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A Study on the Second Order Partial Differential Equation

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Abstract – The purpose of this paper is to discuss numerical solutions of differential equations including the evolution, progress and types of differential equations. Special attention is given to the solution of differential equations by application of spline functions. Here we are interested in differential equation based problems and their solutions using polynomial and no polynomial splines of different orders. It contains crux of various recent research papers based on application of splines of different orders.

Keywords: Differential Equations, Boundary value problems, Spline functions, Polynomial & No polynomial Splines.

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INTRODUCTION

Differential equations are mathematically studied from several different perspectives, mostly concerned with their solutions—the set of functions that satisfy the equation. Time-dependent problems that are modeled by initial-boundary value problems for parabolic or hyperbolic partial differential equations can be treated with the boundary integral equation method. The ideal situation is when the right-hand side in the partial differential equation and the initial conditions vanish, the data are given only on the boundary of the domain, the equation has constant coefficients, and the domain does not depend on time. In this situation, the transformation of the problem to a boundary integral equation follows the same well-known lines as for the case of stationary or time-harmonic problems modeled by elliptic boundary value problems. The same main advantages of the reduction to the boundary prevail: Reduction of the dimension by one, and reduction of an unbounded exterior domain to a bounded boundary.

There are, however, specific difficulties due to the additional time dimension: Apart from the practical problems of increased complexity related to the higher dimension, there can appear new stability problems. In the stationary case, one often has unconditional stability for reasonable approximation methods, and this stability is closely related to variational formulations based on the ellipticity of the underlying boundary value problem. For the description of the general principles, we consider only the simplest model problem of each type. We also assume that the right hand sides have the right structure for the application of a “pure” boundary integral method: The volume sources and the initial conditions vanish, so that the whole system is driven by boundary sources.

In the time dependent case, instabilities have been observed in practice, but due to the absence of ellipticity, the stability analysis is more difficult and fewer theoretical results are available. Like stationary or time-harmonic problems, transient problems can be solved by the boundary integral equation method. When the material coefficients are constant, a fundamental solution is known and the data are given on the boundary, the reduction to the boundary provides efficient numerical methods in particular for problems posed on unbounded domains.

Such methods are widely and successfully being used for numerically modeling problems in heat conduction and diffusion, in the propagation and scattering of acoustic, electromagnetic and elastic waves, and in fluid dynamics. One can distinguish three approaches to the application of boundary integral methods on parabolic and hyperbolic initial-boundary value problems: Space-time integral equations, Laplace-transform methods, and time-stepping methods. Causality implies that the integral equations are of Volterra type in the time variable, and time invariance implies that they are of convolution type in time. Numerical methods constructed from these space-time boundary integral equations are global in time, i. e. they compute the solution in one step for the entire time interval. The boundary is the lateral boundary of the space-time cylinder and therefore has one dimension more than the boundary of the spatial domain. This increase in dimension at first means a substantial increase in complexity:

- To compute the solution for a certain time, one needs the solution for all the preceding times since the initial time.

- The system matrix is much larger. - The integrals are higher-dimensional. For a problem with 3 space dimensions, the matrix elements in a Galerkin method can require 6-dimensional integrals. While the increase in memory requirements for the storage of the solution for preceding times cannot completely be avoided, there are situations where the other two reasons for increased complexity are in part neutralized by special features of the problem:

- The system matrix has a special structure related to the Volterra structure (finite convolution in time) of the integral equations. When low order basis functions in time are used, the matrix is of block triangular Toeplitz form, and for its inversion only one block - which has the size of the system matrix for a corresponding time independent problem - needs to be inverted.

- When a strong Huyghens principle is valid for the partial differential equation, the integration in the integral representation is not extended over the whole lateral boundary of the space-time cylinder, but only over its intersection with the surface of the backward light cone.

This means firstly that the integrals are of the same dimensionality as for time-independent problems, and secondly that the dependence is not extended arbitrarily far into the past, but only up to a time corresponding to the time of traversal of the boundary with the fixed finite propagation speed. These “retarded potential integral equations” are of importance for the scalar wave equation in three space dimensions and to a certain extent for equations derived from them, in electromagnetics and electrodynamics.

