Journal of Advances in Science and Technology
Vol. 14, Issue No. 1, June-2017, ISSN 2230-9659

Unsteady MHD Flow past an Infinite Vertical Porous Plate with Heat and Mass Transfer

Bhagwat Swarup[1]* Satish Kumar[2]

[1]Department of Mathematics, Agra College, Agra

[2]Department of Mathematics, Government P.G. College, Lalitpur (UP)

Abstract – In the present paper we shall discuss unsteady flow, with heat and mass transfer, in an incompressible, electrically conducting, and viscous fluid through a time dependent porous medium past an infinite porous vertical plate with constant suction/injection in the presence of a uniform magnetic field applied perpendicular to the flow region. It is considered that the plate is subjected to a constant suction/injection velocity normal to the plate the flow is through a non-homogeneous porous medium. The effects of various parameters on primary velocity, secondary velocity, temperature field and concentration field have been discussed with the help of figures while the effects of important parameters on in skin-friction due to primary and secondary velocities, rate of heat and mass transfer have been discussed with the help of tables.

---------------------------♦-----------------------------

INTRODUCTION

Free convection problem have attracted a considerable amount of interest because of its importance in atmospheric and oceanic circulations, nuclear reactors, power transformers etc. several authors viz. **Sturat (1954), Greenspan (1969), Jana & Dutta (1977), Sinha & Gupta (1980), Gupta et al. (1983), Purohit & Sharma (1986), Palec & Daguenet (1987), Singh (1994)** have discussed rotating flows, **Seth & Banerjee (1996)** have studied combined free and forced convection flow of a viscous fluid in rotating channel in the presence of a uniform transverse magnetic field applied parallel to the axis of rotation. **Gebhart (1973), Debnath (1973), Acheson and Hide (1973), Reynolds (1975 a, 1975b), Khare (1977), Srinivasan & Kandaswami (1984), Kumar & Mala (1992) Varshney and Johri (1993), Sharma (1995), Varshney and Varshney (1996)** etc. have discussed flow in rotating system in presence magnetic field. **Singh et al. (2001)** have studied free convection in MHD flow of a rotating viscous liquid in porous medium past a vertical porous plate. **Dhiman (2000)** have studied a uniform rotation and uniform magnetic field in thermohaline convection. Recently, **Kumar et al. (2001)** have presented a study of the hydrodynamic lubrication of a micropolar fluid between two rotating rollers. More recently, **Singh et al. (2002)** have studied hydromagnetic oscillatory flow of a viscous fluid past a vertical plate in a rotating system. **Johri (2003)** was investigated approximate solution of the miscible fluid flow through porous media using collocation method.

In the present paper we shall discuss unsteady flow, with heat and mass transfer, in an incompressible, electrically conducting, viscous fluid through a time dependent porous medium past an infinite porous vertical plate with constant suction/injection in the presence of an uniform magnetic field applied perpendicular to the flow region. It is considered that the plate is subjected to a constant suction/injection velocity normal to the plate and the flow is through a non-homogeneous porous medium. The effects of various parameters on primary velocity, secondary velocity, temperature field and concentration field have been discussed with the help of figures while the effects of important parameters on in skin-friction due to primary and secondary velocity, temperature field and concentration field have been discussed with the help of figures while the effects of important parameters on in skin-friction due to primary and secondary velocities, rate of heat and mass transfer have been discussed with the help of tables. There are two figures showing effects of the important parameters on primary and secondary velocities and six tables showing the effects of various parameters on skin-friction due to primary velocity, secondary velocity, rate of heat transfer and rate of mass transfer.

NOMENCLATURE

u, V, w	–	The velocitise along x, y and z axis.
Ω	–	Uniform angular velocity

$\vec{B}_0$ – Uniform magnetic field

Q – Constant heat source

ρ – Density of a linear function

g – Accleration due to gravity

β_0 – Volumetric coefficient of thermal expansion

σ – Electric permeability

μ_e – Magnetic permeability

H_0 – Constant Magnetic Field

k_0 – Constant permeability

µ – Coefficient of viscosity

κ – Thermal conductivity

Cp – Specificant at constant pressure

T – Temperature

T_p – Plate temperature

T_∞ – Temperature far away the plate

C_w – Concentration of species at plate

C_∞ – Concentration of species far away from plate

N_u – Nusslet No.

