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MHD Effect on Viscoelastic Fluid through a Long Vertical Tube

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Abstract – Unsteady flow of dusty viscoelastic fluid through a long uniform tube, whose cross section curvilinear quadrilateral bounded by the areas and radii of two concentric circles under the influence of time varying pressure gradient has been considered expressions for the velocities of liquid and dust particles cases for different pressure gradient have also discussed.

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INTRODUCTION

Soffman (1962) has proposed simple analytical model for the motion of a dusty fluid in term of a large density number of very small particles (uniform in size and shape) distributed in a fluid assuming that the bulk concentration and also the Sedimentation are negligible. Later a large number of dusty flow problems have investigated in the literature and are well documented in a review by Marble (1963), Michael and Miller (1966), Michael and Norey (1968) have considered the unsteady flow problem of a dusty gas in different channels. Dutta (1985) discussed the Temperature field in the flow over stretching surface with uniform heat flux. Khani et al. (2009) gave the Analytic solution for heat transfer of a third grade viscoelastic fluid in non-Darcy porous media with thermo physical effects. In the present paper we consider the unsteady MHD flow of dusty elastic-viscous liquid through a long uniform tube whose cross section is curvilinear quadrilateral bounded by the arcs and radii of two concentric circles *r=l, r=b* and *θ=0, θ=α* under the influence of time varying pressure gradient. Initially the liquid particles are at rest. Some particular cases for different pressure gradient have also been discussed in detail.

NOMENCLATURE

P_{ik}^{*} - stress tensor

u - Velocity of liquid,

λ_0 - Elastic coefficient,

μ - Viscosity of liquid,

P - Pressure

k - The stokes resistance coefficient,

N_0 – number of density of the particles,

v - Kinematic coefficient of viscosity,

ρ - The density of the fluid,

B_0 – Magnetic induction,

r, θ, z – cylindrical coordinates z-axis,

MATHEMATICAL FORMULATION

According to Kuvshiniski (1951), rheological equations satisfied by viscoelastic liquid are

$$P_{ik} = P\delta_{ik} + P_{ik}^{*} \quad \ldots.(1)$$

$$\left(1+\lambda_0 \frac{D}{Dt}\right) P_{ik}^{*} = 2\mu e_{ik} \quad \ldots.(2)$$

$$\frac{D}{Dt} P_{ik}^{*} = \frac{\partial}{\partial t} P_{ik}^{*} + u_m \frac{\partial}{\partial t} P_{ik}^{*} \quad \ldots.(3)$$

$$e_{ik} = \frac{1}{2}\left\{\frac{\partial u_i}{\partial u_k} + \frac{\partial u_k}{\partial u_i}\right\} \quad \ldots.(4)$$

We consider cylindrical polar coordinates (r, θ, z) with the z-axis along the axis of tube. Let u_r, u_θ, u_z, and v_r, v_θ, v_z, of the components of liquid velocity and dust velocity in radial, tangential and axial direction respectively and the boundary are

$$u_r = 0, u_\theta = 0, \quad u_z = u_z(r,t)$$

$$v_r = 0, \ v_\theta = 0, \quad v_z = v_z(r,t)$$

The equations (1) to (4) combined with boundary condition we get the following equation of motion of dusty viscoelastic liquid $\rho\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\partial u_z}{\partial t} =$

$$-\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}\right) +$$

$$KN_0\left(1+\lambda_0\frac{\partial}{\partial t}\right)(v_z-u_z) - \frac{B_0\mu u_z}{r^2}\left(1+\lambda_0\frac{\partial}{\partial t}\right) \quad \ldots\ldots (5)$$

$$m\frac{\partial v_z}{\partial t} = k(u_z - v_z) \quad \ldots. (6)$$

Introducing the following non dimensional quantities are

$$u^* = \frac{u_z l}{\vartheta}, \quad v^* = \frac{v_z l}{\vartheta}, \quad z^* = \frac{z}{l},$$

$$t^* = \frac{t\vartheta}{l^2}, \quad r^* = \frac{r}{l}, \quad p^* = \frac{pl^2}{\vartheta^2},$$

Using the boundary the equation (5) and (6) becomes

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial P}{\partial z} + \left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}\right)$$

$$+\beta(1+\alpha)(v-u) - \frac{B_0 u}{r^2}\left(1+\alpha\frac{\partial}{\partial t}\right) \quad \ldots. \quad (7)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u-v) \quad \ldots. (8)$$