LITERATURE REVIEW:-

Relativity handbooks and papers based on them, as well as science fiction productions exploiting their misleading physical predictions, have flourished ever since. Neither Einstein nor his followers worried about the disastrous impact the chosen dogmatic formulation of SRT would have upon human knowledge and progress in physics and technology: His a fortiori formulation of the light-speed principle and the concepts of space time and time dilation, have broken the logical relation between the typical concepts of space and time, motion and rest, and absolute and relative.

English physicist Isaac Newton (1665) and German mathematician Gottfried Leibnitz (1674). The term differential equation was coined by Leibnitz in 1676 for a relationship between the two

differentials dx and dy for the two variables x and y. Newton solved his first differential equation in 1676 by the use of infinite series, eleven years after his discovery of calculus in 1665. Leibnitz solved his first differential equation in 1693, the year in which Newton first published his results. Hence, 1693 marks the inception for the differential equations as a distinct field in mathematics (Bardo et. al., 2002).

The different phases of 17th, 18th and 19th Centuries played some crucial role in the history of differential equations. In the year 1695 the problem of finding the general solution of what is now called Bernoulli’s equation was proposed by Bernoulli and it was solved by Leibnitz and Johann Bernoulli by different methods. In further development 1724 was important to the early history of ordinary differential equations. Ordinary differential equation acquired it significance when it was introduced in 1724 by Jacopo Francesco, Count Ricatti of Venice in his work in acoustics. Further in the year of 1739 Leonhard Euler solves the general homogeneous linear ordinary differential equation with constant coefficients. L’Hospital came up with separation of variables in 1750, and it is now the physicist’s handiest tool for solving partial differential equations. Since its introduction in 1828, Green’s functions have become a fundamental mathematical technique for solving boundary-value problems.

In 1890 Poincare (Catto et. al., 2002). gave the first complete proof of the existence and uniqueness of a solution of the Laplace Equation for any continuous Dirichlet boundary condition. In 20th Century a lot of quality work has been done in the field of differential equations, but the major concern was the analytic and computational solution of differential equations. In last few decades numerical analysis of differential equations has become a major topic of study. In view of this, this thesis gives a small step towards the development of computational analysis of ordinary differential equations, which have lot of utilities in the field of science and engineering. Relativity handbooks and papers based on them, as well as science fiction productions exploiting their misleading physical predictions, have flourished ever since. Neither Einstein nor his followers worried about the disastrous impact the chosen dogmatic formulation of SRT would have upon human knowledge and progress in physics and technology: His a fortiori formulation of the light-speed principle and the concepts of space time and time dilation, have broken the logical relation between the typical concepts of space and time, motion and rest, and absolute and relative.

The need to abolish the original significance of those concepts could never be proved. Nevertheless, the breaking has violently penetrated the human consciousness for a century now.

The absence of the principle of physical determination of equations in SRT has led to relativistic quantum theories deprived of a large amount of information on the sub-quantum structure of matter. For more than 65 years nobody became aware of this fact. So much more that testifying and exploiting this information needed to develop new techniques.

Instead of developing such techniques, physicists have asked for more and more powerful accelerators of quantum particles. Due to the absent information, they have not succeeded in understanding and systematizing the current data obtained by colliding ultra relativistic particles.

Groups of experimenters who obtained brand new results have associated each of them to separate mathematical explanatory models that have hidden their common nature. Consequently, they could not refine their techniques in order to achieve real advanced technologies. Other radically new technologies remained beyond imagination.

