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Nonlinear Bending Behaviour of Fiber Reinforced Composite Plates

Shreekant Patil[1]*, Dr. Shravan B. Kerur[2]

[1]PG Student, Dept. of Mechanical Engg., BEC. Bagalkot, India

[2]Associate Prof. Dept. of mechanical Engg., BEC. Bagalkot, India

Abstract – This Composites are widely used in aeronautical, mechanical, civil and chemical industries due to their high strength to stiffness ratio and low weight to stiffness ratio. This work discuss about nonlinear bending behavior of the fiber reinforced composite square plate with various boundary conditions. The finite element formulation is based on first order shear deformation theory (FSDT). The finite element analysis has been carried out for a composite plate using an eight noded quadrilateral element with a five degree of freedom (DOF). Nonlinear equations are linearized by Newton-Rapson method. Numerical analysis has been carried by developing a program in MatLab and various results are obtained. The results are compared with those available in the literature for its accuracy and validity. The results are interpreted and discussed - on the effect of no of layer, stacking sequences and different boundary condition on nonlinear deflection composite laminates. The stacking sequence, no of layers and boundary conditions play a vital role on nonlinear bending behaviour of composite structures.

Keywords— FSDT, Composite, Geometric Non-Linear, MatLab.

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1. INTRODUCTION

Composite materials are preferred in places where lighter materials are desired or required without sacrificing strength. They have even become essential for many applications. Composite materials are being used in a variety of applications such as the structural parts of aircrafts, automobiles, chemical equipment, transformer tubes, boats, etc. Some transmission gears make use of plastic materials in many different places such as watches, instruments, washing machines, gear pumps, etc. Composite materials are efficient in applications that required high strength to weight and stiffness to weight ratios. Brebbia and Connor [1] presented the finite element displacement formulation applicable to arbitrary plate and shell elements for geometrically nonlinear problems and also developed appropriate equation for Newton-Raphson iteration. Wood and zienkiewicz [2] studied the geometrically nonlinear analysis of beams frames and arches using total Lagrangian coordinate system by using modified isoparametric element and system nonlinear equation was solved using Newton-Raphson method. Bathe and Bolourchi [3] presented finite element method for linear, geometricand material nonlinear analysis of plates and shell. Pica et al. [4] presented a geometrically nonlinear analysis of plates using finite element Mindilin formulation. Reddy et.al [5] developed a finite element model based on the combined theory of the yang, Norris, Stavsky and Von Karman [6] that, it accounts for the transverse shear strain, large rotations. Puctha and Reddy [7] developed a mixed shear flexible finite element with relaxed continuity, geometrically linear and nonlinear analysis of laminated anisotropic plates. Chia [8] solved the nonlinear bending of an unsymmetrically laminated angle ply rectangular plate analytically under lateral load satisfying the von-karman type strain. Striz et al. [9] investigated the behavior of thin, circular isotropic elastic plates with the immovable edges and undergoing large deflections .they used Newton-Raphson technique to solve the nonlinear systems of equations. Z.G. Azizian and D. J. Dawe [10] studied geometrically nonlinear analysis of rectangular mindlin plates using the finite strip method. Singh et al. [11] investigated the large deflections bending analysis of anti-symmetric rectangular cross-ply plates based on von-karman plate theory, with one–term approximation for the in-plane & transverse displacements, under sinusoidal loading. Clarke et al. [12] described various incremental- iterative technique based on the Newton-Raphson approach to analyze the geometric nonlinear behavior.

II. FINITE ELEMENT FORMULATION AND METHODOLOGY

The nonlinear finite element modelling and analysis is more complicated than linear one, as the stiffness matrix in nonlinear equilibrium equations is dependent on deformations. The first order shear deformation plate theory is considered in the finite element formulation. An eight- noded isoparametric plate/shell element is proposed for the geometrically nonlinear analysis of composite plates subjected to mechanical loading. The geometry and origin of the material coordinates are at the middle of the laminate as shown in Fig.1