The initial boundary conditions are

$$t = 0, u(r,\theta,t) = v(r,\theta,t) = 0$$

$$t > 0, u(r,\theta,t) = v(r,\theta,t) = 0$$

For r=1, r=s, $0 \le \theta \le \alpha$

$$u(r,\theta,t) = v(r,\theta,t) = 0 \ for \ \theta = 0, \theta = \alpha,$$

$$s \le r \le 1, where \ s = \frac{b}{l},$$

Solution of the problem

Putting $\theta = \frac{\alpha}{\pi}\varphi$ and $-\frac{\partial p}{\partial z} = f(t)$ in equation (7) and (8) we get

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\left(1+\alpha\frac{\partial}{\partial t}\right)f(t) + \frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\pi^2}{\alpha^2 r^2}\frac{\partial^2}{\partial \varphi^2}$$

$$+\beta\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u) - \frac{B_0 u}{r^2}\left(1+\alpha\frac{\partial}{\partial t}\right) \quad \ldots. (9)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u-v) \quad \ldots. (10)$$

The boundary conditions are

$$t = 0, u(r,\varphi,t) = v(r,\varphi,t) = 0$$
$$t > 0, u(r,\varphi,t) = v(r,\varphi,t) = 0$$

For r=1, r=s, $0 \le \varphi \le \alpha$

$$u(r,\varphi,t) = v(r,\varphi,t) = 0 \ for \ \varphi = 0, \varphi = \alpha,$$

$$s \le r \le 1, where \ s = \frac{b}{l}$$

After taking finite Fourier's the transformation equations (9) and (10) reduce in the form

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial \bar{u}}{\partial t} = -\frac{2}{q_n}\left(1+\alpha\frac{\partial}{\partial t}\right)f(t) + \frac{\partial^2 \bar{u}}{\partial r^2}+\frac{1}{r}\frac{\partial \bar{u}}{\partial r} - +\beta\left(1+\alpha\frac{\partial}{\partial t}\right)(\bar{v}-\bar{u}) - \frac{B_0\bar{u}}{r^2}\left(1+\alpha\frac{\partial}{\partial t}\right) \quad \ldots. (11)$$

$$\frac{\partial \bar{v}}{\partial t} = \frac{1}{\tau}(\bar{u}-\bar{v}) \quad \ldots. (12)$$

Where

$$\bar{u} = \int_0^\pi u(r,\varphi,t)\sin(q_n\varphi)d\varphi$$

$$q_n = (2n+1), \ m^2 = \frac{\pi^2 q_n^2}{\alpha^2}$$

Taking finite Hankel transformation of (11) and (12) and applying boundary conditions

$$\bar{u}_H = 0, \ \bar{v}_H = 0, at \ t = 0,$$

We get

$$\left(1+\alpha\frac{\partial}{\partial t}\right) = \frac{2}{q_n}\left(1+\alpha\frac{\partial}{\partial t}\right)f(t)\int_0^s r\boldsymbol{B}_m(\xi_i r)\,dr - \xi_i^2\bar{u}_H$$

$$+\beta\left(1+\alpha\frac{\partial}{\partial t}\right)(\bar{v}_H-\bar{u}_H) - \left(1+\alpha\frac{\partial}{\partial t}\right)B_0^2\,\bar{u}_H \quad \ldots. (13)$$

$$\frac{\partial \bar{v}_H}{\partial t} = \frac{1}{\tau}(\bar{u}_H - \bar{v}_H) \quad \ldots. (14)$$

Where

$J_m(\xi_i r)$ *- Bessels function of first kind,*

$Y_m(\xi_i r)$ - *Bessels function of second kind,*

$$B_m(\xi_i r) \quad = J_m(\xi_i r) Y_m(\xi_i) - J_m(\xi_i) Y_m(\xi_i r)$$

$$J_m(\xi_i s) Y_m(\xi_i) - J_m(\xi_i) Y_m(\xi_i s) = 0 \qquad \ldots.(15)$$

ξ_i is the ratios of the equation

Taking Laplace transform of the equations (13) and (14) we get

$$\bar{\bar{u}}_H \quad = \frac{2\delta}{q_n}\left\{\frac{(1+\alpha p)(1+\tau p)\bar{f}(p)}{\alpha\tau p^3 + (\alpha+\tau+\alpha f+\alpha\tau B_0^2)p^2 + [1+f+\tau\xi_i^2+(\alpha+\tau)B_0^2]p + (B_0^2+\xi_i^2)}\right\} \qquad \ldots.(16)$$