The relativism of the last century (unfairly claiming support from Einstein's SRT) continues successfully the dissolution of both scientific and common knowledge, with major consequences upon economy, society, etc. At least strangely, leaders in science and technology policy have opted to ignore this dramatic state of the affairs for at least the next fifty years. The trend to describe the whole physical universe, including the microcosm, in terms of geometry of a claimed physical space time and its quantum nature dominates , against its striking failure. For the description of the general principles, we consider only the simplest model problem of each type. We also assume that the right hand sides have the right structure for the application of a "pure" boundary integral method: The volume sources and the initial conditions vanish, so that the whole system is driven by boundary sources. It has been found that, unlike the parabolic partial differential operator with its time-independent energy and no regularizing property in time direction, the first kind boundary integral operators have a kind of anisotropic space-time ellipticity (Costabel, 1990; Arnold and Noon, 1989; Brown, 1989; Brown and Shen, 1993). This ellipticity leads to unconditionally stable and convergent Galerkin methods (Costabel, 1990; Arnold and Noon, 1989; Hsiao and Saranen, 1993; Hebeker and Hsiao, 1993).

Because of their simplicity, collocation methods are frequently used in practice for the discretization of space-time boundary integral equations. An analysis of collocation methods for second-kind boundary integral equations for the heat equation was given by Costabel et al., 1987. Fourier analysis techniques for the analysis of stability and convergence of collocation methods for parabolic boundary integral equations, including first kind integral equations, have been studied more recently by Hamina and Saranen (1994) and by Costabel and Saranen (2000; 2001; 2003).

The operational quadrature method for parabolic problems was introduced and analyzed by Lubich and Schneider (1992). For hyperbolic problems, the mathematical analysis is mainly based on variational methods as well (Bamberger and Ha Duong, 1986; Ha-Duong, 1990; Ha-Duong, 1996).

There is now a lack of ellipticity which on one hand leads to a loss of an order of regularity in the error estimates. On the other hand, most coercivity estimates are based on a passage to complex frequencies, which may lead to stability constants that grow exponentially in time. Instabilities (that are probably unrelated to this exponential growth) have been observed, but their analysis does not seem to be complete (Becache, 1991; Peirce and Siebrits, 1996; Peirce and Siebrits, 1997; Birgisson et al., 1999).

Analysis of variational methods exists for the main domains of application of space-time boundary integral equations: First of all for the scalar wave equation, where the boundary integrals are given by retarded potentials, but also for elastodynamics (Becache, 1993; Becache and Ha-Duong, 1994; Chudinovich, 1993c; Chudinovich, 1993b; Chudinovich, 1993a), piezoelectricity (Khutoryansky and Sosa, 1995), and for electrodynamics (Bachelot and Lange, 1995; Bachelot et al., 2001; Rynne, 1999; Chudinovich, 1997).

RESEARCH METHODOLOGY

Later workers such as Morgan (1952), Hansen (1964), Krzywoblocki (1963) and Wecker and Hayes (1960) investigated similarity methods by considering the governing equations first and only examining the boundary and initial conditions as a later step, if at cell. Another group of workers developed similarity methods by starting with a complete mathematical formulation and thus motivated to examine less complete (and more general) problems, see for example Coles (1962), Abbott and Kline (1960) and Gukhman (1965).

An examination of these earlier works show that the initial problem statement as far as assumed completeness determined to a large extent the kind of mathematical approach employed. The more information that was known, the more direct was the method developed for finding a similarity solution and at the same time, the less general were both the methods and the conclusions (as regards "general solutions").

A partial differential equation (PDE) is called dispersive if, when no boundary conditions are imposed, its wave solutions spread out in space as they evolve in time. As an example consider $iu_t + u_{xx} = 0$. If we try a simple wave of the form $u(x,t) = Ae^{i(kx-\omega t)}$, we see that it satisfies the equation if and only if $\omega = k^2$. This is called the dispersive relation and shows that the frequency is a real valued function of the wave number.

If we denote the phase velocity by $v = \frac{\omega}{k}$ we can write the solution as $u(x,t) = Ae^{ik(x-v(k)t)}$ and notice that the wave travels with velocity k. Thus the wave propagates in such a way that large wave numbers travel faster than smaller ones. (Trying a wave solution of the same form to the heat equation $u_t - u_{xx} = 0$, we obtain that the lj is complexed valued and the wave solution decays exponential in time.

On the other hand the transport equation $u_t - u_x = 0$ and the one dimensional wave equation $u_{tt} = u_{xx}$ are traveling waves with constant velocity.)