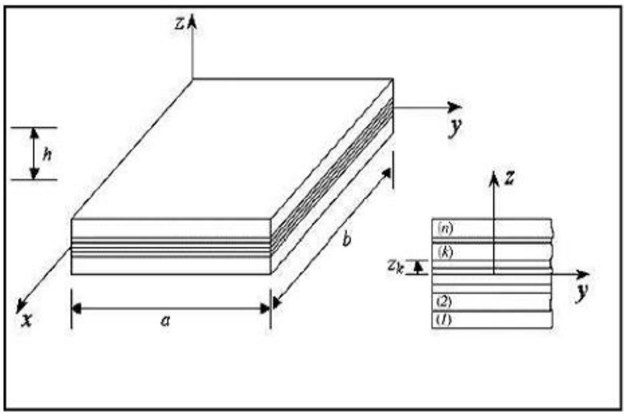

Fig. 1. Geometry of the Composite Laminate (Ref: Reddy 2004)

Displacement Field

The first order shear deformation theory is used to describe the kinematics of deformation for the present analysis. The displacements components u, v and w along x, y and z respectively are expressed as in given below,

Finally, for the deformation theory to finding the displacement expressions are given below,

$$u(x, y, z, t) = u_0(x, y, t) + z\ \theta_x(x, y, t)$$
$$v(x, y, z, t) = v_0(x, y, t) + z\ \theta_y(x, y, t)$$
$$w(x, y, t) = w_0(x, y, t) \qquad (1)$$

Strain- displacement relation

$$\varepsilon_{xx} = \varepsilon^0_{xx} + z\,k_{xx}$$
$$\varepsilon_{yy} = \varepsilon^0_{yy} + z\,k_{yy}$$
$$\gamma_{xy} = \gamma^0_{xy} + z\,k_{xy}$$
$$\gamma_{xz} = \gamma^0_{xz}$$
$$\gamma_{yz} = \gamma^0_{yz} \qquad (2)$$

where, the strains of the middle surface for the displacements.

$$\varepsilon^0_{xx} = \partial u/\partial x + ½(\partial w/\partial x)^2$$
$$\varepsilon^0_{yy} = \partial v/\partial y + ½(\partial w/\partial y)^2$$
$$\gamma^0_{xy} = \partial u/\partial y + +\partial v/\partial x + (\partial w/\partial x)((\partial w/\partial y)$$
$$\gamma^0_{xz} = \theta_x + (\partial w/\partial x)$$
$$\gamma^0_{y} = \theta_y + (\partial w/\partial y) \qquad (3)$$

Constitutive relations

Composite square plate is made up with the number of laminates, each lamina is having the different orientations and equation related fiber orientation are given as follow with respect to the principal axis as shown in below.

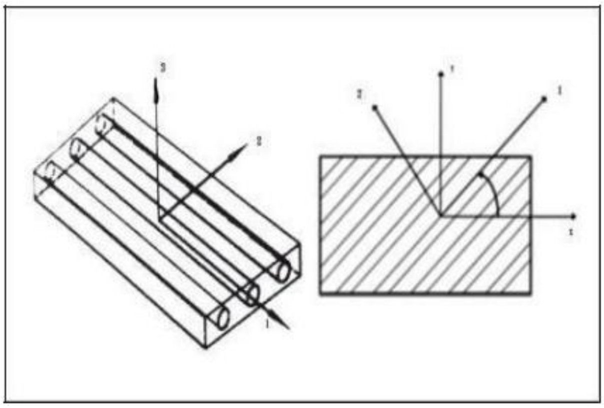

Fig. 2. Lamina with Material Axes System (Ref: Reddy 2004)

Stress strain relationship for a substrate lamina is given by

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & K_iQ_{44} & 0\\ 0 & 0 & 0 & 0 & K_iQ_{55}\end{bmatrix} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \upsilon_{xy}\\ \upsilon_{yz}\\ \upsilon_{xz}\end{Bmatrix} \qquad (4)$$

$$Q_{11} = \frac{E_{11}}{1-\upsilon_{12}\upsilon_{21}} \qquad Q_{12} = \frac{\upsilon_{12}E_{22}}{1-\upsilon_{12}\upsilon_{21}}$$

$$Q_{22} = \frac{E_{22}}{1-\upsilon_{12}\upsilon_{21}} \quad Q_{66} = G_{12},\ Q_{44} = G_{13},\ Q_{55} = G_{23} \qquad (5)$$

The subscripts x, y, z represent the principle material axes. E12, E21, G12, G13 and G23 are elastic longitudinal, transverse and shear moduli of laminated plate respectively. Transformed reduced stiffness matrix is calculated by following formula.

