Dufour Effect on Radiative MHD Flow of a Viscous Fluid in a Parallel Porous Plate Channel under the Influence of Slip Condition

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Abstract – The objective of the present paper is to study the Dufour and chemical reaction effects on an unsteady heat and mass transfer MHD flow of a viscous, incompressible and electrically conducting fluid between two parallel porous plates under the influence of slip condition. Exact solution of the governing equations for the fluid velocity, temperature and concentration are obtained. The numerical values of fluid velocity are displayed graphically for various values of pertinent flow parameters.

Keywords: MHD fluid , Dafour effect, Chemical reaction, Parallel plate channel. 2010 Mathematics Subject Classification: 76D05, 76S05.

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I. INTRODUCTION

The Dufour and Soret effects in the combined heat and mass transfer processes, due to the thermal energy flux resulting from concentration gradients and the thermal diffusion flux resulting from the temperature gradients, may be significant in the areas of geosciences and chemical engineering, Eckert and Drake [1]. Such physical effects were explored in the papers by Kafoussias and Williams [2], by Anghel et al. [3] and by Lin et al. [4], amongst others. Tai and Char [5] examined the Soret and Dufour effects on free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium with thermal radiation. Venkateswarlu et al. [6-9] presented heat and mass transfer characteristics on MHD flows with chemical reaction and thermal radiation.

II. MATHEMATICAL FORMATION OF THE PROBLEM

We consider the unsteady laminar slip flow of an incompressible, viscous and electrically conducting fluid through a channel with non-uniform wall temperature bounded by two parallel plates separated by a distance a . The channel is assumed to be filled with a saturated porous medium. A uniform magnetic field of strength B_o is applied perpendicular to the plates. The above plate is heated at constant temperature and thermal radiation effect is also taken in to account. It is assumed that there exist a homogeneous chemical reaction of first order with constant rate K_r^* between the diffusing species and the fluid. Geometry of the problem is presented in Figure. 1. We choose a Cartesian coordinate system (x, y) where $x-$ lies along the centre of the channel, $y-$ is the distance measured in the normal section such that $y = a$ is the channel's width as shown in the figure below. Under the usual Boussinesq's approximation, the equations of conservation of mass, momentum, energy and concentration governing the natural convective nonlinear boundary layer flow over a laminar porous plate in porous medium can be expressed as:

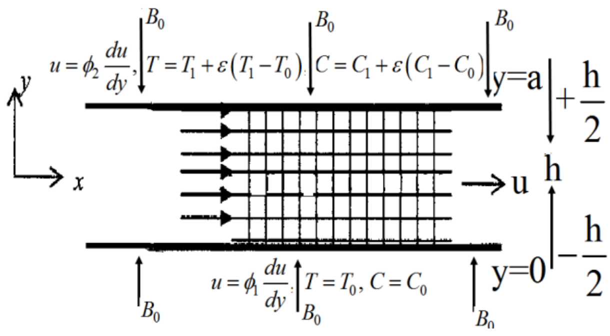

Figure 1: Geometry of the problem

Continuity equation:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T\left(T - T_0\right) + g\beta_C\left(C - C_0\right) - \frac{\sigma_e B_0^2}{\rho}u - \frac{\nu}{K}u \qquad (2)$$

Energy equation:

$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{4\alpha^2}{\rho c_p}\left(T - T_0\right) + \frac{D_m K_T}{C_s c_p}\frac{\partial^2 C}{\partial y^2} \qquad (3)$$

Diffusion equation;

$$\frac{\partial C}{\partial t} = D_m\frac{\partial^2 C}{\partial y^2} - K_r^*\left(C - C_0\right) \qquad (4)$$

where $u-$ fluid velocity in $x-$direction, $v-$ fluid velocity along $y-$direction, $p-$fluid pressure, $g-$ acceleration due to gravity, $\rho-$ fluid density, β_T- coefficient of thermal expansion, β_C- coefficient of concentration volume expansion, $t-$ time, $K-$ permeability of porous medium, B_o- magnetic induction, $T-$ fluid temperature, T_o- temperature at the cold wall, K_T- thermal diffusivity of the fluid, $\alpha-$ dimensional radiation parameter, $C-$ species concentration in the fluid, C_o- concentration at the cold wall, σ_e- fluid electrical conductivity, c_p- specific heat at constant pressure, D_m- chemical molecular diffusivity, c_s- concentration susceptibility, $\nu-$kinematic viscosity of the fluid and K_r^*- dimensional chemical reaction parameter respectively.