If we add nonlinear effects and study $iu_t + u_{xx} = f(u)$, we will see that even the existence of solutions over small times requires delicate techniques.

Going back to the linear equation, consider $u_0(x) = \int_R \hat{u}_0(k)e^{ikx}dk$. For each fixed k the wave solution becomes $u(x,t) = \hat{u}_0(k)e^{ik(x-kt)} = \hat{u}_0(k)e^{ikx}e^{-ik^2}$. Summing over k (integrating) we obtain the solution to our problem $u(x,t) = \int_R \hat{u}_0(k)e^{ikx-ik^2t}dk$

Since $|\hat{u}(k,t)| = |\hat{u}_0(k)|$ we have that $\|u(t)\|_{L^2} = \|u_0\|_{L^2}$.

Thus the conservation of the L^2 norm (mass conservation or total probability) and the fact that high frequencies travel faster, leads to the conclusion that not only the solution will disperse into separate waves but that its amplitude will decay over time.

Furthermore, they provide an excellent substitute in estimates that are known to fail on Lebesgue spaces. This is not entirely surprising, if we consider their analogy with Besov spaces, since modulation spaces arise essentially replacing dilation by modulation.

The equations that we will investigate are:

$$(NLS)\, i\frac{\partial u}{\partial t} + \Delta_x u + f(u) = 0, u(x,0) = u_0(x)$$

$$(NLW)\frac{\partial^2 u}{\partial t^2} - \Delta_x u + f(u) = 0, u(x,0) = u_0(x),$$

$$\frac{\partial u}{\partial t} u(x,0) = u_1(x),$$

$$(NLKG)\frac{\partial^2 u}{\partial t^2} + (I - \Delta_x)u + f(u) = 0,$$

$$u(x,0) = u_0(x),\ \frac{\partial u}{\partial t} u(x,0) = u_1(x),$$

where $u(x,t)$ is a complex valued function on $\mathbb{R}^d \times \mathbb{R}$, $f(u)$ (the nonlinearity) is some scalar function of u, and u_0, u_1 are complex valued functions on $\mathbb{R}^d$ The nonlinearities considered in this study have the generic form

$$f(u) = g(|u|^2)u,$$

where $g \in A_+(\mathbb{C})$; here, we denoted by $A_+(\mathbb{C})$ the set of entire functions $g(z)$ with expansions of the form

$$g(z) = \sum_{k=1}^{\infty} c_k z^k, c_k \geq 0$$

As important special cases, we highlight nonlinear it lies that are either power-like

$$p_k = \lambda |u|^{2k}\, u, k \in \mathbb{N}, \lambda \in \mathbb{R}$$

or exponential-like

$$e_\rho(u) = \lambda\left(e^{\rho|u|^2} - 1\right)u, \lambda, \rho \in \mathbb{R}$$

The nonlinearities considered have the advantage of being smooth. The corresponding equations having power-like nonlinearities pk are sometimes referred to as algebraic nonlinear (Schrodinger, wave, Klein-Gordon) equations.

ANALYSIS OF THE DATA:-

A different approach in which particle methods were used for approximating solutions of the heat equation and related models (such as the Fokker-Planck equation and a Boltzmann-like equation :the Kac equation), was introduced by Russo (2003).

In these works, the diffusion of the particles was described as a deterministic process in terms of a mean motion with a speed equal to the osmotic velocity associated with the diffusion process.

In a following work, the method was shown to be successful for approximating solutions to the two-dimensional Navier-Stokes (NS) equation in an unbounded domain. In this setup, the particles were convected according to the velocity field while their weights evolved according to. the diffusion term in the vorticity formulation of the NS equations.

Another deterministic approach for approximating solutions of the parabolic equations with particle methods was introduced by Degond and Mustieles (2000). Their so-called diffusion-velocity method was based on defining the convective field associated with the heat operator which then allowed the particles to convect in a standard way.

For example, the one-dimensional heat equation $u_t = u_{xx}$ is rewritten as $u_t + (a(u)u)_x = 0,$ where the velocity a(u) is taken as –ux/u. Particles carrying fixed masses will be then convicted with speed a(u). The convergence properties of the diffusion-velocity method were investigated, where short time existence and uniqueness of solutions for the resulting diffusion-velocity transport equation were proved.

