), (b,0.5), (c,0.2)\}$, $C = \{(a,0.8), (b,0.5), (c,0.9)\}$. Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. then (X, T) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is f -continuous but not perfectly f $g^{##}$ -continuous as the fuzzy set $1-C = \{(a, 0.2), (b, 0.5), (c, 0.1)\}$ is $g^{##}$ -open fuzzy set in Y and $f^{-1}(1-C) = 1-C$ which is not both open and closed fuzzy set in X .

Theorem 4.05: Every perfectly f $g^{##}$ -continuous function is a f -perfectly continuous function.

Proof: let $f: X \rightarrow Y$ be perfectly f $g^{##}$ -continuous. Let V be open fuzzy set in Y , then V be $g^{##}$ -open fuzzy set in Y . Since f is perfectly f $g^{##}$ -continuous. Then $f^{-1}(V)$ is both open and closed fuzzy set in X . And hence f is f -perfectly continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 4.06: Example: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows. $A = \{(a,0.7), (b,0.5), (c,0.8)\}$, $B = \{(a,0.3), (b,0.5), (c,0.2)\}$ $C = \{(a,0.8), (b,0.5), (c,0.9)\}$.

Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. then (X, T) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then f is f -perfectly function. As the fuzzy set in B is open fuzzy set in Y , and its inverse image $f^{-1}(B) = B$ is both open and closed fuzzy set in X . But f is not perfectly $fg^{##}$ -continuous as the fuzzy set $1-C = \{(a, 0.2), (b, 0.5), (c, 0.1)\}$ is $g^{##}$ -open fuzzy set in Y and $f^{-1}(1-C) = 1-C$ which is not both open and closed fuzzy set in X .

Theorem 4.07: Every perfectly $f g^{##}$ -continuous function is strongly $f g^{##}$ -continuous function.

Proof: Let $f: X \rightarrow Y$ be perfectly $f g^{##}$ -continuous. Let V be $g^{##}$ -open fuzzy set in Y . Then $f^{-1}(V)$ is both open and closed fuzzy set in X . Therefore $f^{-1}(V)$ is open fuzzy set in X . Hence f is strongly $f g^{##}$ -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 4.08: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A_1, A_2, A_3, A_4, A_5 and A_6 be defined as follows.

$A_1 = \{(a, 1), (b, 0), (c, 0)\}, A_2 = \{(a, 0), (b, 1), (c, 0)\}, A_3 = \{(a, 0), (b, 0), (c, 1)\}, A_4 = \{(a, 1), (b, 1), (c, 0)\}, A_5 = \{(a, 1), (b, 0), (c, 1)\}$ and $A_6 = \{(a, 0), (b, 1), (c, 1)\}$.

Consider $T = \{0, 1, A_1, A_2, A_4\}$ and $\sigma = \{0, 1, A_4\}$. Then (X, T) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = b, f(b) = a$ and $f(c) = c$. Then f is strongly $f g^{##}$ -continuous but not perfectly $f g^{##}$ -continuous as the fuzzy set A_3 is $g^{##}$ -closed fuzzy set in Y and $f^{-1}(A_3) = A_3$ is not both open and closed fuzzy set in X .

Theorem 4.09: Let $f: X \rightarrow Y, g: Y \rightarrow Z$ be two perfectly $f g^{##}$ -continuous function then $gof: X \rightarrow Z$ is perfectly $f g^{##}$ -continuous function.

Proof: Let V be $g^{##}$ -open fuzzy set in Z . Then $g^{-1}(V)$ is both open and closed fuzzy set in Y , since g is perfectly $f g^{##}$ -continuous. Therefore $g^{-1}(V)$ is $g^{##}$ -open fuzzy set in Y . Also since f is perfectly $f g^{##}$ -continuous. $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is both open and closed fuzzy set in X . Hence gof is perfectly $f g^{##}$ -continuous function.

Theorem 4.10: Let $f: X \rightarrow Y$ be perfectly $f g^{##}$ -continuous and $g: Y \rightarrow Z$ be $g^{##}$ -irresolute function then $gof: X \rightarrow Z$ is perfectly $f g^{##}$ -continuous function.

Proof: Omitted.

5. COMPLETELY $g^{##}$ -CONTINUOUS FUNCTION IN FUZZY TOPOLOGICAL SPACES

Definition 5.01: A map $f: X \rightarrow Y$ is called completely fuzzy $g^{##}$ -continuous (briefly completely $g^{##}$ -continuous) if the inverse image of every $g^{##}$ -open fuzzy set in Y is regular-open fuzzy set in X .

Theorem 5.02: A map $f: X \rightarrow Y$ is completely $f g^{##}$ -continuous. Iff the inverse image of every $g^{##}$ -closed fuzzy set in Y is regular-closed fuzzy set in X .

Proof: The proof follows from the definition.

Theorem 5.03: Every completely $fg^{##}$ -continuous function is a f -continuous function.

Proof: Let $f: X \rightarrow Y$ be completely $f g^{##}$ -continuous function. Let V be open fuzzy set in Y . Then V is $g^{##}$ -open fuzzy set in Y . And then $f^{-1}(V)$ is both regular-open fuzzy set in X , and therefore $f^{-1}(V)$ is open fuzzy set in X . Hence f is f -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 5.04: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows.

