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Analysis of Deflection of Beam Using Finite Element Method and Approximate Analytical Methods - A Comparative Study

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Abstract – The present paper demonstrates the deflection analysis of simply supported beam subjected to uniformly distributed load. This work includes a comparative study of deflection of beam obtained by using 4th degree polynomial, 2nd degree polynomial for the field variable and finite element approach. This paper helps the engineering students & research scholars to understand and interpret the results of analytical method, weak form of weighed residual method and FEA analysis of beam.

Keywords — Weighed residual Method, BEAM Analysis

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I. INTRODUCTION

The beams and frames are widely used in construction of structures. To predict their behavior accurately, a fair analysis is needed for different parameters like deflection, bending stresses and strain energy absorbed by them. A variety of theories and techniques are widely used by engineers to calculate the basic as well as derived parameters. The present paper compares the deflection of simply supported beam obtained by three different methods

1. Using analytical method involving use of 4th degree polynomial for field variable

2. Using weak form of weighed residual method (2nd degree of polynomial) and

3. Finite element approach

Each method has its own advantages and dis advantages. As a matter of fact, the analytical method is useful if the loading and geometry is simple whereas use of weak form of weighed residual method allows designer to impose less continuity requirements on field variable. [1] As can be seen in the following analysis. But their application requires mostly use of even degree of polynomial to be defined for field variable. On the contrary the use of FEA method accepts geometric & material discontinuity problems, problems involving three dimensional complex loading pattern[3][4]. The paper compares the results of these three methods for relatively simple problem of simply supported beam subjected to UDL. The readers of this paper are further advised to identify the applicability of these methods for fairly critical loading and geometric conditions encountered in practical applications.

II. PROBLEM DEFINITION

In given problem, analysis of simply supported beam is carried out. The beam is made up of steel with E = 210 Gpa, μ = 0.33. The beam spans over a length of 1 m and is subjected to a uniformly distributed load of intensity 750 N/m the cross section of beam is square with side 10 mm.

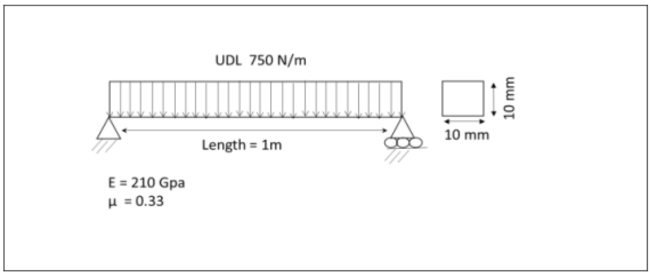
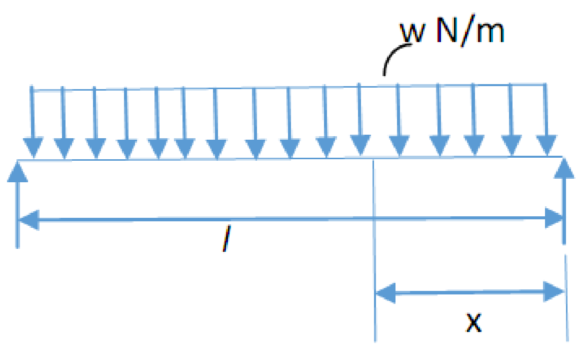


Figure 1 Simply supported beam with UDL

III. DEFLECTION USING 4TH DEGREE POLYNOMIAL (ANALYTICAL METHOD)

Consider a section at a distance x from right support of beam, the bending moment Mx is given by



$M_x = wx^2/2 - R_2x$

$M_x = wx^2/2 - wlx/2$ ---------------------------(1)

Thus,

$EI\, d^2y/dx^2 = wx^2/2 - wlx/2$ --------------------------------(2)

Integrating, we get,

$\therefore EI\, dy/dx = wx^3/6 - wlx^2/4 + C1$ --------------------------(3)

Applying boundary conditions at x = l/2, dy/dx = 0

Hence $C1 = wl^3/24$

Thus,

$EI\, dy/dx = wx^3/6 - wlx^2/4 + wl^3/24$ ----------------------(4)

Again integrating, We get

$E\,I\, y = wx^4/24 - wlx^3/12 + wl^3x/24 + C2$ ------------------(5)

At x = 0, y = 0 Thus, C2 = 0

$y = \delta = 1/EI\, [wx^4/24 - wlx^3/12 + wl^3x/24]$ --------------------(6)

IV. DEFLECTION USING WEAK FORM OF WEIGHED RESIDUAL METHOD

Governing Differential Equation

$EI\, d^4y/dx^4 - q = 0$

Where q is the intensity of UDL

Let, $y = a_0 + a_1x + a_2x^2$ (Second degree polynomial)

Applying Boundary conditions (Slope and Deflection)

We get,

$$a_0 = 0$$
$$a_1 = -a_2 l$$

Thus, $y = a_2x\,(x-l)$

Let y* be the approximate solution thus,

$R_d(x) = EI\, d^4y^*/dx^4 - q$, where $R_d(x)$ = residual

According to weighed Residual Method,

$\int_0^l w(x)\, R_d(x) = 0$, Where w(x) is weighing function.