Assuming that slipping occurs between the plate and fluid, the corresponding initial and boundary conditions of the system of partial differential equations for the fluid flow problem are given below

$$\left.\begin{array}{l} u = \phi_1\dfrac{du}{dy}, \quad T = T_0, \quad C = C_0 \quad \text{at} \quad y = 0 \\ u = \phi_2\dfrac{du}{dy}, T = T_1 + \varepsilon\left(T_1 - T_o\right)\exp\left(\text{int}\right), \\ C = C_1 + \varepsilon\left(C_1 - C_o\right)\exp\left(\text{int}\right) \text{ at } y = a \end{array}\right\} \qquad (5)$$

where T_1-fluid temperature at the heated plate, C_1-species concentration at the heated plate, ϕ_1-cold wall dimensional slip parameter, ϕ_2-heated wall dimensional slip parameter, $n-$ frequency of oscillation and $\varepsilon << 1$ is a very small positive constant.

We introduce the following non-dimensional variables

$$\psi = \frac{x}{h}, \eta = \frac{y}{h}, U = \frac{h}{\nu}u, \; P = \frac{h^2}{\rho\nu^2}p, \gamma = \frac{\phi_1}{h}, \sigma = \frac{\phi_2}{h},$$
$$\omega = \frac{h^2}{\nu}n, \tau = \frac{\nu}{h^2}t, \theta = \frac{T - T_o}{T_1 - T_o}, \phi = \frac{C - C_o}{C_1 - C_o} \qquad (6)$$

Equations (2), (3) and (4) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial \tau} = -\frac{dP}{d\psi} + \frac{\partial^2 U}{\partial \eta^2} + Gr\theta + Gm\phi - \left[M + \frac{1}{Da}\right]U \qquad (7)$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} + Du\frac{\partial^2 \phi}{\partial \eta^2} - H\theta \qquad (8)$$

$$\frac{\partial \phi}{\partial \tau} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial \eta^2} - Kr\phi \qquad (9)$$

Here $Gr = \frac{g\beta_T\left(T_1 - T_o\right)h^3}{\nu^2}$ is the thermal buoyancy force, $Gm = \frac{g\beta_C\left(C_1 - C_0\right)h^3}{\nu^2}$ is the concentration buoyancy force, $M = \frac{\sigma_e B_o^2 h^2}{\rho\nu}$ is the magnetic parameter, $Da = \frac{K}{h^2}$ is the Darcy parameter, $\Pr = \frac{\rho c_p \nu}{K_T}$ is the Prandtl number, $N = \frac{4\alpha^2 h^2}{\rho c_p \nu}$ is the thermal radiation parameter, $Du = \frac{D_m K_T\left(C_1 - C_o\right)}{c_s c_p \nu\left(T_1 - T_o\right)}$ is the Dufour number, $Sc = \frac{\nu}{D_m}$ is the Schmidt number and $Kr = \frac{h^2}{\nu}K_r^*$ is the chemical reaction parameter respectively.

Corresponding initial and boundary conditions are given by

$$\left.\begin{array}{l} U = \gamma U', \quad \theta = 0, \quad \phi = 0 \text{ at } \eta = 0 \\ U = \sigma U', \theta = 1 + \varepsilon\exp\left(i\omega\tau\right), \phi = 1 + \varepsilon\exp\left(i\omega\tau\right) \text{at } \eta = 1 \end{array}\right\} \qquad (10)$$

Following Adesanya and Makinde [12], for purely an oscillatory flow we take the pressure gradient of the form

$$\lambda = -\frac{dP}{d\psi} = \lambda_0 + \varepsilon\exp(i\omega t)\lambda_1 \qquad (11)$$

where λ_0- and λ_1- are constants and ω is the frequency of oscillation.