$A = \{(a, 0.7), (b, 0.5), (c, 0.8)\},$

$B = \{(a, 0.3), (b, 0.5), (c, 0.2)\}, C = \{(a, 0.8), (b, 0.5), (c, 0.9)\}.$

Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. then (X, T) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then f is f -continuous function but not completely $fg^{##}$ -continuous as the fuzzy set $1-C = \{(a, 0.2), (b, 0.5), (c, 0.1)\}$ is $g^{##}$ -open fuzzy set in Y and $f^{-1}(1-C) = 1-C$ which is not regular open fuzzy set in X .

Theorem 5.05: Every completely $fg^{##}$ -continuous function is a f -completely continuous function.

Proof: Let $f: X \rightarrow Y$ be completely $fg^{##}$ -continuous. Let V be open fuzzy set in Y . Then V be $g^{##}$ -open fuzzy set in Y . Then $f^{-1}(V)$ is regular-open fuzzy set in X . Hence f is f -completely continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 5.06: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A, B and C be defined as follows.

$A = \{(a, 0.7), (b, 0.5), (c, 0.8)\}, B = \{(a, 0.3), (b, 0.5), (c, 0.2)\}, C = \{(a, 0.8), (b, 0.5), (c, 0.9)\}.$

Consider $T = \{0, 1, A, B\}$ and $\sigma = \{0, 1, B\}$. then (X, T) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then f is f -completely continuous function as the fuzzy set B is open fuzzy set in Y , and its inverse image $f^{-1}(B) = B$ is regular-open fuzzy set in X . But not completely $fg^{##}$ -continuous as the fuzzy set $1-C = \{(a, 0.2), (b, 0.5), (c, 0.1)\}$ is $g^{##}$ -open fuzzy set in Y and $f^{-1}(1-C) = 1-C$ which is not regular open fuzzy set in X .

Theorem 5.07: every completely $fg^{##}$ -continuous function is strongly $fg^{##}$ -continuous function.

Proof: Let $f: X \rightarrow Y$ be completely $fg^{##}$ -continuous. Let V be $g^{##}$ -open fuzzy set in Y . Then $f^{-1}(V)$ is regular-open fuzzy set in X . Therefore $f^{-1}(V)$ is open fuzzy set in X . Hence f is strongly $fg^{##}$ -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 5.08: Let $X = Y = \{a, b, c\}$ and the fuzzy sets A_1, A_2, A_3, A_4, A_5 and A_6 be defined as follows.

$A_1 = \{(a, 1), (b, 0), (c, 0)\}$ $A_2 = \{(a, 0), (b, 1), (c, 0)\}$, $A_3 = \{(a, 0), (b, 0), (c, 1)\}$, $A_4 = \{(a, 1), (b, 1), (c, 0)\}$, $A_5 = \{(a, 1), (b, 0), (c, 1)\}$ and $A_6 = \{(a, 0), (b, 1), (c, 1)\}$. Consider $\tau = \{0, 1, A_1, A_2, A_4\}$ and $\sigma = \{0, 1, A_4\}$. Then (X, τ) and (Y, σ) are fts. Define $f: X \rightarrow Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is strongly $fg^{##}$ -continuous but not completely $fg^{##}$ -continuous as the fuzzy set A_3 is $g^{##}$ -closed fuzzy set in Y and its inverse image $f^{-1}(A_3) = A_3$ is not regular-closed fuzzy set in X .

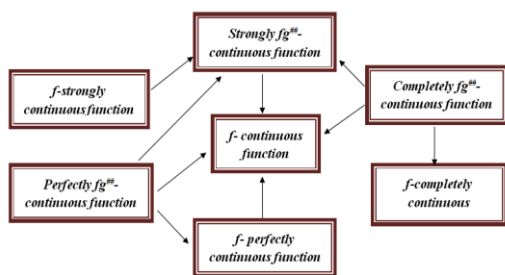
Theorem 5.09: If $f: X \rightarrow Y$ is completely $fg^{##}$ -continuous and $g: Y \rightarrow Z$ is $fg^{##}$ -irresolute function then $g \circ f: X \rightarrow Z$ is completely $fg^{##}$ -continuous function.

Proof: Let V be $g^{##}$ -open fuzzy set in Z . Then $g^{-1}(V)$ is $g^{##}$ -open fuzzy set in Y , since g is $fg^{##}$ -irresolute function. Also since f is completely $fg^{##}$ -continuous. $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is regular-open fuzzy set in X . Hence $g \circ f$ is completely $fg^{##}$ -continuous function.

Theorem 5.10: If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two completely $fg^{##}$ -continuous function then $g \circ f: X \rightarrow Z$ is completely $fg^{##}$ -continuous function.

Proof: Omitted.

Figure 1: The following diagram shows the relationship of stronger maps with some other fuzzy maps.



Where $A \rightarrow B$ ($A \dashv\vdash B$) represents A implies B but not conversely. (A & B are independent).

CONCLUSION:

It's interesting to work on the stronger form of fuzzy continuous mapping and their properties. The

relationship of stronger maps with some other fuzzy maps and compositions of mapping have been investigated.

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Corresponding Author

Kiran G Potadar*

Assistant Prof., Department of Mathematics, Angadi Institute of Technology and Management, Belagavi, Karnataka (India)

E-Mail – kgpotadar@gmail.com