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# A Study on Bending and Free Vibration Behavior of Fiber Reinforced Composite Plate Based On First Order Shear Deformation Theory

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***Abstract – Composite laminated plates are used in various fields now days due to their high advantages and better performance. These advanced materials help us to explore the new possible things without relying on existing ordinary materials like metals. The present paper discusses the bending and free vibration behavior of laminated composite plate under different loading conditions and boundary conditions. In this study, the hypothesis is based on first order shear deformation theory. The composite plate domain is discretized by using 8 node quadrilateral element. Numerical analysis has been done by creating MATLAB code and results are obtained. The outcomes of results are compared with results available in the literature. The present paper work discusses the effect of number of layers, different boundary conditions and different loading condition on bending and frees vibration behavior. For free vibration analysis Eigen value problem is considered and it is solved using vector iteration method.***

***Keywords— Composite Materials, First Order Shear Deformation Theory, Vector Iteration Method.***

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## 1. INTRODUCTION

Materials are having greater impact on the mankind due to their effective use, since from ancient times. Considering the extent of composites for the use of mankind from the last forty year, has remarkable applications of the composites in various fields. Nowadays it has been found that composite materials are having the tremendous applications compared to the other materials. Such an impact made us to look through the composites for the betterment of the mankind. Composite materials are made up of two or more materials that will give desired properties which cannot be achieved by using any one of the constituents alone itself. For example fiber reinforced composite materials consists of more than two materials which will give desired properties like high strength to weight ratio and also gives high modulus. The fiber reinforced composite materials are idle for structural applications where high strength to weight and stiffness to weight ratios are required. A variety of structural components made of composite materials such as aircraft wing, helicopter blade, vehicle axles and turbine blades. The requirement to study the bending and free vibration behavior in structural components plays an important role

## 2. LITERATURE REVIEW

A number of researchers have been developed numerous solution methods in recent 20-30 years.

Cawley and Adams (1978) investigated the natural modes of square aluminium plates and square composite plates with different ply orientations for free-free boundary conditions, both theoretically as well as experimentally. Cawley and Adams (1979) also used dynamic analysis to detect, locate and roughly quantify damage to components fabricated from fiber reinforced plastic.

Maiti and Sinha (1994) used the first order shear deformation theory to develop FEM methods to study the bending and free vibration behavior composite beam. The free vibration frequencies of cross ply laminated square plates for twelve different boundary conditions were determined using Ritz method by Aydogdu and Timarci (2003).

G.Rajeshkumar et al.(2014) presented the free vibration characteristics of newly identified Phoenix Sp fiber reinforced polymer matrix composite beams, moreover the physical, chemical and mechanical properties of fiber was determined by using standard experimental methods. The modal analysis is carried out on composite beams having different fiber gauge lengths such as 10mm, 20mm, 30mm, 40mm and 50mm.

G.Abdollahzadeh and M. Ahmadi (2015) made an attempt to study the non linear behavior of composite beams undergoing prebuckling snd postbuckling. Analytical solutions are obtained to analyze the free non linear vibrations. The numerical results are obtained which have good agreement. The influences of the parameters of laminated composite beams on the free vibrations are studied. J. Alexander and B. S. Augustine (2015) presented the free vibration and damping characteristics of GFRP and BFRP laminated composites at various boundary conditions. Basically this study made on aircraft wing because of severe vibration during the motion of aircraft. Vibration analysis carried out by using modal analysis set up for various end conditions. Natural frequencies and damping coefficients were determined by for all materials, end conditions and for fiber orientations. Fabrication of GFRP and BFRP composites is done by using compression moulding technique.

The present work is mainly focused on the numerical study of fiber reinforced composite plate subjected to different boundary conditions and loading conditions first order shear deformation theory The first order shear deformation theory (FSDT), also called Mindlin's plate theory is based on the assumptions that the transverse shear strains are constant which leads to a particular kinematics of deformation. The transverse normals do not remain perpendicular to the mid surface after deformation. The displacement of this theory is in the form,

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \qquad (1)$$

Where $(u_0, v_0, w_0, \theta_x, \theta_y)$ are unknown functions to be calculated. As before $(u_0, v_0, w_0)$ denote the displacement of a mid surface i.e. z=0. Note that which indicates that $du/dz = \theta_x$ and $dv/dz = \theta_y$ are the rotations of a transverse normal relating to y and x-axes respectively. The notation $\theta_x$ denotes the rotation about y-axis and $\theta_y$ denotes the rotation about x-axis.