III. SOLUTION OF THE PROBLEM

Due to the selected form of pressure gradient we assume the solution of equations (7) to (9) of the form

$$U(\eta,\tau)=U_0(\eta)+\varepsilon\exp(i\omega\tau)U_1(\eta)+o(\varepsilon^2) \quad (12)$$

$$\theta(\eta,\tau)=\theta_0(\eta)+\varepsilon\exp(i\omega\tau)\theta_1(\eta)+o(\varepsilon^2) \quad (13)$$

$$\phi(\eta,\tau)=\phi_0(\eta)+\varepsilon\exp(i\omega\tau)\phi_1(\eta)+o(\varepsilon^2) \quad (14)$$

Substituting equations (12) to (14) into equations (7) to (9), then equating the harmonic and non–harmonic terms and neglecting the higher order terms of $o(\varepsilon^2)$, we obtain

$$U_0''-\left[M+\frac{1}{Da}\right]U_0=-[Gr\theta_0+Gm\phi_0+\lambda_0] \quad (15)$$

$$U_1''-\left[M+\frac{1}{Da}+i\omega\right]U_1=-[Gr\theta_1+Gm\phi_1+\lambda_1] \quad (16)$$

$$\theta_0''-\Pr N\theta_0=-\Pr Du\phi_0'' \quad (17)$$

$$\theta_1''-\Pr(N+i\omega)\theta_1=-\Pr Du\phi_1'' \quad (18)$$

$$\phi_0''-ScKr\phi_0=0 \quad (19)$$

$$\phi_1''-Sc(Kr+i\omega)\phi_1=0 \quad (20)$$

where the prime denotes the ordinary differentiation with respect to η.

Initial and boundary conditions in equation (10), can be written as

$$\left.\begin{array}{l}U_o=\gamma U_o',U_1=\gamma U_1',\theta_o=0,\theta_1=0,\phi_o=0,\phi_1=0 \text{ at } \eta=0\\ U_o=\sigma U_o',U_1=\sigma U_1',\theta_o=1,\theta_1=1,\phi_o=1,\phi_1=1 \text{ at } \eta=1\end{array}\right\} \quad (21)$$

We obtained the analytical solutions for the fluid velocity, temperature and concentration and are presented in the following form

$$U(\eta,\tau)=\left[\begin{array}{l}B_{22}\exp(A_5\eta)+B_{21}\exp(-A_5\eta)+\\ B_8-\dfrac{B_6\sinh(A_3\eta)}{\sinh(A_3)}+\dfrac{B_7\sinh(A_1\eta)}{\sinh(A_1)}\end{array}\right]+ \varepsilon\exp(i\omega\tau)\left[\begin{array}{l}B_{39}\exp(A_6\eta)+B_{38}\exp(-A_6\eta)+\\ B_{25}-\dfrac{B_{23}\sinh(A_4\eta)}{\sinh(A_4)}+\dfrac{B_{24}\sinh(A_2\eta)}{\sinh(A_2)}\end{array}\right] \quad (22)$$

$$\theta(\eta,\tau)=\left[\frac{B_3\sinh(A_3\eta)}{\sinh(A_3)}-\frac{B_2\sinh(A_1\eta)}{\sinh(A_1)}\right]+ \varepsilon\exp(i\omega\tau)\left[\frac{B_5\sinh(A_4\eta)}{\sinh(A_4)}-\frac{B_4\sinh(A_2\eta)}{\sinh(A_2)}\right] \quad (23)$$

$$\phi(\eta,\tau)=\left[\frac{\sinh(A_1\eta)}{\sinh(A_1)}\right]+\varepsilon\exp(i\omega\tau)\left[\frac{\sinh(A_2\eta)}{\sinh(A_2)}\right] \quad (24)$$

Here the constants are not given under the brevity

IV. GRAPHICAL RESULTS AND DISCUSSION

In order to investigate the influence of various physical parameters such as thermal Grashof number Gr, solutal Grashof number Gm, Darcy parameter Da, pressure gradient λ, magnetic parameter M, cold wall slip parameter γ, heated wall slip parameter σ, Prandtl number $\Pr$, chemical reaction parameter Kr, mass diffusion parameter Sc and Dufour effect Du on the flow-field, the fluid velocity U, temperature θ and concentration ϕ have been studied analytically and computed results of the analytical solutions from equations (22) to (24) are displayed graphically from Figs.2 to 20 for various values of these physical parameters. In the present study following default parameter values are adopted for computations: $Du=4$,

$Gr=5, Gm=10, M=5, Da=3, \lambda=1, N=2$, Air $(\Pr=0.71)$, Water $(\Pr=7.0)$, Hydrogen $(Sc=0.22)$, Ammonia $(Sc=0.78)$ $Kr=5, \omega=0.5, \gamma=0.1, \sigma=0.1, \tau=0, \varepsilon=0.005$.

