Journal of Advances in Science and Technology
Vol. 14, Issue No. 1, June-2017, ISSN 2230-9659

Flow of a Dusty Revlin – Ericksen Fluid Due To an Impulsively Started Flate Plate

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Abstract – In the present problem a theoretical analysis of two dimensional flow of Revlin – Ericksen fluid through a porous medium. The unsteady laminar flow of an elastico-viscous fluid suspended with a uniform distribution of non-conducting dust particles in presence of uniform transverse magnetic field fixed relative to the plate which is applied perpendicular to the flow of fluid is considered. The velocity field for the conducting dusty fluid, non-conducting dust particles and the skin friction at the plate are obtained for large value of time. The effect of magnetic field and the mass concentration of the dust particle an velocity and skin friction are discussed numerically.

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INTRODUCTION

Many researchers worked on flow of a viscous incompressible and electrically conducting fluid over a flat plate in the presence of uniform transverse magnetic fluid which is fixed relative to the plate or to the fluid. Soundalagakas (1965), Pop (1968), Pande (1970), Ahmadi and Manvi (1971), Happel (1959) have solved the hydromagnetic flow due to accelerated motion of an infinite flat plate in the presence of magnetic field fixed relative to the plate.

Later a large No. of dusty flow problems have been investigated in the literature and are well documented in a review by **Marble (1963), Michecel and Micus (1966), Michael and Norey (1968)** considered the unsteady flow problem of dusty fluid in different ways.

In the present chapter we consider the MHD flow of dusty **Revlin- Ericksen** fluid due to an impulsively started flat plate.

NOMENCLATURE

m : Mass of the dust particle

N_0 : The number density of dust particle

t : Time

U : Constant velocity of the plate

u,v : Velocity component of the conducting dusty elastic viscous liquid and dust particle parallel to the plate respectively.

y : Rectangular coordinate normed to the plate.

ν : Kinematic coefficient of viscosity of the conducting dusty fluid.

μ : Viscosity of the conducting dusty fluid.

σ : Electrical conductivity of the conducting dusty fluid.

τ : The relaxation time of the dust particle.

ρ : The density of the conducting dusty fluid.

α^1 : Kinematic viscosity.

β^1 : The kinematic viscoelasticity

l, M : Non-dimensional magnetic field parameters. **(Hartman No.)**

k : Stokis resistance coefficient

B_0 : Uniform transverse magnetic field

a : Radius of spherical dust particle

λ_0 : The elastic coefficient

MATHEMATICAL ANALYSIS

Consider the x-axis along the plate and y-axis normal to it then eqn. of motion are :

$$\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\upsilon\frac{\partial^2 u}{\partial y^2}+\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{kN_0}{\rho}(v-u)-\frac{\sigma B_0^2}{\rho}\left(1+\lambda_0\frac{\partial}{\partial t}\right)u \quad .....(A)$$

and $$\tau\frac{\partial v}{\partial t}=(u-v) \quad .....(B)$$

where $$\tau=\frac{m}{K}$$

eqn A & B can be put in non-dimensional forms with the help of following substitutions

$$t^*=\frac{t}{\tau},\quad y^*=\frac{y}{\sqrt{\upsilon\tau}},\quad u^*=\frac{u}{U},\quad v^*=\frac{v}{U}$$

then eqn. A & B respectively reduced to

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+l\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u)-M^2\left(1+\alpha\frac{\partial}{\partial t}\right)u \quad .....(1)$$

and $$\frac{\partial v}{\partial t}=(u-v) \quad .....(2)$$

where l & M are non-dimensional parameters given as

$$l=\frac{MN_0}{\rho},\qquad M=\sqrt{\frac{\sigma\tau}{\rho}}B_0,\qquad \alpha=\frac{\lambda_0}{\tau}$$

Now when the magnetic field is fixed relative to the plate then eqn. (1) will be modified at time t = 0, the conducting dusty fluid, the plate and the magnetic field are assumed to be stationary everywhere for all later time the plate and the magnetic field are moving with velocity u = 1. Because the magnetic field is moving and the conducting dusty fluid is initially at rest. The relative motion must be accounted for the origin of coordinate system is fixed in space. Hence by transformation of coordinates the eqn (1) becomes

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+l\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u)-M^2\left(1+\alpha\frac{\partial}{\partial t}\right)(u-1) \quad .....(3)$$

eqn. (3) and (2) are solved under the initial and the boundary conditions

$$t=0,\ u=0,\ v=0\ \ for\ y>0 \quad .....(4)$$

$$t>0,\ u=l\ at\ y=0,\ u\to\ finiteas\ y\to\infty \quad .....(5)$$

$$Let\ \bar{u}=\int_0^\infty ue^{-st}dt \qquad (S>0)$$

$$\bar{v}=\int_0^\infty ue^{-st}dt \qquad (S>0)$$

be respectively the laplace transforms of u & v on taking laplace transforms of (3) & (4) and using (4) we get

$$\frac{\partial^2\bar{u}}{\partial^2 y^2}-p^2\bar{u}=-\frac{M^2}{S} \quad .....(6)$$

and $$\bar{v}=\frac{\bar{u}}{1+S}\simeq\bar{u} \qquad (S<<1) \quad .....(7)$$

where

$$p^2=(1+\alpha s)\left[s+\frac{ls}{1+s}+M^2\right] \quad .....(8)$$

The boundary condition (5) transform to

$$\bar{u}=\frac{1}{s}\ \ at\ \ y=0,\ \ \bar{u}\to\ finite\ as\ \ y\to\infty \quad .....(9)$$

the solution of (6) subject to the condition (9) is

$$\bar{u}=\left(\frac{1}{s}-\frac{M^2}{sp^2}\right)e^{-py}+\frac{M^2}{sp^2} \quad .....(10)$$