α_0 – Heat source parameter

E – Rotation parameter

n – Frequency parameter

FORMULATION OF THE PROBLEM

We consider an unsteady heat and mass transfer flow of an incompressible, electrically conducting, viscous liquid flowing through porous medium, which depends on time such that $k(t)=k_0(1+\in e^{\text{int}})$ past an infinite, vertical, porous plate with constant heat source in the presence of transverse uniform magnetic field. Further we consider a Cartesian coordinate system choosing x-axis and y-axis in the plane of the porous plate and z-axis normal to the plate with velocity components u,v, w in these directions respectively. Both the liquid and the plate are considered in a state of rigid body rotation about z-axis with uniform angular velocity Ω. We also assume that the uniform magnetic field $\vec{B}_0=\mu_e\vec{H}$, where $\vec{H}=(0,0,H_0)$ is applied in the z-direction and the magnetic Reynolds number is small. The constant heat source Q is assumed at z = 0. We take the heat source of absorption type $Q=Q_0(T-T_\infty)$. The suction velocity at the plate is $w=-w_0$ where w_0 is a positive real number and negative sign indicates that the suction is towards the plate. In this analysis buoyancy force, hall effect, effect due to perturbation of the field, induced magnetic field and polarization effect are ignored. Initially at t ≤ 0 the plate and the fluid are at the same temperature T_∞ and species concentration is uniformly distributed in the flow region such that it is everywhere C_∞. When t > 0 the temperature of the plate is raised to $T_w(1+\in e^{\text{int}})$ and the concentration level is raised to $C_w(1+\in e^{\text{int}})$. For formulation of mathematical equations the following assumption have been made :

(i) The physical properties of the fluid are constant excluding density in the buoyancy force term in the momentum equation.

(ii) The density is a linear function of temperature and species concentration given by $\rho=\rho_0[\{1-\beta_0(T-T_\infty)\quad+\beta(T-T_\infty)\}]$ so that Boussinesq's approximation is taken into account.

Following, Gebhart & Pera (1971), the species concentration is very low so that the **Soret** and **Dofour** effects are negligible.

(i) The induced magnetic field and the heat due to viscous dissipation are negligible.

(ii) The plate is infinite in length so that the physical quantities involved in the governing equations depend on z and t only.

(iii) The magnetic field is not strong enough to cause Joule heating so that the term due to electrical dissipation is neglected in energy equation.

Under above stated restrictions the equations of motion and energy are :

$$\frac{\partial u}{\partial t}-w_0\frac{\partial u}{\partial z}-2\Omega v=\upsilon\frac{\partial^2 u}{\partial z^2}+g\beta_0(T-T_\infty)+g\beta(C-C_\infty)$$

$$-\frac{\upsilon}{k_0(1+\in e^{int})}u-\frac{\sigma}{\rho}\mu_e^2H_0^2u \quad ......(1)$$

$$\frac{\partial v}{\partial t}-w_0\frac{\partial v}{\partial z}-2\Omega v=\upsilon\frac{\partial^2 v}{\partial z^2}-\frac{\upsilon}{k_0(1+\in e^{int})}v-\frac{\sigma}{\rho}\mu_e^2H_0^2v \quad ......(2)$$

$$\frac{\partial T}{\partial t}-w_0\frac{\partial T}{\partial z}=\frac{K}{\mu C_p}\frac{\partial^2 T}{\partial z^2}-\frac{Q_0(T-T_\infty)}{\rho C_p} \quad ......(3)$$

$$\frac{\partial C}{\partial t}-w_0\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2} \quad ......(4)$$

The boundary conditions relevant to the problem are :

$$u=U_0(1+\in e^{int}), v=0, T=T_w(1+\in e^{int}),$$

$$C=C_w(1+\in e^{int}) as\ z=0$$

$$u=U(t)\to 0,\quad v\to 0,\quad T\to T_\infty,$$

$$C\to C_\infty,\quad as\quad z\to\infty \quad ......(5)$$

We introduce the following non-dimensional quantities :

$$z^*=\frac{w_0 z}{\upsilon},\quad t^*=\frac{w_0^2 t}{\upsilon},\quad u^*=\frac{u}{U_0},\quad n^*=\frac{\upsilon n}{w_0^2},\quad v^*=\frac{v}{U_0},$$

$$k_0^*=\frac{w_0^2 k_0}{\upsilon},\quad C^*=\frac{C-C_\infty}{C_w-C_\infty}\quad and\quad T^*=\frac{T-T_\infty}{T_w-T_\infty}$$

Using the above stated non-dimensional quantities, the equations (1), (2), (3) and (4) after ignoring the stars over them, reduce to :