Therefore all the graphs are corresponding to these values unless specifically indicated on the appropriate graph.

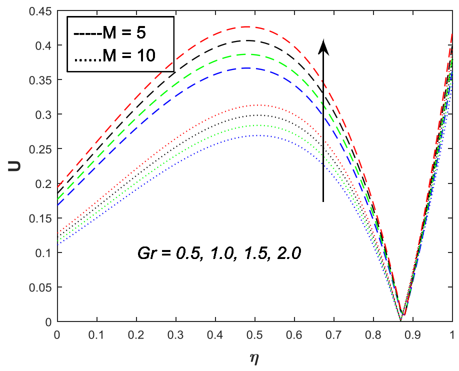

Figure 2: Velocity U against η for varying Gr and M.

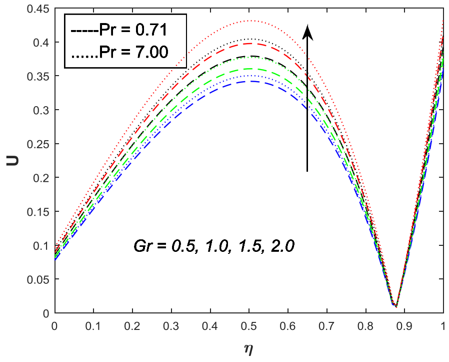

Figure 3: Velocity U against η for varying Gr and $\Pr$.

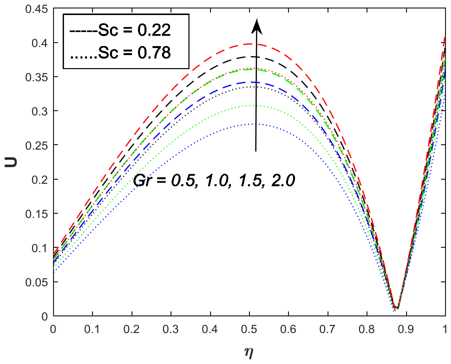

Figure 4: Velocity U against η for varying Gr and Sc .

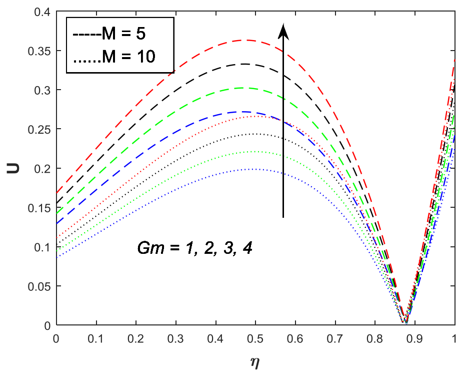

Figure 5: Velocity U against η for varying Gm and M .

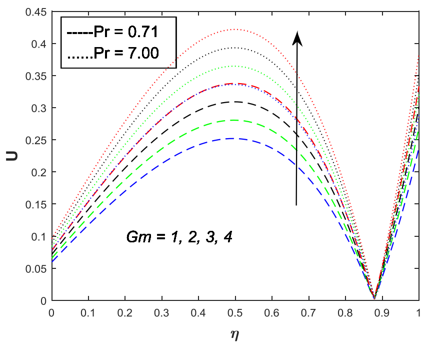

Figure 6: Velocity U against η for varying Gm and $\Pr$.

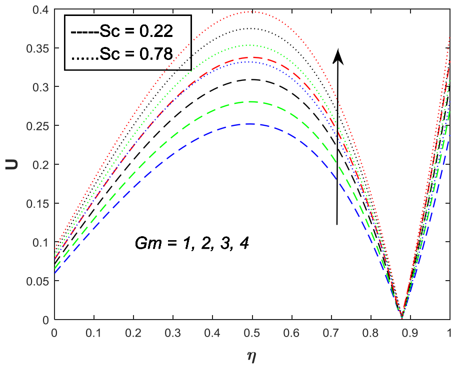

Figure 7: Velocity U against η for varying Gm and Sc .