Shear correction factor used is 5/6.

finite element formulation and methodology The laminated composite plate consists of number of thin elastic substrate laminates. The fibers are oriented with different angles and they are embedded in matrix material. A lamina is assumed as elastically orthotropic.

Consider a fiber reinforced laminated composite plate having length a, width b and thickness h, shown figure 1. The geometry and material coordinates are at the middle of the laminated plate.

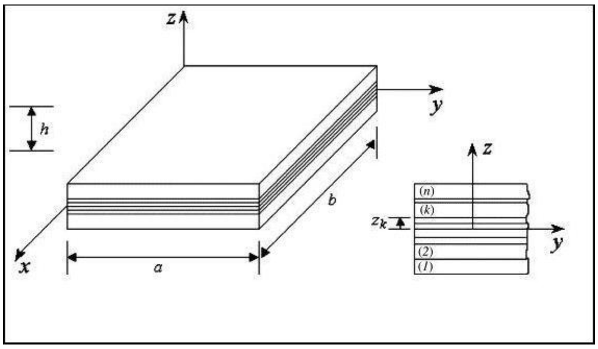



**Fig.1 Geometry of the laminated composite plate**

To determine the reduced stiffness matrix for a lamina, longitudinal elastic modulus, transverse elastic modulus, major poisons ratio, minor poisons ratio and shear modulus are required for the given composite. Formulas used for determining reduced matrix are as follows.

$$Q_{11} = \frac{E_{11}}{1-\upsilon_{12}\upsilon_{21}}$$
$$Q_{12} = \frac{\upsilon_{12}E_{22}}{1-\upsilon_{12}\upsilon_{21}}$$
$$Q_{22} = \frac{E_{22}}{1-\upsilon_{12}\upsilon_{21}} \qquad (2)$$

$$Q_{66} = G_{12}; \quad Q_{44} = G_{13}; \quad Q_{55} = G_{23};$$

Transformed reduced stiffness matrix is calculated by following formula.

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta \\
\bar{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\cos^4\theta\sin^4\theta) \\
\bar{Q}_{22} &= Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta \\
\bar{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})\cos^3\theta\sin\theta + (Q_{12}-Q_{22}+2Q_{66})\cos\theta\sin^3\theta \\
\bar{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})\cos\theta\sin^3\theta + (Q_{12}-Q_{22}+2Q_{66})\cos^3\theta\sin\theta \\
\bar{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\cos^2\theta\sin^2\theta + Q_{66}(\cos^4\theta+\sin^4\theta)
\end{aligned} \qquad (3)$$

The generalized [ABD] matrix is written as follows

$$[ABD] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \qquad (4)$$

Where [A]= In-plane stiffness matrix, [B]= Coupling stiffness matrix, [D]= Bending stiffness matrix and are calculated using following formula

$$A_{ij} = \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k (h_k - h_{k-1})$$
$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k (h_{k-1}^2 - h_{k-1}^2)$$
$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k (h_{k-1}^3 - h_{k-1}^3)$$
$$i, j = 1, 2, 6. \qquad (5)$$

An eight node quadrilateral isoparametric element is used in this analysis. Let {u, v, w, $\theta_x$, $\theta_y$} be the displacements field variables. The plate is discretized into finite elements. The analysis is based on first order shear deformation theory.

Shape functions for 8 node quadrilateral element are as follows

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)(-1-\xi-\eta)$$
$$N_2 = \frac{1}{2}(1-\xi^2)(1-\eta)$$
$$N_3 = \frac{1}{4}(1+\xi)(1-\eta)(-1+\xi-\eta)$$
$$N_4 = \frac{1}{2}(1+\xi)(1-\eta^2)$$
$$N_5 = \frac{1}{4}(1+\xi)(1+\eta)(-1+\xi+\eta)$$
$$N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$$
$$N_7 = \frac{1}{4}(1-\xi)(1+\eta)(-1-\xi+\eta)$$
$$N_8 = \frac{1}{2}(1-\xi)(1-\eta^2) \qquad (6)$$