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial z}-2Ev=\frac{\partial^2 u}{\partial z^2}+G_rT+G_mC-\left[M^2+\frac{1}{k_0(1+\in e^{int})}\right]u \quad ......(6)$$

$$\frac{\partial v}{\partial t}-\frac{\partial v}{\partial z}-2Eu=\frac{\partial^2 v}{\partial z^2}-\left[M^2+\frac{1}{k_0(1+\in e^{int})}\right]v \quad ......(7)$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial z}=\frac{1}{\rho r}\frac{\partial^2 T}{\partial z^2}-\alpha_0 T \quad ......(8)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial z}=\frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} \quad ......(9)$$

where $P_r=\frac{\mu C_p}{K}$ (Prandtl number), $S_C=\frac{\upsilon}{D}$ (Schmidt number)

$$G_r=\frac{g\beta_0\upsilon(T_w-T_\infty)}{U_0w_0^2}$$ (Grashof number)

$$G_m=\frac{g\beta\upsilon(C_w-C_\infty)}{U_0w_0^2}$$ (modified Grashof number),

$$E=\frac{\Omega\zeta}{w_0^2}$$ (Rotation parameter),

$$M^2=\frac{\sigma\mu_e^2H_0^2\upsilon}{\rho w_0^2}$$ (Magnetic parameter)

$$\alpha_0=\frac{Q_0\upsilon^2}{Kw_0^2}$$ (Heat source parameter)

Using q = u +iv in (6) and (7), we obtain

$$\frac{\partial q}{\partial t}-\frac{\partial q}{\partial z}+\left[M^2+\frac{1}{k_0(1+\in e^{int})}+2iE\right]q=\frac{\partial^2 u}{\partial z^2}+G_rT+G_mC \quad ..... (10)$$

The equation (8) can be written in the following form :

$$\frac{\partial^2 T}{\partial z^2}+P_r\frac{\partial T}{\partial z}-P_r\frac{\partial T}{\partial t}-\alpha_0 T=0 \quad ..... (11)$$

$$\frac{\partial^2 C}{\partial z^2}+S_C\frac{\partial C}{\partial z}-S_C\frac{\partial C}{\partial t}=0 \quad ..... (12)$$

The boundary conditions (5) are transformed to :

$$q=1+\in e^{int},\quad T=1+\in L_1e^{int},\quad C=1+\in L_2e^{int},\quad at\ z=0$$

$$q\to 0,\quad T\to 0,\quad C\to 0,\quad as\ z\to\infty \quad ..... (13)$$

$$where\ L_1=\frac{T_w}{T_w-T_\infty}\quad and\quad L_2=\frac{C_w}{C_w-C_\infty}$$

SOLUTION OF THE PROBLEM

In order to solve the equations (10), (11) and (12), we assume the velocity, temperature and concentration in the neighbourhood of the plate as follows:

$$q(z,t)=q_0+\in q_1(z)e^{int} \quad ..... (14)$$

$$T(z,t)=T_0+\in T_1(z)e^{int} \quad ..... (15)$$

and $$C(z,t)=C_0+\in C_1(z)e^{int} \quad ..... (16)$$

Using equation (14), (15) and (16) in equations (10), (11) and (12), we obtain following equations :

$$q_0''(z)+q_0'(z)-(M_1+2iE)q_0(z)=-G_rT_0(z)-G_mC_0(z) \quad ..... (17)$$

$$q_0''(z)+q_0'(z)-[M_1+(2E+n)]q_1(z)=-G_rT_1(z)$$

$$-G_mC_1(z)-\frac{1}{k_0}q_0(z) \quad \text{..... (18)}$$

$$T_0''(z)+P_rT_0'(z)-\alpha_0T_0(z)=0 \quad \text{..... (19)}$$

$$T_1''(z)+P_rT_1'(z)-(inP_r+\alpha_0)T_1(z)=0 \quad \text{..... (20)}$$

$$C_0''(z)+S_CC_0'(z)=0 \quad \text{..... (21)}$$

$$C_1''(z)+S_CC_1'(z)inS_CC_1(z)=0 \quad \text{..... (22)}$$

Using (14), (15) and (16) in (13) the boundary conditions are reduced to :

$$q_0=1,\ q_1=1,\ T_0=1,\ T_1=L_1,\ C_0=1,\ C_1=L_2 \quad at\ z=0$$

$$q_0=0,\ q_1=0,\ T_0=0,\ T_1=0,\ C_0=0,\ at\ z\to\infty \quad \text{..... (23)}$$

The solution of equations (17) to (22), under the boundary conditions (23) are :

$$T_0(z)=e^{-H_2z} \quad \text{..... (24)}$$

$$T_1(z)=L_1e^{-H_4z} \quad \text{..... (25)}$$

$$C_0(z)=e^{-S_Cz} \quad \text{..... (26)}$$

$$C_1(z)=L_2e^{-R_4z} \quad \text{..... (27)}$$

$$q_0(z)=e^{-H_6z}+D_1\left(e^{-H_2z}-e^{-H_6z}\right)+R_5\left(e^{-S_Cz}-e^{-H_6z}\right) \quad \text{..... (28)}$$

and $q_1(z)=D_2e^{-H_2z}+D_3e^{-H_4z}+D_4e^{-H_6z}$

$$+R_6e^{-R_4z}+R_7e^{S_Cz}+R_8e^{-H_6z}$$

$$+(1-D_2-D_3-D_4-R_6-R_7-R_8)e^{-H_8z} \quad \text{..... (29)}$$

Substituting the values of q_0 (z) and q_1 (z) in (14), the values of T_0 (z) and T_1 (z) in the equation (15) and the values of C_0 (z) and C_1 (z) in the equation (16) we obtain.