Thus

$\int_0^l w(x)\, [EI\, d^4y^*/dx^4 - q]\, dx = 0$

Integrating by parts and using standard formula,

$\int_0^l u\, dv = [uv]_0^l - \int_0^l v\, du$ (in order to reduce continuity requirement of field variable)

Thus,

$[\, w(x)\, EI\, d^3y^*/dx^3\,]_0^l - \int_0^l \{EI\, (d^3y^*/dx^3)\, dw(x)/dx\}\, dx = \int_0^l w(x)\, q\, dx$

Again integrating by parts,

$[\, w(x)\, EI\, d^3y^*/dx^3\,]_0^l - \{[dw(x)/dx\;\; EI\, d^2y^*/dx^2]_0^l - \int_0^l [EI\, d^2y^*/dx^2\, d^2w(x)/dx^2]dx\} = \int_0^l w(x)\, q\, dx$

As w(x) and y(x) are similar (as in galerkins method)

$w(0) = 0 \qquad y(0) = 0$ ----------------(a)

$w(l) = 0 \qquad y(l) = 0$ ----------------(b)

$EI\, d^2y/dx^2 = M = 0,$

at x = 0 and x = $l/2$ --------(c)

Thus,

$\int_0^l EI\, d^2y^*/dx^2\, d^2w(x)/dx^2 = \int_0^l w(x)\, q\, dx$

The continuity requirement has reduced and equally distributed between field variable and interpolation function requires lower order polynomial (i.e. 2nd order)

Now, $y = a_2x\,(x-l)$ and $w(x) = x\,(x-l)$

$d^2y/dx^2 = 2a_2 \qquad d^2w(x)/dx^2 = 2$

Thus above equation becomes

$$4\, a_2\, EI\, l = q\, [l^3/3 - l^3/2]$$
$$a_2 = -1/24\, ql^2/EI$$

Thus ,

$y = -1/24\, ql^2/EI\, x\,(x-l)$

$y = \delta = 1/24\, ql^2/EI\, x\,(l-x)$ -------------(7)

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V. DEFLECTION USING ANSYS

Problem in ANSYS 14.0 APDL is modeled using BEAM 188 Element. The loading conditions are applied i.e. left end of beam is fixed but ROTZ (rotation along axis perpendicular plane of paper is) unblocked whereas the right end ROTZ and UX is unblocked as the case is simply supported beam [2][5]. The beam is divided into eight equal parts and is subjected to UDL of intensity 750N/m in negative Y direction as shown in figure below