Elemental stiffness matrix for the plate element is given by,

$$[K_e] = \iint [B]^T [D][B] |J| d\xi d\eta \qquad (7)$$

Elemental load vector is given by

$$\{F\}_e = \iint [N]^T q |J| d\xi d\eta \qquad (8)$$

Where,

$[K_e]$=Element stiffness matrix

$\{F\}_e$ =Element load vector

$|J|$ =Determinant of Jocobian

Jacobian matrix, $[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$ (9)

$$[B] = \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_r}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_r}{\partial y} \\ 0 & 0 & 0 & \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & -N_r & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial y} & 0 & -N_r \end{bmatrix}$$
$$r = 1, 2....8 \qquad (10)$$

For free vibration analysis equations used are

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \qquad (11)$$

Where,

$\{u\}$ = Displacement vector

$\{\dot{u}\}$ =Velocity vector

$\{\ddot{u}\}$ =Acceleration vector

[K]= Global stiffness and [M]=Global mass matrix.

The element mass matrix is given by

$$[M]_e = \int_{Ae} \int_{-h/2}^{h/2} [N]^T [\rho][N] dz dA \qquad (12)$$

For equations (7) and (11), [*N*] is given by

$$[N] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}$$
$$i = 1, 2.......8 \qquad (13)$$

The above equations are used to find the bending deflection and fundamental frequency for a fiber reinforced composite plate.

## 3. RESULTS AND DISCUSSION

In the present work, MATLAB software is used to perform the objective. The methodology described above is used to study the bending and free vibration behavior of composite plate. Material properties for composite plate are referred from Reddy (2004) and are given below

$E_1/E_2 = 25,\ G_{12} = G_{13} = 0.5 * E_2,\ G_{23} = 0.2 * E_2$ ,Density (□)=1800,

Plate length (a) and width (b) = 100, thickness (h)=1

*Validation study*

In the present work, the validation study is carried out by comparing the present results with the Reddy (2004). It is concluded from the table1 that the present results are good agreement with the Reddy for both loading conditions (CPL and UDL).

**Table1. Comparison of present results with Ref. (Reddy)**

| Laminate | Uniformly distributed load(UDL) | | Central point load(CPL) | |
|---|---|---|---|---|
| | Present results | Ref. | Present results | Ref. |
| 0/90 | 1.6985 | 1.6955 | 4.6886 | 4.6664 |
| 90/0 | 1.6985 | 1.6955 | 4.6886 | 4.6664 |
| $(0/90)_2$ | 0.8116 | 0.8085 | 2.2372 | 2.2105 |
| $(0/90)_4$ | 0.7181 | 0.7150 | 1.9806 | 1.9536 |

(a) *Bending analysis of fiber reinforced composite square plate*

A study on bending behavior of composite square plate with different fiber orientations, loadings and boundary conditions are carried out and the results are computed by considering the following load types and boundary conditions.

**Load types:**

Uniformly distributed load (UDL) and central point load (CPL).

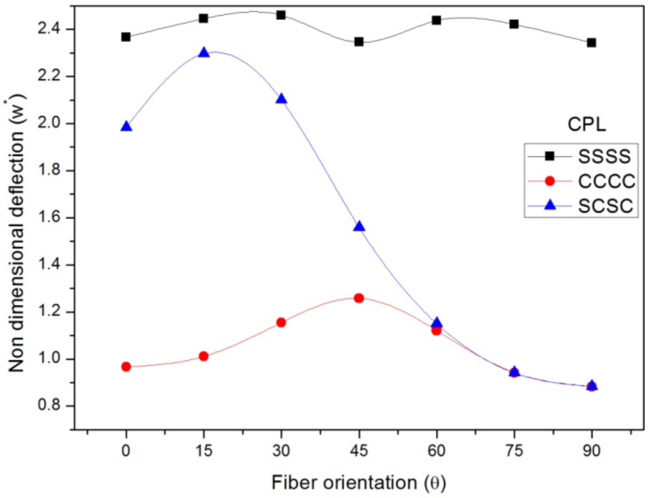
**Boundary conditions (BCs):**

SSSS: plate with all edges simply supported i.e. edges along x=0 and a, and also for y=0 and b, simply supported.