$$q(z,t)=e^{-H_6z}+D_1\left(e^{-H_2z}-e^{-H_6z}\right)+R_5\left(e^{-S_Cz}-e^{-H_6z}\right)e^{-H_6z}$$

$$+R_6e^{-R_4z}+R_7e^{S_Cz}+R_8e^{-H_6z}$$

$$+(1-D_2-D_3-D_4-R_6-R_7-R_8)e^{-H_8z} \quad \text{..... (30)}$$

$$T(z,t)=e^{-H_2z}+\in\left[L_1e^{-H_4z}\right]e^{\text{int}} \quad \text{..... (31)}$$

$$C(z,t)=e^{-S_Cz}+\in\left[L_2e^{-R_4z}\right]e^{\text{int}} \quad \text{..... (32)}$$

From (30), the steady part of the primary velocity (u_0) and the steady part of the secondary velocity (v_0) are :

$$u_0(z)=F_5e^{-H_2z}+P_2e^{-S_Cz}-e^{-A_2z}(P_2\cos B_2z+Q_2\sin B_2z)$$

$$+e^{-A_2z}(\cos B_2z-F_5\cos B_2z-F_6\sin B_2z) \quad \text{..... (33)}$$

and $v_0(z)=F_6e^{-H_2z}+Q_2e^{-S_Cz}-e^{-A_2z}(Q_2\cos B_2z+P_2\sin B_2z)$

$$-e^{-A_2z}(\sin B_2z-F_6\cos B_2z-F_5\sin B_2z) \quad \text{..... (34)}$$

From (30), the unsteady part i.e. time dependent part of the primary velocity (u_1) and time dependent part of the secondary velocity (v_1) are :

$$u_1(z)=(F_9\cos B_1z+F_{10}\sin B_1z)e^{-A_1z}(F_{10}\cos B_2z+F_{12}\sin B_2z)e^{-A_1z}$$

$$+(F_{13}\cos B_3z-F_{14}\sin B_3z)e^{-A_1z}+F_7e^{-H_2z}$$

$$+e^{-P_1z}(P_3\cos Q_1z+Q_3\sin Q_1z)+P_4e^{-S_Cz}$$

$$+(P_5\cos B_2z+Q_5\sin B_2z)e^{-A_2z} \quad \text{..... (35)}$$

$$v_1(z)=(F_{10}\cos B_1z+F_9\sin B_1z)e^{-A_1z}+(F_{12}\cos B_2z+F_{11}\sin B_2z)e^{-A_2z}$$

$$-(F_{14}\cos B_3z+F_{13}\sin B_3z)e^{-A_1z}+F_8e^{-H_2z}$$

$$+e^{-P_1z}(Q_3\cos Q_1z-P_3\sin Q_1z)+Q_4e^{-S_Cz}$$

$$+(Q_5\cos B_2z-P_5\sin B_2z)e^{-A_2z} \quad \text{..... (36)}$$

Therefore substituting these values of u_0 (z), v_0 (z), u_1 (z) and v_1 (z) the primary velocity u (z, t) and secondary velocity v (z, t) can be written as

$$u(z,t)=u_0(z)+\in(u_1\cos nt-v_1\sin nt) \quad \text{..... (37)}$$

$$v(z,t)=v_0(z)+\in(v_1\cos nt-u_1\sin nt) \quad \text{..... (38)}$$

Hence, from (31) and (32), the primary and secondary velocities at $nt=\frac{\pi}{2}$ are :

$$u\left(z, \frac{\pi}{2n}\right) = u_0(z) - \in v_1(z) \qquad \text{..... (39)}$$

$$v\left(z, \frac{\pi}{2n}\right) = v_0(z) - \in u_1(z) \qquad \text{..... (40)}$$

$$H_2 = \frac{P_r + \sqrt{P_r^2 + 4\alpha_0}}{2}, \qquad M_1 = M^2 + \frac{1}{k_0},$$

$$H_4 = A_1 + iB_1 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 + 4\alpha_0 + i4nP_r}\right],$$

$$H_6 = A_2 + iB_2 = \frac{1}{2}\left[1 + \sqrt{P_r^2 + 4M_1 + i8E}\right],$$

$$H_8 = A_3 + iB_3 = \frac{1}{2}\left[1 + \sqrt{P_r^2 + 4M_1 + i2(4E + n)}\right],$$