$$\bar{Q}_{11} = Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta$$
$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\cos^4\theta\sin^4\theta)$$
$$\bar{Q}_{22} = Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta$$
$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos\theta\sin^3\theta$$
$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos^3\theta\sin\theta$$
$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos^2\theta\sin^2\theta + Q_{66}(\cos^4\theta + \sin^4\theta)$$
(6)

Shreekant Patil[1]*, Dr. Shravan B. Kerur[2]

$$[ABD]=\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \quad (7)$$

$$A_{ij}=\sum_{k=1}^{n}\left(\bar{Q}_{ij}\right)_k (h_k-h_{k-1})$$

$$B_{ij}=\frac{1}{2}\sum_{k=1}^{n}\left(\bar{Q}_{ij}\right)_k (h_{k-1}^2-h_{k-1}^2)$$

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{n}\left(\bar{Q}_{ij}\right)_k (h_{k-1}^3-h_{k-1}^3)$$

$$i,j=1,2,6. \qquad (8)$$

[A] = In plane stiffness matrix,

[B]= Coupling stiffness matrix,

[D]= Bending stiffness matrix and are calculated using following formula

Elemental stiffness matrix for the plate element is given by,

$$[K_e]=\iint [B]^T[D][B]|J|d\xi d\eta \qquad (9)$$

Elemental load vector is given by

$$\{F\}_e=\iint [N]^T q|J|d\xi d\eta \qquad (10)$$

Where,

$[K_e]$=Element stiffness matrix

$\{F\}_e$=Element load vector

$|J|$ =Determinant of Jocobian

Determinant of Jocobian is given by,

$$[J]=\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} (11)$$

$$[B]=\begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_r}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_r}{\partial y} \\ 0 & 0 & 0 & \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & N_r & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial y} & 0 & N_r \end{bmatrix} (12)$$

III. FINITE ELEMENT FORMULATION

Geometrically nonlinear finite element is developed for the static analysis of laminated composite plate using first order shear deformation theory. An eight noded continuous element is employed.

Isoparametric element

In the present analysis eight node isoparametric elements is considered and the shape function of the element is given below. The independent field variables based on the [u, v, w, θx, θy] values.

Here θx and θy are local coordinates of the element. For the isoparametric element concept, the element geometry, displacement vectors over each element are represented by the interpolation functions.

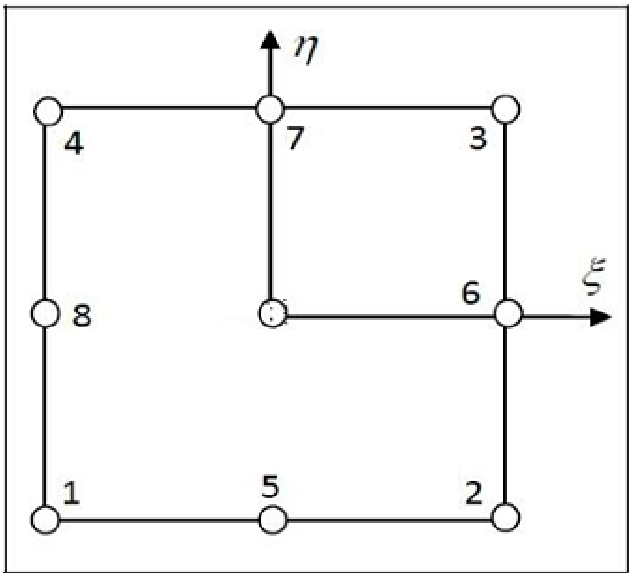

Fig. 3. Lamina with Material Axes System (Ref: Reddy 2004)

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)(-1-\xi-\eta)$$
$$N_2 = \frac{1}{2}(1-\xi^2)(1-\eta)$$
$$N_3 = \frac{1}{4}(1+\xi)(1-\eta)(-1+\xi-\eta)$$
$$N_4 = \frac{1}{2}(1+\xi)(1-\eta^2)$$
$$N_5 = \frac{1}{4}(1+\xi)(1+\eta)(-1+\xi+\eta)$$
$$N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$$
$$N_7 = \frac{1}{4}(1-\xi)(1+\eta)(-1-\xi+\eta)$$
$$N_8 = \frac{1}{2}(1-\xi)(1-\eta^2) \qquad (13)$$

$$[N] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}$$
$$i = 1,2.......8 \qquad (14)$$

Ni is the interpolation function,

IV. RESULTS AND DISCUSSION

Geometrically nonlinear analysis of square plate is formulated by using the finite element modeled for the bending behavior. The laminated square plate is discretized using the eight nodded element .The development of a program was successfully done .In the present work nonlinear problem/ equation is linearized using Newton-Rapson method, now based on the numerical analysis carried out for the non-linear analysis. The finite element model is converted into MatLab code.