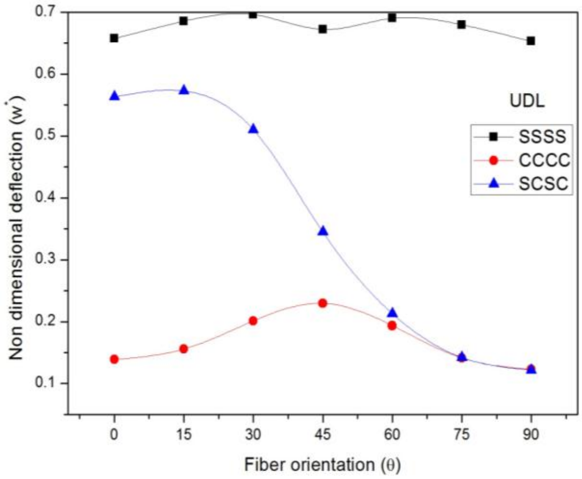
CCCC: plate with all edges clamped.

SCSC: plate with edges along x=0 and a, simply supported and edges along y=0 and b, clamped.

The effect of loading conditions, boundary conditions and number of layers on bending behavior of fiber reinforced composite square plate is studied based on following figures,



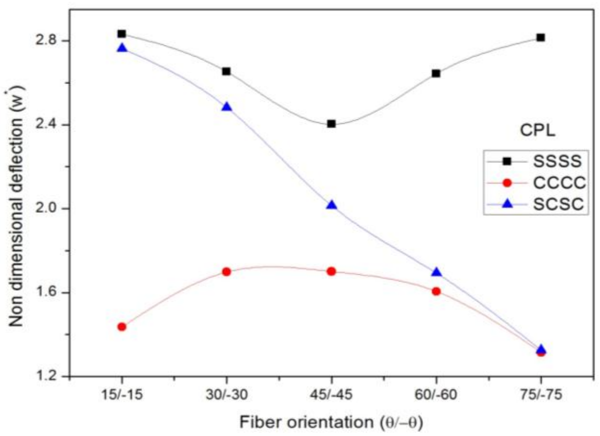

**(a) Central point load (CPL)**



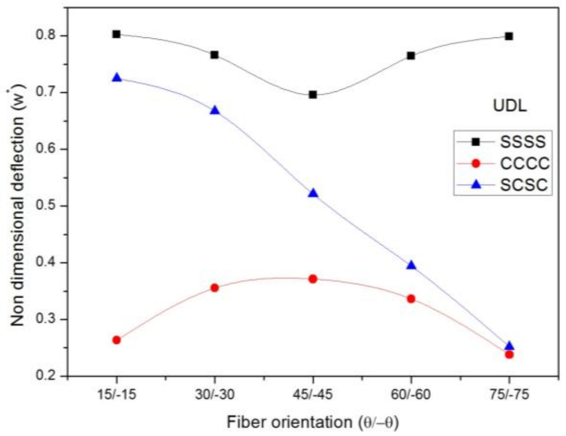

**(b) Uniformly distributed load (UDL)**

**Fig.2. Non dimensional central deflection (w*) of a single layer composite square plate for different fiber orientation($\theta$) with respect to different boundary conditions (Both CPL and UDL).**

The variation of non dimensional central deflections (w*) for a single layer fiber reinforced composite square plate having different fiber orientation (0, 15, 30, 45, 60, 75, 90) with different load and boundary conditions are presented in Fig. 2. The non dimensional central deflections of CCCC type boundary conditions are smaller than that of both SSSS and SCSC boundary conditions (both UDL and CPL), this is because stiffness of the square plate is more under CCCC boundary condition and for SCSC boundary condition non dimensional central deflection decreases with increase in the fiber orientation(both UDL and CPL).