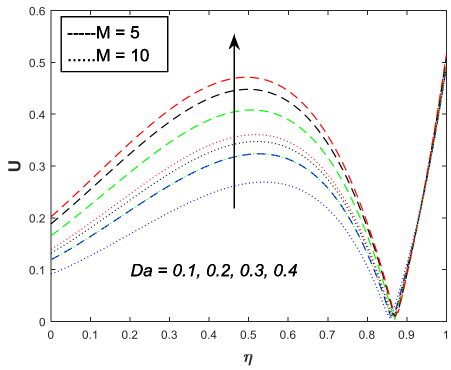

Figure 8: Velocity U against η for varying Da and M .

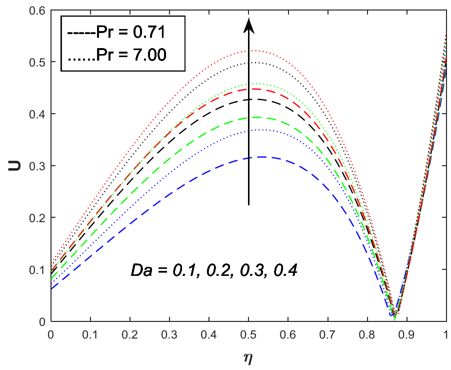

Figure 9: Velocity U against η for varying Da and $\Pr$.

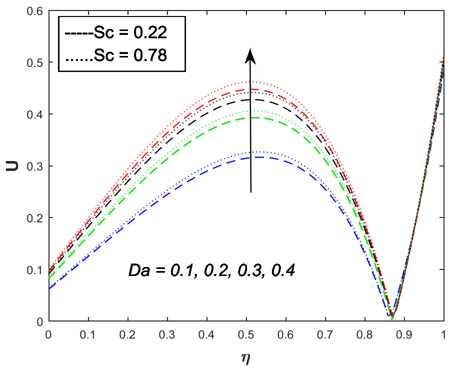

Figure 10: Velocity U against η for varying Da and Sc .

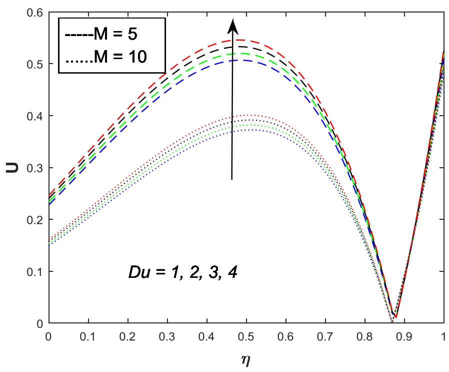

Figure 11: Velocity U against η for varying Du and M .

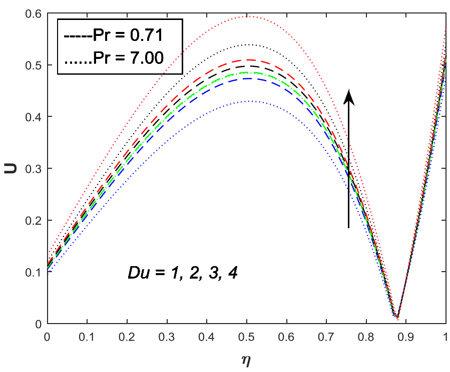

Figure 12: Velocity U against η for varying Du and $\Pr$.

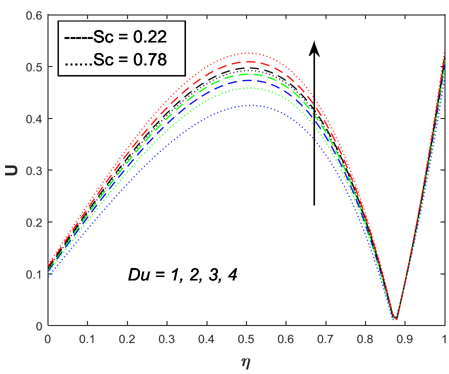

Figure 13: Velocity U against η for varying Du and Sc .