1. Non dimensional zed center Deflection :

$$q^* = \frac{q}{E_2}\left(\frac{a}{h}\right)^4, \qquad \overline{w} = \frac{w}{h} \qquad (15)$$

Governing Equations for Nonlinear Bending Analysis

The governing equilibrium equations are obtained by using the Lagrangian virtual work principal and the resulting nonlinear finite element equation is written as

[K{d}] {d} = {F} (16)

Here the nonlinear term [K {d}] in the above nonlinear finite element equation (16), depends on the unknown solution {d}. Thus the above equation needs to be solved iteratively till the solution convergence is achieved with the said tolerance. Here nonlinear finite element equation (17), is linearized by using Newton-Raphson iterative method.

[KT]n {∂d}n+1 = {F} – [Ks]n {d}n (17)

The tangent stiffness matrix [KT]n and the secant stiffness matrix [Ks]n are given in above the boundary conditions and material properties are respectively as shown in the table 1.

Finite element model derived into investigate the nonlinear bending behavior of plate subjected to the mechanical load. A wide variety of numerical examples considering with the plate with the different support conditions and laminates loading conditions, validated formulation is further employed to study the nonlinear bending analysis of plates by varying the loading condition .The nonlinear linear equations are solved using Newton-Rapson iterative method.

For the analysis carried out here, the boundary conditions considered are given in the table 1.The obtained result, unless otherwise mentioned are presented in the following non-dimensional form:

Table I. Boudary Conditions

Simplysupported (S)	Constraints	axis
Cross ply	u = w = θx = 0	Along x axis
	v = w = θy = 0	Along y axis
Angle support	u = w = θx = 0	Along x axis
	u = w = θy = 0	Along y axis
Clamped Support (C)	u = v = w = θx = θy = 0	Along x and y axis

V. VALIDATION STUDY

The present nonlinear FE model is converted to MatLab code and is validated with available results for its accuracy and correctness. The comparison of present results with available results is presented in below table.

Nonlinear deflection for simply supported angle ply [θ/ -θ] plate with θ =45, 30, 15 are presented in table 4.2 with respect results are available in open literature. Present study are having the material properties shown in table 4.3.Results are good agreement with available reference

Table 4.2 : Comparison of non- linear deflection of simply support two layer angle ply square plates under uniform loading (a/h=100) , Ref* Turvey and Wittrick (1973) and Ref¥ Singh et al,(1994).

Load Q	45/-45			30/-30		
	Ref[*]	Ref[¥]	Present	Ref[*]	Ref[¥]	Present
200	0.77	0.7556	0.7564	0.75	0.7391	0.7411
400	1.02	1.0055	1.0071	1.00	0.9748	0.9781
600	1.20	1.1736	1.1757	1.15	1.1322	1.1365
800	1.32	1.3049	1.3075	1.27	1.2551	1.2599
1000	1.42	1.4145	1.4174	1.38	1.3581	1.3628

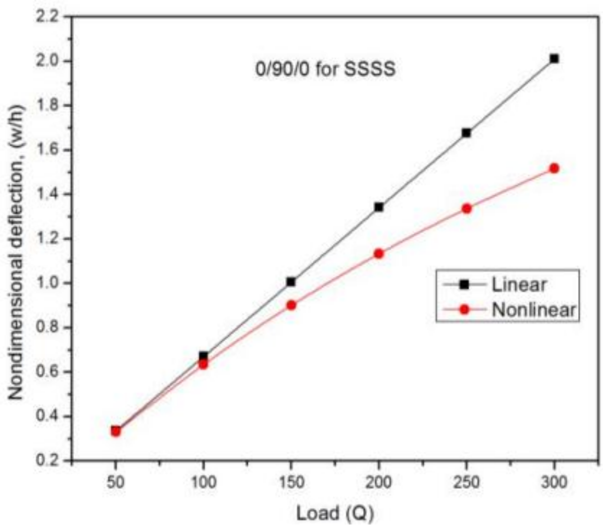

Fig. 4. Comparison of linear and nonlinear results for a simply supported.