And

$$\bar{\bar{v}}_H = \frac{\bar{\bar{u}}_H}{p\tau+1} \qquad \ldots.(17)$$

Where $\bar{\bar{u}}_H$, $\bar{\bar{v}}_H$ and are $\bar{f}(p)$ the Laplace transform of and respectively and

$$\delta = \int_0^s r B_m(\xi_i r) dr$$

Now we obtain u and v from the equation (16) and (17), first invert the Laplace transform by inversion theorem, then applying version formulation for Hankel and sine transformation we get

$$u = \frac{8}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\delta\xi_i^2 J_m^2(\xi_i s) B_m(\xi_i r)}{q_n[J_m^2(\xi_i) - J_m^2(\xi_i s)]}\sin(q_n\varphi) Q(p_n^j)\left[\int_0^t e^{p_n^j\lambda} f(t-\lambda) d\lambda\right] \qquad \ldots.(18)$$

$$v = \frac{8}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\delta\xi_i^2 J_m^2(\xi_i s) B_m(\xi_i r)}{q_n[J_m^2(\xi_i) - J_m^2(\xi_i s)]}\sin(q_n\varphi) R(p_n^j)\left[\int_0^t e^{p_n^j\lambda} f(t-\lambda) d\lambda\right] \qquad \ldots.(19)$$

Where p_n^j the root of the cubic education

$$\alpha\tau(p_n^j)^3 + (\alpha+\tau+\alpha f+\alpha\tau B_0^2)(p_n^j)^2 + [1+f+\tau\xi_i^2+(\alpha+\tau)B_0^2]p_n^j + (B_0^2+\xi_i^2) = 0$$

$$Q(p_n^j) \; = \frac{(1+\alpha p_n^j)(1+\tau p_n^j)}{3\alpha\tau(p_n^j)^2 + 2(\alpha+\tau+\alpha f+\alpha\tau B_0^2)p_n^j + [1+f+\tau\xi_i^2+(\alpha+\tau)B_0^2]}$$

$$R(p_n^j) \; = \frac{(1+\alpha p_n^j)}{3\alpha\tau(p_n^j)^2 + 2(\alpha+\tau+\alpha f+\alpha\tau B_0^2)p_n^j + [1+f+\tau\xi_i^2+(\alpha+\tau)B_0^2]}$$

Some special cases:

Case I: flow under constant pressure gradient, let us consider *f (t) =c*

Where c is the positive constant now substituting *f (t) =c* in equation (18) and (19) get we

$$u = \frac{8c}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\delta\xi_i^2 J_m^2(\xi_i s) B_m(\xi_i r)}{q_n[J_m^2(\xi_i) - J_m^2(\xi_i s)]}\sin(q_n\varphi) Q(p_n^j)\left[\frac{e^{p_n^j t} - 1}{p_n^j}\right] \qquad \ldots.(20)$$

$$v = \frac{8c}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\delta\xi_i^2 J_m^2(\xi_i s) B_m(\xi_i r)}{q_n[J_m^2(\xi_i) - J_m^2(\xi_i s)]}\sin(q_n\varphi) R(p_n^j)\left[\frac{e^{p_n^j t} - 1}{p_n^j}\right] \qquad \ldots.(21)$$

Case II: flow under exponentially pressure gradient, let consider $f(t) = \; ce^{-wt}$

$$u = \frac{8c}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\delta\xi_i^2 J_m^2(\xi_i s) B_m(\xi_i r)}{q_n[J_m^2(\xi_i) - J_m^2(\xi_i s)]}\sin(q_n\varphi) Q(p_n^j)\left[\frac{e^{p_n^j t} - e^{-wt}}{p_n^j + w}\right] \qquad \ldots.(22)$$

$$v = \frac{8c}{\pi}\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=1}^{3}\frac{\delta\xi_i^2 J_m^2(\xi_i s) B_m(\xi_i r)}{q_n[J_m^2(\xi_i) - J_m^2(\xi_i s)]}\sin(q_n\varphi) R(p_n^j)\left[\frac{e^{p_n^j t} - e^{-wt}}{p_n^j + w}\right] \qquad \ldots.(23)$$

Substitute in (18) and (19), we get

CONCLUSION:

If the mass of the dust particles are small then there influence and the fluid flow is reduced as *m → 0* then fluid becomes ordinary viscous. If we put α=0, B_0=0 then all the results are in agreement with these of Gupta (1979).

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