**(a) Central point load (CPL)**

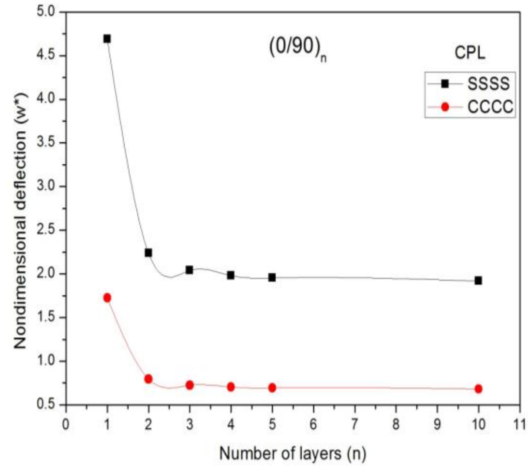



**(b) Uniformly distributed load (UDL)**

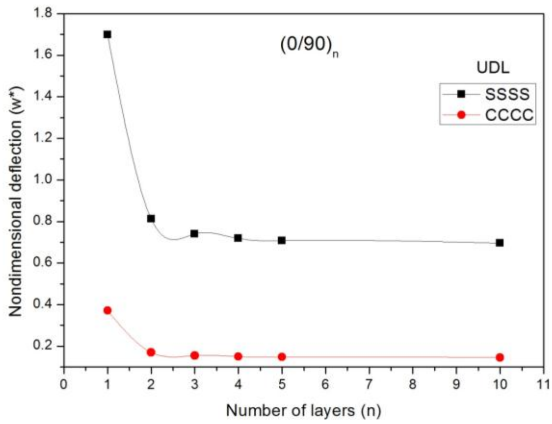
**Fig.3.Non dimensional central deflection (w*) of two layer composite square plate for different fiber orientation (□□□□) with respect to different boundary conditions (Both CPL and UDL).**

The variation of non dimensional central deflections (w*) for different layers (2, 4, 6, 8, 10, 20) of anti-symmetric □□□□□□□composite square plate having different fiber orientation; □=15, 30, 45, 60, and 75, with different load and boundary conditions are presented e.g. Fig.3 shows non dimensional central deflection for two layer composite square plate. The non dimensional central deflections of CCCC type boundary conditions are smaller than that of both SSSS and SCSC boundary conditions (both UDL and CPL) and for SCSC boundary condition non dimensional central deflection decreases with increase in the fiber orientation(both UDL and CPL). For SSSS boundary conditions the non dimensional central deflection decreases with increase in the fiber orientation up to (45/-45) and increases, further increase in the orientation(for both CPL and UDL).

The deformation behavior of angle ply plates are influenced by presence of extension-twisting and shear bending coupling due to existence of $B_{16}$ and $B_{26}$.

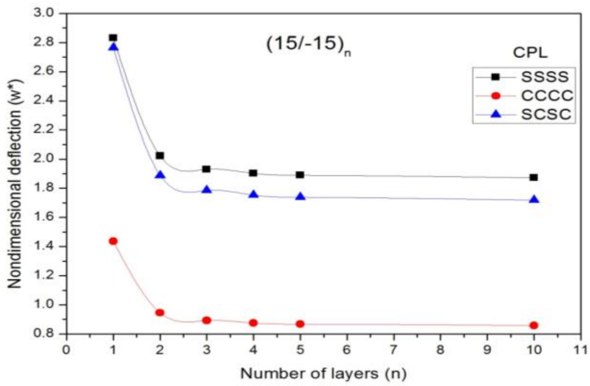



**(a) Central point load (CPL)**

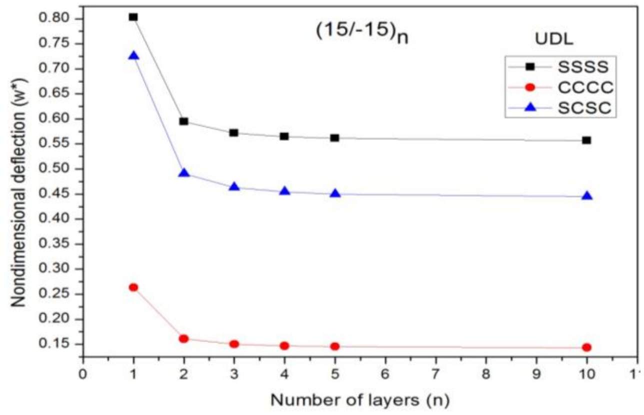



**(b) Uniformly distributed load (UDL)**

**Fig.4. Non dimensional central deflection (w*) for cross ply $(0/90)_n$ fiber orientation under central point load (CPL) and uniformly distributed load (UDL).**