Numerical examples:

In this section numerical analysis is carried out by considering different boundary conditions to know the geometric nonlinear behavior of anti-symmetric cross ply laminated plates. In the present analysis the effect of number of layers and the lamination scheme on the center deflection is examined for anti-symmetric cross-ply laminates. A square plate with clamped, simply supported, hinged boundary condition and material. The material properties of the laminates are taken as follows:

$E1=25GPa$, $E2=1GPa$, $G12=0.5GPa$, $G13=0.4GPa$, $G23=0.5GPa$, $\upsilon 12=0.26$, $\rho=1824\ kg/m^3$.

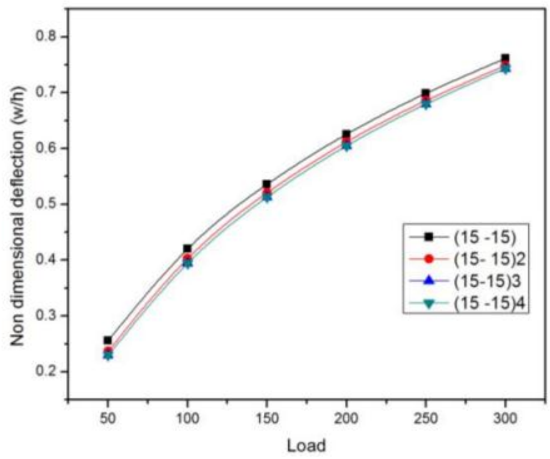

Fig. 5. Effect on number of layers on transverse displacement of uniform distributed load with CSCS boundary conditions for angle ply (15/-15).

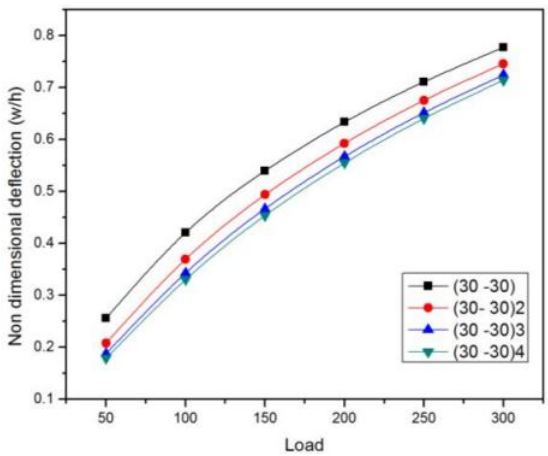

Fig. 6. Effect on number of layers on transverse displacement of uniformly distributed load with CSCS boundary conditions for angle ply (30/-30).

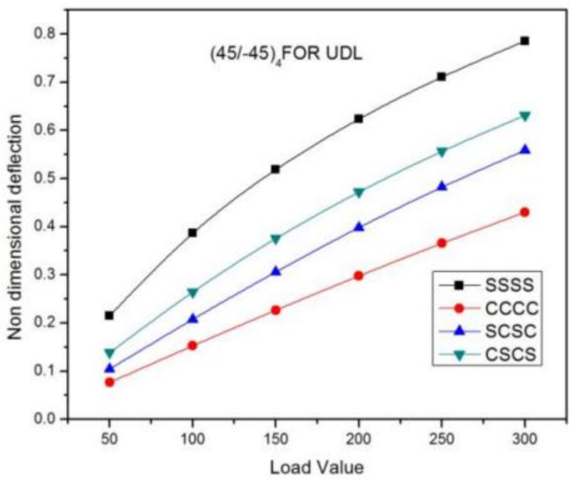

Fig. 7. Effect on number of layers on transverse displacement of uniformly distributed load all boundary condition for anti-symmetric angle ply

Shreekant Patil[1]*, Dr. Shravan B. Kerur[2]