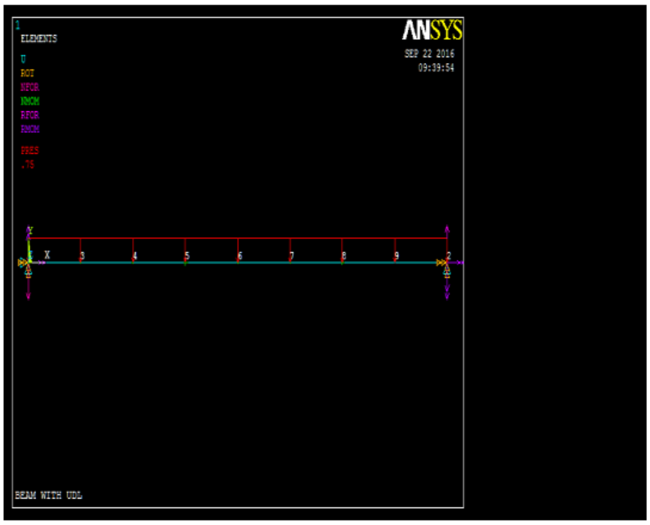

Figure 2. Modeled beam with forces and reaction

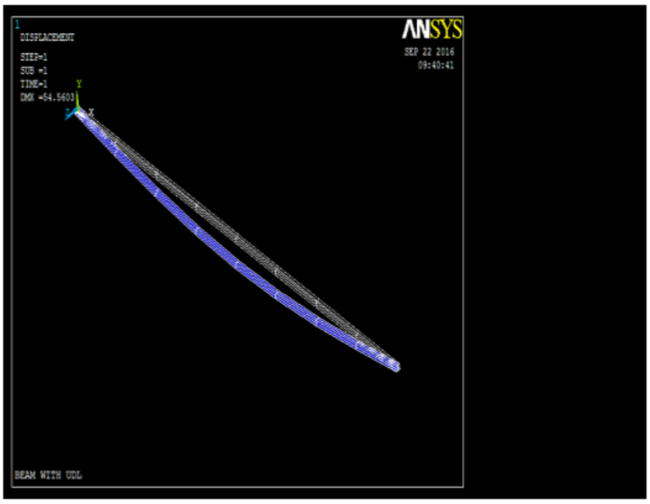

Figure 3. Solution showing maximum deflection of beam

It is clear that the deflection is maximum at midpoint and δmax = 54.56 mm

VI. DISCUSSION AND CONCLUSIONS

The table below compares the result obtained in all three methods

TABLE I COMPARATIVE VALUES OF DEFLECTION

Location of nodal point	Deflection δ using 4^{th} order polynomial (mm)	Deflection δ using 2^{nd} order polynomial (mm)	Deflection δ using ANSYS (mm)	% Error in analytical and ANSYS	% Error in analytical and WR Method
0 % L	0	0	0	0.00	0.00
12.5 % L	21.66	19.53	21.124	2.47	9.83
25.0 % L	39.76	33.48	38.828	2.34	15.79
37.5 % L	51.66	41.85	50.496	2.25	18.99
50.0 % L	55.80	44.46	54.56	2.22	20.32
62.5 % L	51.66	41.85	50.496	2.25	18.99
75.0 % L	39.76	33.48	38.826	2.35	15.79
87.5 % L	21.66	19.53	21.124	2.47	9.83
100 % L	0	0	0	0.00	0.00

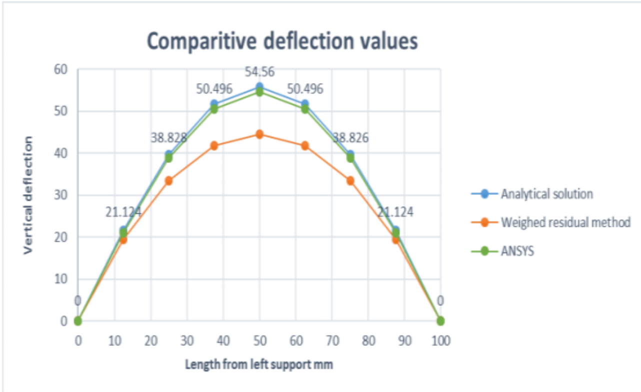

Figure 4 Comparative deflection values

It is seen that the weighed residual method lowers the continuity requirement on field variable (2nd order polynomial can also be used instead of 4th order) but at the same time, results obtained from it deviates from actual results. This deviation is maximum towards the point of maximum deflection. See highlighted cell in table. Also the variation of % error can also be seen from graph. The ANSYS predicts much closer results to actual solution (4th order polynomial) but the values are lower bound (due to high stiffness matrices in FEA formulations) one can increase no of nodes to obtain more close results to actual using either P or H formulations. i.e either use BEAM 189 three noded element with less no of nodes or use two noded BEAM 188 element using more no of nodes. For practical purpose where material discontinuity and geometric discontinuity is present along with complex three dimensional loading, suitable P & H formulation in FEA can lead to more accurate results. Also from table one can predict the accuracy of results remains almost constant over entire domain which ensures more realistic predictions than any other method.

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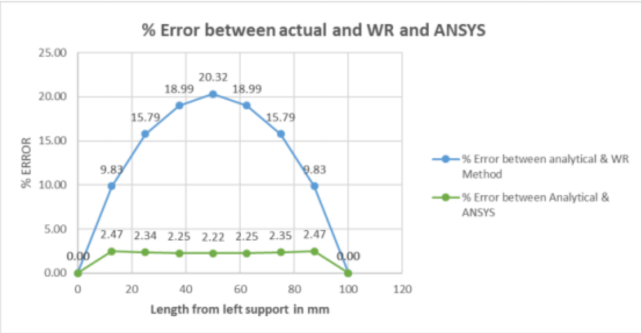

Figure 5 % Error between Actual and WR and ANSYS Method

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