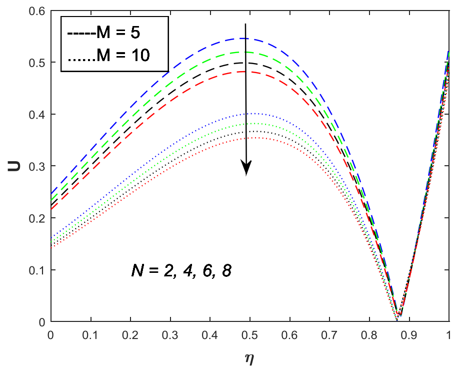

Figure 14: Velocity U against η for varying N and M .

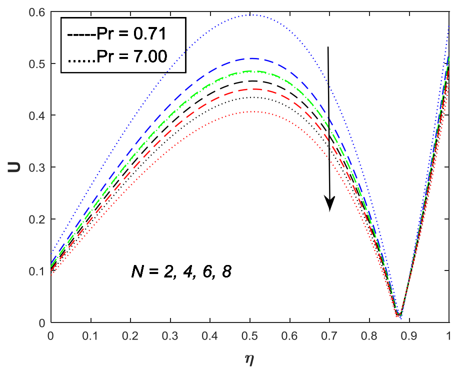

Figure 15: Velocity U against η for varying N and $\Pr$.

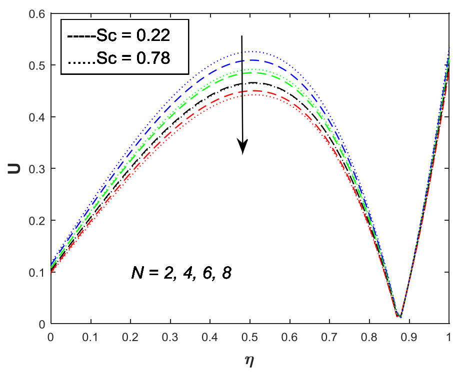

Figure 16: Velocity U against η for varying N and Sc .

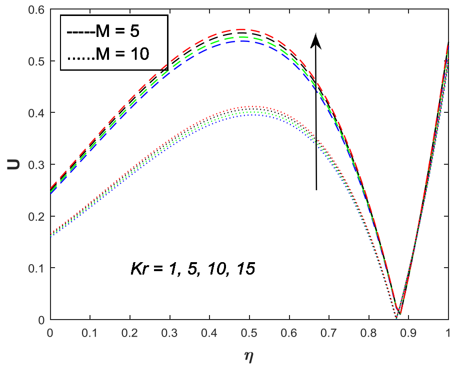

Figure 17: Velocity U against η for varying Kr and M .

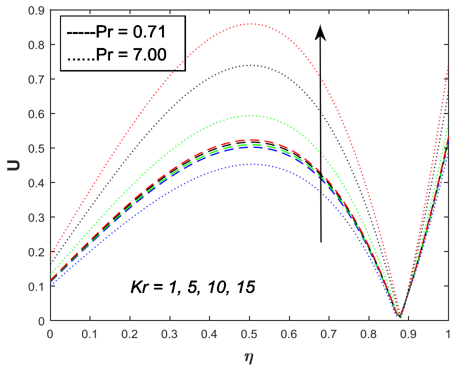

Figure 18: Velocity U against η for varying Kr and $\Pr$.

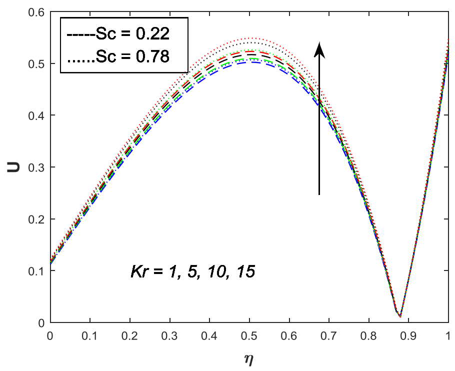

Figure 19: Velocity U against η for varying Kr and Sc .