$$R_4 = P_1 + iQ_1 = \frac{1}{2}\left[S_C + \sqrt{S_C^2 + i4nS_C}\right],$$

$$R_5 = P_2 + iQ_2 = \frac{-G_m}{S_C^2 - S_C - M_1 - 2iE},$$

$$R_6 = P_3 + iQ_3 = \frac{-G_m L_2}{R_4^2 - R_4 - M_1 - i(2E + n)},$$

$$R_7 = P_4 + iQ_4 = \frac{-R_5}{k_0\left[S_C^2 - S_C - M_1 - (2E + n)\right]},$$

$$R_8 = P_5 + iQ_5 = \frac{-R_5}{k_0\left[F_4 - i(2E + n)\right]},$$

$$D_1 = F_5 + iF_6 = \frac{-G_r}{\left[F_1 - i2E\right]},$$

$$D_2 = F_7 + iF_8 = \frac{-D_1}{\left[F_2 - ik_0(2E + n)\right]},$$

$$D_3 = F_9 + iF_{10} = \frac{-G_r L_1}{\left[F_3 - i(2E + n)\right]},$$

$$D_4 = F_{11} + iF_{12} = \frac{-(1 - D_1)}{k_0\left[F_4 - i(2E + n)\right]},$$

$$A_1 = \frac{P_r}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\left(P_r^2 + 4\alpha_0\right)^2 + 16n^2P_r^2} + \left(P_r^2 + 4\alpha_0\right)\right]^{1/2},$$

$$B_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(P_r^2 + 4\alpha_0\right)^2 + 16n^2P_r^2} - \left(P_r^2 + 4\alpha_0\right)\right]^{1/2},$$

$$A_2 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1 + 4M_1)^2 + 64E^2} + (1 + 4M_1)\right]^{1/2},$$

$$B_2 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1 + 4M_1)^2 + 64E^2} - (1 + 4M_1)\right]^{1/2},$$

$$A_3 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1 + 4M_1)^2 + 16(2E + n)^2} + (1 + 4M_1)\right]^{1/2},,$$

$$B_3 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1 + 4M_1)^2 + 16(2E + n)^2} - (1 + 4M_1)\right]^{1/2},$$

$$P_1 = \frac{S_c}{2} + \frac{1}{2\sqrt{2}}\left[S_c\sqrt{S_c^2 + 16n^2} + S_c^2\right]^{1/2},$$

$$Q_1 = \frac{1}{2\sqrt{2}}\left[S_c\sqrt{S_c^2 + 16n^2} - S_c^2\right]^{1/2},$$

$$P_2 = \frac{-G_m d_1}{d_1^2 + 4E^2}, \qquad Q_2 = \frac{-2G_m E}{d_1^2 + 4E^2},$$

$$P_3 = \frac{-G_m L_2 d_2}{d_2^2 + d_3^2}, \qquad Q_3 = \frac{G_m L_2 d_3}{d_2^2 + d_3^2},$$

$$P_4 = \frac{-d_4}{k_0\left[d_1^2 + (2E + n)^2\right]}, \qquad Q_4 = \frac{-d_5}{k_0\left[d_1^2 + (2E + n)^2\right]},$$

$$P_5 = \frac{d_6}{k_0\left[a_2^2 + (2E + n - b_2)^2\right]}, \qquad Q_5 = \frac{d_7}{k_0\left[a_2^2 + (2E + n - b_2)^2\right]}$$

$$F_1 = H_2^2 - H_2 - M_1, \qquad F_2 = k_0\left(H_2^2 - H_2 - M_1\right),$$

$$F_3 = (a_1 + ib_1) = H_4^2 - H_4 - M_1, \qquad F_4 = (a_2 + ib_2) = H_6^2 - H_6 - M_1,$$

$$F_5 = \frac{-G_r F_1}{F_1^2 + 4E^2}, \qquad F_6 = \frac{-2G_r E}{F_1^2 + 4E^2},$$

$$F_7 = \frac{-F_2F_5 + k_0F_6(2E + n)}{F_2^2 + k_0^2(2E + n)^2}, \qquad F_8 = \frac{-F_2F_6 - k_0F_5(2E + n)}{F_2^2 + k_0^2(2E + n)^2},$$

$$F_9 = \frac{-G_r L_1 a_1}{a_1^2 + (2E + n - b_1)^2} \qquad F_{10} = \frac{-G_r L_1(2E + n - b_1)}{a_1^2 + (2E + n - b_1)^2},$$

$$F_{11} = \frac{a_2(F_5 - 1) - F_6(2E + n - b_2)}{k_0\left[a_2^2 + (2E + n - b_1)^2\right]},$$

$$F_{12} = \frac{a_2F_6 + (F_5 - 1)(2E + n - b_2)}{k_0\left[a_2^2 + (2E + n - b_1)^2\right]},$$