**(a) Central point load (CPL)**




**(b) Uniformly distributed load (UDL)**

**Fig.5. Non dimensional central deflection (w*) of a composite square plate for number of layers (15/-15)$_n$ with respect to different boundary conditions (Both CPL and UDL).**

Fig.4 and 5 presents plots of non dimensional central deflection (w*) for cross ply and angle ply composite square plate versus different number of layers under different loading conditions (both CPL and UDL) and for different boundary conditions. From fig.5 and 6 it is observed that, the increase in number of layers (n) decreases the non dimensional central deflection of the square plate. It is also observed that the decrease is small with the increase of the total number of layers beyond six. This is because of decrease in the bending stretching coupling effect with increase in the number of layers. It is also noticeable that the non dimensional central deflections of CCCC type boundary conditions are smaller than that of both SSSS and SCSC boundary conditions (both UDL and CPL).

(b) *Free vibration behavior of fiber reinforced composite square plate*

**Boundary conditions (BCs):**

SSSS: plate with all edges simply supported i.e. edges along x=0 and a, and also for y=0 and b, simply supported.

CCCC: plate with all edges clamped.

SCSC: plate with edges along x=0 and a, simply supported and edges along y=0 and b, clamped.

SFSF: plate with edges along x=0 and a, simply supported and edges along y=0 and b, free.

Non dimensional fundamental frequency is given by

$$\omega^* = \omega(\frac{a^2}{h})\sqrt{\frac{\rho}{E_2}} \qquad (13)$$

**Table2. Non dimensional fundamental frequencies of single layer composite square plate with various fiber orientation(□) and boundary conditions (BCs).**

| Laminate\BCs | SSSS | CCCC | SCSC | SFSF |
|---|---|---|---|---|
| 0 | 15.1366 | 32.7245 | 16.7381 | 14.1596 |
| 15 | 15.0251 | 31.2471 | 16.6650 | 12.6348 |
| 30 | 15.0319 | 28.0235 | 17.6885 | 8.9435 |
| 45 | 15.2519 | 26.3951 | 21.3656 | 5.4592 |
| 60 | 15.1041 | 28.4814 | 26.6965 | 3.3797 |
| 75 | 15.0896 | 32.3307 | 31.4033 | 2.8755 |
| 90 | 15.1899 | 34.0732 | 33.2395 | 2.8486 |

**Table3. Non dimensional fundamental frequencies of two layer composite square plate with various fiber orientation and boundary conditions (BCs).**

| Laminate\BCs | SSSS | CCCC | SCSC | SFSF |
|---|---|---|---|---|
| 15/-15 | 13.9354 | 24.3875 | 15.5944 | 8.9909 |
| 30/-30 | 14.4248 | 21.2378 | 16.3949 | 6.3033 |
| 45/-45 | 15.1493 | 20.8051 | 18.2006 | 4.4064 |
| 60/-60 | 14.4398 | 21.7122 | 20.1407 | 3.2367 |
| 75/-75 | 13.9672 | 25.3439 | 23.7065 | 2.8744 |

The tables2 and 3 shows the fundamental frequencies of single and two layers of composite square plate with different fiber orientation and different boundary conditions. From tables it is observed that for SSSS boundary conditions, the increase in fundamental frequency is marginal. For SFSF boundary condition the fundamental frequency decreases with increase in fiber orientation. For SCSC boundary conditions, the fundamental frequency increases with increase in the fiber orientation. It is noted that the fundamental frequencies of CCCC boundary conditions are maximum compared to other boundary conditions , this is because increase in the number of boundary conditions will increase the fundamental frequency.

## CONCLUSIONS

The finite element modeling is done by MATLAB software to study the bending and free vibration behavior of fiber reinforced composite plate. A study on the effect of boundary conditions, ply orientation and number of layers are examined and bending, free vibration analysis is carried out for fiber reinforced composite plate. Finite element model is formulated efficiently for linear bending and free vibration behavior of composite square plate. Bending deflections and fundamental frequencies for fiber reinforced composite plate with different ply orientation and boundary conditions are efficiently derived from the analysis using MATLAB software. From bending analysis, it is came to know that the central deflection of composite plate dependent on the ply orientation, number of layers used and type of loading.

From free vibration analysis it is concluded that the increase in the number of boundary constraints will increase the fundamental frequency.

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