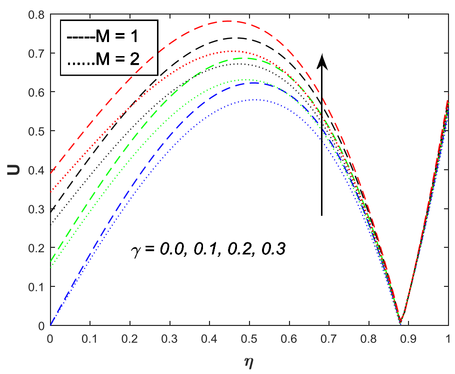

Figure 20: Velocity U against η for varying γ and M .

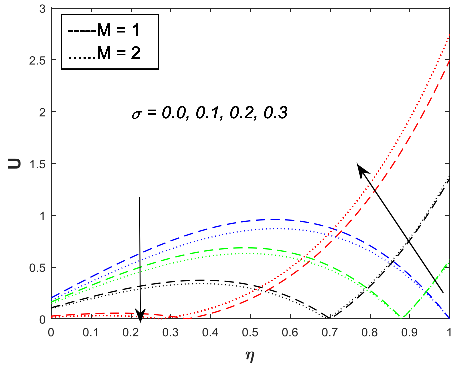

Figure 21: Velocity U against η for varying σ and M .

The effect of thermal Grashof number Gr on the velocity U of the flow field is presented in Figs. 2 to 4. Physically, thermal Grashof number Gr signifies the relative strength of thermal buoyancy force to viscous hydrodynamic force in the boundary layer. A study of the curves shows that thermal Grashof number Gr accelerates the velocity of the flow field at all points. This is due to the reason that there is an enhancement in thermal buoyancy force. The effect of solutal Grashof number Gm on the velocity U of the flow field is presented in Figs. 5 to7. Physically, Solutal Grashof number Gm signifies the relative strength of species buoyancy force to viscous hydrodynamic force in the boundary layer. A study of the curves shows that solutal Grashof number Gm accelerates the velocity of the flow field at all points. This is due to the reason that there is an enhancement in concentration buoyancy force.

Figs. 8 to 10 show the variation of fluid velocity U with the Darcy parameter Da . The graph shows that an increase in the Darcy parameter increases the fluid flow except at the flow reversal point at the heated wall. The influence of Dufour number Du on velocity U is plotted in Figs. 11 to 13. The Dufour number signifies the contribution of the concentration gradients to the thermal energy flux in the flow. It is found that an increase in Dufour number causes a rise in the velocity.It is observed that from Figs.14 to 16, the fluid velocity U decreases on increasing the radiation parameter N . Figs. 17 to 19 demonstrate the effects of chemical reaction parameter Kr on the velocity. It is observed that, velocity U increases on increasing the chemical reaction parameter Kr .

Figs. 20 and 21 shows the fluid velocity profile variations with the cold wall slip parameter γ and the heated wall slip parameter σ . It is observed that, the fluid velocity U increases on increasing the cold wall slip parameter γ thus enhancing the fluid flow. The cold wall slip parameter did not cause any appreciable effect on the heated wall. An increase in the heated wall slip parameter σ decreases the fluid velocity minimally at the cold wall and increasing the heated wall slip parameter causes a flow reversal towards the heated wall. It is observed that $\sigma = 0$ corresponds to the pulsatile case with no slip condition at the heated wall in Fig 21.

IV. CONLUSIONS

In this paper we have studied the Dufour Effect on Radiative MHD Flow of a Viscous Fluid in a Parallel Porous Plate Channel under the Influence of Slip Condition. From the present investigation the following conclusions can be drawn:

An increase in the magnetic parameter M decreases the fluid velocity U due to the resistive action of the Lorenz forces except at the heated wall where the reversed flow induced by wall slip caused an increase in the fluid velocity. This implies that magnetic field tends to decelerate fluid flow. Velocity profiles are increases on increasing the Prandtl number. Influence of thermal buoyancy force decreases on velocity on increasing the Schmidt number. An increase in the heated wall slip parameter σ decreases the fluid velocity minimally at the cold wall and increasing in the heated wall slip parameter causes a flow reversal towards the heated wall. The cold wall slip parameter did not cause any appreciable effect on the heated wall.

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