$$F_{13} = 1 - F_7 - F_9 - F_{11} - P_3 - P_4 - P_5,$$

$$F_{14} = F_8 + F_{10} + F_{12} + Q_3 + Q_4 + Q_5,$$

$$a_1 = A_1^2 - B_1^2 - A_1 - M_1, \qquad a_2 = A_2^2 - B_2^2 - A_2 - M_1,$$

$$b_1 = 2A_1B_1 - B_1, \qquad b_2 = 2A_2B_2 - B_2,$$

$$d_1 = S_c^2 - S_c - M_1, \qquad d_2 = P_1^2 - Q_1^2 - P_1 - M_1,$$

$$d_3 = 2P_1Q_1 - Q_1 - 2E - n, \qquad d_4 = P_2d_1 - Q_2(2E + n),$$

$$d_5 = Q_2d_1 - P_2(2E + n), \qquad d_6 = P_2a_1 - Q_2(2E + n - b_2),$$

and $d_7 = Q_2a_2 - P_2(2E + n - b_2)$,

SKIN – FRICTION AND HEAT TRANSFER

The non-dimensional skin-friction at the plate is :

$$\tau = \left(\frac{\partial q}{\partial z}\right)_{z=0} e^{\text{int}} = \left[\left(\frac{\partial q_0}{\partial z}\right)_{z=0} + i \in \left(\frac{\partial q_1}{\partial z}\right)_{z=0}\right] e^{\text{int}} = \tau_p + i\tau_s \qquad \text{.....(41)}$$

Hence, primary skin-friction (τ_p) due to primary velocity is :

$$\tau_p = F_{15} + \in \left(F_{17} \cos nt + F_{18} \sin nt\right) \qquad \text{.....(42)}$$

Also, secondary skin-friction (τ_s) due to secondary velocity is :

$$\tau_s = F_{16} - \in \left(F_{18} \cos nt - F_{17} \sin nt\right) \qquad \text{.....(43)}$$

where

$$F_{15} = A_2F_5 - B_2F_6 - H_2F_5 - A_2 + P_2(A_2 - S_c) - B_2Q_2,$$

$$F_{16} = A_2F_6 - B_2F_5 - H_2F_6 - B_2 + Q_2(A_2 - S_c) + B_2P_2,$$

$$F_{17} = A_3F_{13} - B_2F_{14} - H_2F_7 - A_1F_9 + B_1F_{11} - A_2F_{11} + B_2F_{12},$$

$$F_{18} = B_3F_{13} - A_3F_{14} + H_2F_8 + B_1F_9 + A_1F_{10} + B_2F_{11} + A_2F_{12},$$

The rate of heat transfer at the plate in terms of Nusselt number (N_u) is :

$$N_u = \left(\frac{\partial T}{\partial z}\right)_{z=0} e^{int} = \left[\left(\frac{\partial T_0}{\partial z}\right)_{z=0} + i \in \left(\frac{\partial T_1}{\partial z}\right)_{z=0}\right] e^{int} \qquad \text{.....(44)}$$

Hence, considering that real part only is of significance, the rate of heat transfer is :

$$N_u = -H_2 - \in \left[A_1L_1 \cos nt - B_1L_1 \sin nt\right] e^{int} \qquad \text{.....(45)}$$

The rate of mass transfer at the plate in terms of Sherwood number (S_h) is

$$S_h = \left(\frac{\partial C}{\partial z}\right)_{z=0} e^{int} = \left[\left(\frac{\partial C_0}{\partial z}\right)_{z=0} + i \in \left(\frac{\partial C_1}{\partial z}\right)_{z=0}\right] e^{int} \qquad \text{.....(46)}$$

Hence, considering that real part only is of significance, the rate of mass transfer is :

$$S_h = -S_c - \in \left[P_1L_2 \cos nt - Q_1L_2 \sin nt\right] e^{int} \qquad \text{.....(47)}$$

Table – 1

Skin friction due to primary velocity

(Cooling case $G_r > 0$)

at n = 5.0, t = 1.0 and $\in$ = 0.002)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	τ_S
0.71	0.66	1.0	20.0	1.0	5.00	8.0	1.0	4.39557
7.00	0.66	1.0	20.0	1.0	5.00	8.0	1.0	3.02178
0.71	0.78	1.0	20.0	1.0	5.00	8.0	1.0	4.19363
0.71	0.66	2.0	20.0	1.0	5.00	8.0	1.0	2.33361
0.71	0.66	1.0	40.0	1.0	5.00	8.0	1.0	4.40933
0.71	0.66	1.0	20.0	2.0	5.00	8.0	1.0	4.14551
0.71	0.66	1.0	20.0	1.0	10.00	8.0	1.0	6.39133
0.71	0.66	1.0	20.0	1.0	5.00	12.0	1.0	6.52598
0.71	0.66	1.0	20.0	1.0	5.00	8.0	2.0	2.48726