Since the inversion of (10) presents some difficulty so we restrict over selves to large value of t. Now when t is large then s is very small in this case.

$$p=\left[(1+l+\alpha M^2)S+M^2\right]^{1/2} \quad .....(11)$$

them from (10) we get (for large t)

$$\bar{u}=\frac{1}{S}-\frac{1}{\left[S+\frac{M^2}{(1+l+\alpha M^2)}\right]}+\frac{e^{-\left[(1+l+\alpha M^2)S+M^2\right]^{1/2}y}}{\left[S+\frac{M^2}{(1+l+\alpha M^2)}\right]} \quad .....(12)$$

By inversion theorem we get

$$u = 1 - e^{-\left[\left(\frac{M^2}{1+l+\alpha M^2}\right)\right]t} + e^{-\left[\left(\frac{M^2}{1+l+\alpha M^2}\right)\right]t} erfc\left[\frac{y\sqrt{1+l+M^2\alpha}}{2\sqrt{t}}\right] \quad .....(13)$$

similarly taking inverse transform for large value of t, we get from (7)

$$v = 1 - \left(\frac{n_1}{n_1-1}\right)e^{-t} + \frac{e^{-t}}{(n_1-1)} erfc\left[\frac{y\sqrt{1+l+M^2\alpha}}{2\sqrt{t}}\right] + \frac{e^{-n_1 t}}{(n_1-1)}\left[1 - erfc\left\{\frac{y\sqrt{1+l+M^2\alpha}}{2\sqrt{t}}\right\}\right] \quad .....(14)$$

where $n_1 = \frac{M^2}{1+l+\alpha M^2}$

eqn. (13) and (14) represent the velocities of the conducting dusty elastic-viscous liquid and dust particles for large t in the case when the magnetic field is fixed relative to the plate.

If we put $\lambda_0 = 0$ then all the results agree with the results obtained by **Mitra (1985)**.

CALCULATION OF SKIN FRICTION

Shearing stresses at the plate where t is large is given by

$$\left(-\frac{\partial u}{\partial y}\right)_{y=0} = \sqrt{\frac{1+l+\alpha M^2}{\pi t}}\, e^{-\left(\frac{M^2}{1+l+\alpha M^2}\right)t} \quad .....(15)$$

RESULT AND DISCUSSION

In order to get a physical understanding of the problem which are generally valid for large value of t numerical calculations are carried out for the velocities of the conducting dusty liquid non conducting dust particles and the skin friction at the plate from eqn. (13), (14) and (15) respectively.

In **table I and II** one finds the velocity distribution of the conducting dusty fluid and non-conducting dust particles respectively for different value of land M and at different points of the flow field. It is noted that for fixed *l* = 0.1, t = 2 and α = 1.1 the velocity of the conducting dusty fluid and non-conducting dust particles gradually decreases with the increasing values of M. Similarly for fixed M = 0.3 and t =2, α = 1.1 the velocities decreases with the increase of *1*.

TABLE – I

Velocity of the conducting dusty fluid for t = 2, α = 1.1.

S. No.		*l* = 0.1		M = 0.3	
	y	M = 0.1	M = 0.5	*l* = 0.1	*l* = 0.5
1	0.0	1.0000	1.0000	1.0000	1.0000
2	0.1	0.9839	0.9747	0.9773	0.9675
3	0.2	0.9288	0.9357	0.9290	0.9173
4	0.3	0.8895	0.8971	0.9032	0.8677
5	0.4	0.8546	0.8693	0.8554	0.8331
6	0.5	0.8009	0.8451	0.8169	0.7798
7	0.6	0.7891	0.80037	0.7927	0.7244

TABLE – II

Velocity of the conducting dusty fluid for t = 2, α = 1.1.

S. No.		*l* = 0.1		M = 0.3	
	y	M = 0.1	M = 0.5	*l* = 0.1	*l* = 0.5
1	0.0	1.0000	1.0000	1.0000	1.0000
2	0.1	0.8506	0.8397	0.8439	0.8354
3	0.2	0.8027	0.7919	0.7999	0.7903
4	0.3	0.7685	0.7361	0.7764	0.7457
5	0.4	0.7381	0.7357	0.7329	0.7146
6	0.5	0.6914	0.7111	0.6978	0.6666
7	0.6	0.6532	0.6782	0.6581	0.6102

Finally in **table III** the skin friction at the plate are shown for different value of M, *1* and t. It is noted that for fixed *1* = 0.1 and t =2, α = 1.1, the skin friction decreases with the increasing value of M and for fixed (t =2, M = 0.3, α = 1.1) the skin friction increasing with increasing value of *l*, also for fixed value of *1* = 0.1, M = 0.3, α = 1.1 the skin friction at the plate decreases as time t increases.

TABLE – III

Skin friction at the plate for α = 1.1.

S.No.	t = 2	*l* = 0.1	t = 2	M = 0.3	M = 0.3	*l* = 0.1
1	M = 0.1	0.4129	*l* = 0.1	0.3758	t = 2	0.3758
2	M = 0.2	0.3978	*l* = 0.2	0.3957	t = 3	0.2847
3	M = 0.3	0.3758	*l* = 0.3	0.4148	t = 4	0.2287
4	M = 0.4	0.3506	*l* = 0.4	0.4330	t = 5	0.1897
5	M = 0.5	0.3251	*l* = 0.5	0.4506	t = 6	0.1607
6	M = 0.6	0.3001	*l* = 0.6	0.4778	t = 7	0.1327
7	M = 0.7	0.2731	*l* = 0.7	0.4925	t = 8	0.1000

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