The diffusion velocity method serves as the basic tool for the derivation of our particle methods in the dispersive world.

We focus our attention on linear and nonlinear dispersive partial differential equations. Our model problem in the linear setup is the linear Airy equation,

$$u_t = u_{xxx}$$

The success of particle methods in approximating the oscillatory solutions that develop in this dispersive equation, provide us with valuable insight regarding the potential embedded in our approach.

CONCLUSION

The existence theory in ID was given in and the analysis in 2D was recently announced in. Another interesting problem is the existence and uniqueness of the ground states, i.e. the solutions which minimize the total energy functional under the normalization constraint.

For the most simple-looking equation, i.e. the SN equation without external potential, the existence of a unique spherically symmetric ground state in 3D was proven by Lieb and in any dimension $d \leq 6$ was given.

There is no global minimum of the energy functional for the repulsive SP equation without external potential since the infimum of its energy is always zero. When the Slater term is considered and in the absence of any external potential, the existence analysis of ground states in 3D was given in, and in particular the existence of a unique spherically symmetric ground state is proven in for the attractive case. To our knowledge, so far the existence analysis of higher bound states remains open.

Along the numerical front, self-consistent solutions of the SPS equation are important in the simulations of a quantum system. For example, time-independent SP equation was solved in for the Eigen states of the quantum system, and time-dependent spherically symmetric SP equation was considered in and time-dependent SN equation was treated in with three kinds of symmetry: spherical, axial and translational symmetry.

Most of the previous work apply Crank Nicholson time integration and finite difference for space discretization. Also, note that in general the ground states of the SPS equation will lose the symmetric profile due to the external potential and therefore one cannot obtain a reduced quasi-ID model as for the SN system, by studying which the SN equation was extensively investigated in . On the other hand, the computation of stationary states and dynamics of the NLS equation without Hartree potential, has been extensively studied. Among the numerical methods proposed in the literature, discretizations based on a gradient flow with discrete normalization (GFDN) show more efficient in finding the ground and excited states of NLS modeling the Bose-Einstein condensates (BEC).

RESULTS & DISCUSSION

Differential equations, such as whether or not solutions exist, and should they exist, whether the are unique. Applied mathematicians emphasize differential quations from applications, and in addition to existence/uniqueness questions, are also concerned with rigorously justifying methods for approximating solutions. Physicists and engineers are usually more interested in computing approximate solutions to differential equations. These solutions are then used to simulate celestial motions, simulate neurons, design bridges, automobiles, aircraft, sewers, etc. Often, these equations do not have closed form solutions and are solved using numerical methods. Mathematicians also study weak solutions (relying on weak derivatives), which are types of solutions that do not have to be differentiable everywhere. This extension is often necessary for solutions to exist, and it also results in more physically reasonable properties of solutions, such as shocks I hyperbolic (or wave) equations. Numerical techniques to solve the boundary value problems include some of the following methods:

1) These are initial value problem methods. In this method, we convert the given boundary value problem to an initial value problem by adding sufficient number of conditions at one end and adjust these conditions until the given conditions are satisfied at the other end.

2) In finite difference method (FDM), functions are represented by their values at certain grid points and derivatives are approximated through differences in these values. For the finite difference method, the domain under consideration is represented by a finite subset of points. These points are called “nodal points” of the grid. This grid is almost always arranged in (uniform or non-uniform) rectangular manner. The differential equation is replaced by a set of difference equations which are solved by direct or iterative methods.

3) In finite element method (FEM), functions are represented in terms of basic functions and the ODE is solved in its integral (weak) form. In this method the domain under consideration is partitioned in a finite set of elements. In this the differential equation is discretized by using approximate methods with the piecewise polynomial solution (Ch. Lubich, 2008)..

4) In spline based methods, the differential equation is discretized by using approximate methods based on spline. The end conditions are derived for the definition of spline. The algorithm developed not only approximates the solutions, but their higher order derivatives as well.

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