Table – 2

Skin friction due to secondary velocity

(Cooling case $G_r > 0$)

(n = 5.0, t = 1.0 and $\in$ = 0.002)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	τ_S
0.71	0.66	1.0	20.0	1.0	5.00	8.0	1.0	-3.4703
7.00	0.66	1.0	20.0	1.0	5.00	8.0	1.0	-2.88124
0.71	0.78	1.0	20.0	1.0	5.00	8.0	1.0	-3.22668
0.71	0.66	2.0	20.0	1.0	5.00	8.0	1.0	-1.42073
0.71	0.66	1.0	40.0	1.0	5.00	8.0	1.0	-3.50291
0.71	0.66	1.0	20.0	2.0	5.00	8.0	1.0	-3.30506
0.71	0.66	1.0	20.0	1.0	10.00	8.0	1.0	-4.11685
0.71	0.66	1.0	20.0	1.0	5.00	12.0	1.0	-4.50417
0.71	0.66	1.0	20.0	1.0	5.00	8.0	2.0	-3.99068

Table – 3

Skin friction due to primary velocity

(Heating case G < 0)

(n = 5.0, t = 1.0 and ∈ = 0.003)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	τ_S
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	0.40407
7.00	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	1.77784
0.71	0.78	1.0	20.0	1.0	-5.00	8.0	1.0	0.20212
0.71	0.66	2.0	20.0	1.0	-5.00	8.0	1.0	-0.8868
0.71	0.66	1.0	40.0	1.0	-5.00	8.0	1.0	0.41182
0.71	0.66	1.0	20.0	2.0	-5.00	8.0	1.0	0.65414
0.71	0.66	1.0	20.0	1.0	-10.0	8.0	1.0	-1.5916
0.71	0.66	1.0	20.0	1.0	-5.00	12.0	1.0	2.53447
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	2.0	-0.7074

Table – 4

Skin friction due to secondary velocity

(Heating case Gr < 0)

(n = 5.0, t = 1.0 and ∈ = 0.002)

P_r	S_C	M	k_0	α_0	G_r	G_m	E	τ_s
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	-2.17660
7.00	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	-2.76516
0.71	0.78	1.0	20.0	1.0	-5.00	8.0	1.0	-1.93319
0.71	0.66	2.0	20.0	1.0	-5.00	8.0	1.0	-0.92693
0.71	0.66	1.0	40.0	1.0	-5.00	8.0	1.0	-2.19954
0.71	0.66	1.0	20.0	2.0	-5.00	8.0	1.0	-2.34162
0.71	0.66	1.0	20.0	1.0	-10.0	8.0	1.0	-1.52985
0.71	0.66	1.0	20.0	1.0	-5.00	12.0	1.0	-3.21067
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	2.0	-2.49533

Table – 5

Rate of heat transfer in terms of Nusselt Number

(∈ = 0.005)

S.No.	P_r	n	t	N_u
1.	0.71	5.00	1.0	-1.42517
2.	0.025	5.00	1.0	-1.01767
3.	7.00	5.00	1.0	-7.18105
4.	11.4	5.00	1.0	-11.5488
5.	0.71	10.0	1.0	-1.42642
6.	0.71	5.00	2.0	-1.42457

Table – 6

Rate of mass transfer in terms of Sherwood Number

(∈ - 0.005)

S.No.	P_r	n	t	N_u
1.	0.66	5.0	1.0	-0.67355
2.	0.22	5.0	1.0	-0.22733
3.	0.30	5.0	1.0	-0.30869
4.	0.60	5.0	1.0	-0.61283
5.	0.78	5.0	1.0	-0.79493
6.	0.66	10.0	1.0	-0.67654
7	0.66	5.0	2.0	-0.67229

DISCUSSION AND CONCLUSION

We have observed the effects of Prandtl number parameter (P_r), Schmidt number (S_C), constant permeability parameter (k_0), heat source parameter (α_0) magnetic parameter (M), grashof number (G_r), modified Grash of number (G_m) and rotation parameter (E) on primary and secondary velocities. These effects are shown in figures. The effects of important parameter on rate of heat transfer, rate of mass transfer and skin-friction due to primary and secondary velocities have also been observed. These effects are computed in table.

Figure -1 shows effects of magnetic parameter *(M)*, Grahsof number *(G_r)*, modified Grashof number *(O_m)* and rotation parameter *(E)*, on primary velocity *(u)* at P_r=0.71, S_c=0.66, k_0=20.0, α_o=1.0, n=5.0, t=1.0, and ∈ = 0.005. It is observed that primary velocity *(u)* increases as *z* increases and after attaining a maximum value near the plate, it decreases rapidly as *z* increases. It is also noted that an increase in *M*, G_r, or

G_m results in an increase while an increase in E result in a decrease in primary velocity.