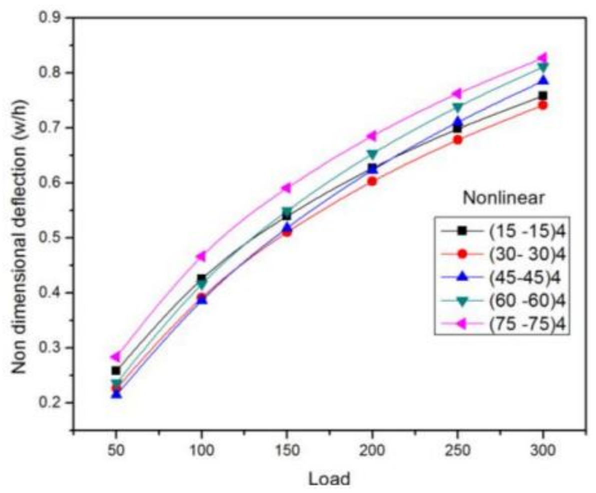

Fig. 8. Effect on number of layers on transverse displacement of uniformaly distributed load SSSS boundary condition for anti-symmetric angle-ply.

VI. CONCLUSION

Geometrically nonlinear analysis of square plate is formulated by using the finite element method for the bending behavior. The laminated square plate is discretized using the eight nodded element. The formulated FE model is successfully converted to MatLab code and is been validated. It is concluded that the number of layers, stacking sequence, boundary condition have significant effect on the nonlinear deflection of the plate. In the present work nonlinear problem/equation is linearized using Newton-Rapson method, now based on the numerical analysis carried out for the non-linear bending analysis conclusions are as follows

- The effect of fiber orientation and number of layers on geometric nonlinear behavior as the number of ply increase then the nonlinearity increases.
- The effect on same number of layers with having different fiber orientation then the deflection will increases.

REFERENCES

Brebbia, C. and Connor, J. (1969). Geometrically nonlinear finite-element analysis, Journal of Engineering Mechanics Division 95: 463-483.

Wood, R. D. and Zienkiewicz, O. C. (1997). Geometrically nonlinear finite element analysis of beams, frames, arches and axisymmetric shells, Computer & Structure 7(6): 725-735.

Bathe, K. J. and Bolourchi, S. (1980). A geometric and material nonlinear plate and shell element, Computer & Structure 11(1): 23-48.

A. Pica R. D. Wood and E. Hinton (1979). Finite element analysis of geometrically nonlinear plate behaviour using mindlin formulation. Computer & Structure 11: 203-215

J. N. Reddy and W. C. Chao (1981). Nonlinear bending of thick rectangular, laminated composite plates. Int. J. Nonlinear Mechanics 11(3/4):291-301.

P. C. Yang, C. H. Norris and Y. Stavsky (1966). Elastic wave propagation in heterogeneous plates. Int. J. Solids and structure 2:665-685.

N. S. Puctha and J. N. Reddy (1986). A refined mixed shear felexible finite element for the nonlinear analysis of laminated plates, Computer & Structure 22(4): 529-538.

Chia CY. (1985). Large deflection of unsymmetrical laminates with mixed boundary conditions. Int J Non-Linear Mech 20(4):273-82.

Striz AG, Jang SK, Bert CW. (1988). Nonlinear bending analysis of thin circular plates by differential quadrature. Thin-Walled Struct 6(1):51-62.

Z. G. Azizian and D. J. Dawe (1992). Geometrically nonlinear analysis of rectangular Mindilin plates using the finite strip method. Computer & structure 21(3): 423-436.

Gajbir Singh, G. Venkateswara Rao, N.G.R. Iyengar. (1991).Some observations on the large deflection bending of rectangular anti-symmetric cross-ply plates. Compos Struct 18:77-91.

Murray J. Clarke, Gregory J. Hancock. (1990) A study of incremental-iterative strategies for non-linear analyses. Int J Numer Methods Eng. 29:1365-91.

Madhulit Mukhopadhya,(2012). Mechanics of Composite materials and structures, Universites press, ISBN-978 81 7371 4771.

Zaghloul SA, Kennedy JB. Nonlinear behavior of symmetrically laminatedplates.J.Appl.Mech,42:2.

Corresponding Author

Shreekant Patil*

PG Student, Dept. of Mechanical Engg., BEC. Bagalkot, India

E-Mail – Shreekantbpatil007@gmail.com