Figure-2 shows effects of Prandtl number (P_r), Schmidt number (S_e), permeability parameter (k_o) and heat source parameter αo on secondary velocity (v) at M = 1.0, G_r=5.0, G_m=8.0, $\in$ =1.0, n=5.0, t=1.0, and $\in$ = 0.005. It is observed that secondary velocity (v) decreases as z increases and after attaining a maximum value near the plate, it increases rapidly as z increases. It is also noted that an decrease in P_a or k_o results in an decrease and an increase in secondary velocity respectively while an increase in α_o, k_0 or S_c result in an increase in secondary velocity.

The effects of the parameter namely Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), permeability parameter (k_o), heat source parameter Grashof number (G_r), modified Grashof number (G_m) and rotation parameter (E), at n=5.0, t=1.0, and $\in$ = 0.005, on skin friction (τ_p) due to primary velocity and skin-friction (τ_s) due to secondary velocity. In the computation of numerical values for skin-friction due to primary velocity and secondary velocity, in the computation of numerical values for skin-friction due to primary velocity and secondary velocity. We have taken two important cases namely cooling case and heating case. These cases are of immense importance in astrophysical problems and industrial technology, where heating and cooling of the plates have economic applications. Therefore the cases of externally cooled plate (G_r>0) and externally heated plate (G_r<0) are studied taking numerical values of various parameter encountered in the equations of the skin-friction. The value of Prandtl number (P_r) is chosen as P_r=0.71 which corresponds to water, which correspond to air. The numerical values of the remaining parameters are choosen arbitrary. These effects are shown in tables (l to 4). The effects of Prandtl number (P_r), frequency parameter (n) and time parameter (t) on rate of heat transfer [expressed in terms of Nusselt number (N_u)] and the effects of Schmidt number (S_c), frequency parameter (n) and time parameter (t) on rate of mass transfer [expressed in terms of Sherwood number (S_h)] are numerically expressed in table-5 and table-6 respectively.

Table-I represents the skin-friction (τ_p) to show the effects of Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), permeability parameter (k_0), heat source parameter (α_o) Grahsof number (G_r), modified

Grashof number (G_m) and rotation parameter (E), at n = 5.0, t =1.0, and $\in$ = 0.005 for cooling case. It is observed an increase in P_r, S_c, M, α_o or E decreases skin-friction due to primary velocity while an increase in k_o, G_r or G_m increases skin-friction due to primary velocity in cooling case.

Table-2 represents the skin-friction (τ_s) due to secondary velocity to show the effects of Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), permeability parameter (k_0), heat source parameter (α_o) Grashofnumber (G_r), modified Grashof number (G_m) and rotation parameter (E), at n = 5.0, t=1.0, and $\in$ = 0.005 for cooling case. It is observed increase in P_n S_o M or α_o increases skin-friction due to secondary velocity while an increase in k_0, G_r, G_m or E decreases skin-friction due to secondary velocity in cooling case.

Table-3 represents the skin-friction (τ_p) due to primary velocity to show the effects of Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), permeability parameter (k_0), heat source parameter (α_o) Grashof number (G_r), modified Grahsof number (G_m) and rotation parameter (E), at n=5.0, t=1.0, and $\in$ = 0.005 for heating case. It is observed an increase in P_r, k_0, α_o or G_m increases skin-friction due to primary velocity while an increase in S_c, M, G_r, or E decreases skin-friction due to primary velocity in heating case. For G_r, we are considering magnitude only due to heating case.

Table-4 represents the skin-friction (τ_s) due to secondary velocity to show the effects of Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), permeability parameter (k_0), heat source parameter (α_o) Grahsof number (G_r), modified Grashof number (G_m) and rotation parameter (E), at n=5.0, t=1.0, and $\in$ = 0.005 for heating case. It is observed an increase in S_c, M or G_r increases skin-friction due to secondary velocity while an increase in P_r, k_o, α_o G_m or E decreases skin-friction due to secondary velocity in heating case. For G_r, we are considering magnitude only due to heating case.

The effects of P_r, n and t on the rate of heat transfer, expressed in terms of Nusselt number at $\in$ = 0.005 are numerically represented in table-5. It is observed that a decrease in P_r increases the rate of heat transfer and vice-versa. It is also observed that the effects of increase in n or t are opposite to each others.

Table-6 shows the effects of S_C, n and t on the rate of mass transfer, expressed in terms of Sherwood number at $\in$ = 0.005. It is observed that a decrease in Sc increases the rate of mass transfer. It is also observed that the effects of increase in n or t mass transfer are reciprocal to each other.

Corresponding Author

Bhagwat Swarup*

Department of Mathematics, Agra College, Agra

E-Mail – drbswarup@gmail.com