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# An Optimal Solution for Fuzzy Assignment Problem

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## INTRODUCTION

The present research paper describes the method called fuzzy assignment method for finding an optimal solution for fuzzy assignment problem where the assignment matrix is in a triangular fuzzy numbers. The optimal solution is also in a triangular fuzzy number.

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operation research in mathematics. It is a special case of transportation problem in which the objective is to assign a number of origins to the equal number of destinations at the minimum cost (or maximum profit). It involves assignment of people to projects, jobs to machines, workers to jobs and teachers to classes etc., while minimizing the total assignment costs. One of the important characteristics of assignment problem is that only one job (or worker) is assigned to one machine (or project). Hence the number of sources are equal to the number of destinations and each requirement and capacity value is exactly one unit.

Fuzzy sets were introduced by L. A. Zadeth [7] as an enter of classical notion of the set. Later many researchers [2, 3, 5] also use fuzzy set theory and fuzzy numbers in different field. After this J.J. Buckly[1] used triangular fuzzy numbers in linear programming. P. Pandian and G. Natarajan [4] use fuzzy zero point method in trapezoidal fuzzy numbers for finding a fuzzy optimal solution for a fuzzy transportaion problem.

In this paper, we propose a new method called fuzzy assignment method for finding a fuzzy optimal solution for a fuzzy assignment problem where all parameters are triangular fuzzy numbers. The optimal solution of fuzzy assignment problem by the fuzzy aasignment method is a triangular fuzzy number. The solution procedure is illustrated with the numerical examples.

When we use the assignment method for finding an optimal solution for a fuzzy assignment problem, we have the following advantages.

- We don't use linear programming techniques.
- We don't use goal and parametric programming techniques.
- The optimal solution is a fuzzy number.
- The proposed method is very easy to understand and to apply.

**Fuzzy number and fuzzy assignment problem**

We use the following definitions of triangular fuzzy number and membership function and also, definitions of basic arithmetic operation on fuzzy triangular numbers.

Definition A fuzzy number $\tilde{x}$ is a triangular fuzzy number denoted by ($x_1$, $x_2$, $x_3$) where $x_1$, $x_2$ and $x_3$ are real numbers and its membership function $\mu_{\tilde{x}}(a)$ is given below.

$$\mu_{\tilde{x}}(a) = \begin{cases} (a - x_1)/(x_2 - x_1) \text{ for } x_1 \le a \le x_2 \\ (x_3 - a)/(x_3 - x_2) \text{ for } x_2 \le a \le x_3 \\ 0 \quad \text{oterwise} \end{cases}$$

Definition Let $\tilde{a}$ = ($a_1$, $a_2$, $a_3$) and $\tilde{b}$ = ($b_1$, $b_2$, $b_3$) be two triangular fuzzy numbers. Then

(i) $\tilde{a} \oplus \tilde{b}$ = ($a_1 + b_1$ , $a_2 + b_2$ , $a_3 + b_3$) ;

(ii) $\tilde{a} - \tilde{b}$ = ($a_1 - b_3$ , $a_2 - b_2$ , $a_3 - b_1$) ;

(iii) $k\tilde{a}$ = ($ka_1$, $ka_2$, $ka_3$), for $k \ge 0$;

(iv) $k\tilde{a}$ = $(ka_3, ka_2, ka_1)$, for $k < 0$;

(v) $\tilde{a} \otimes \tilde{b}$ = $(t_1, t_2, t_3)$, where $t_1$= min. $\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$; $t_2 = a_2b_2$ and $t_3$ = max. $\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$;

(vi) $\frac{1}{\tilde{b}} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where $b_1$, $b_2$ and $b_3$ are all non-zero positive real numbers.

(vii) $\frac{\tilde{a}}{\tilde{b}} = \tilde{a} \otimes \frac{1}{\tilde{b}}$, where $b_1$, $b_2$ and $b_3$ are all non-zero positive real numbers.

Definition The magnitude of the triangular fuzzy number $\tilde{u}$ = $(a, b, c)$ is given by Mag ($\tilde{u}$) = $\frac{a+10b+c}{12}$.

Definition Let $\tilde{u}$ and $\tilde{v}$ be two triangular fuzzy numbers.

The ranking of $\tilde{u}$ and $\tilde{v}$ by the Mag (.) on E is defined as follows:

(i) Mag($\tilde{u}$) > Mag($\tilde{v}$) if and only if $\tilde{u} > \tilde{v}$;

(ii) Mag($\tilde{u}$) < Mag($\tilde{v}$) if and only if $\tilde{u} < \tilde{v}$;

(iii) Mag($\tilde{u}$) = Mag($\tilde{v}$) if and only if $\tilde{u} = \tilde{v}$.

**Fuzzy assignment problem:-**

Consider the following fuzzy assignment problem,

(P) Min. $\tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$ ; [$i$= 1, 2, 3, …, n; $j$= 1, 2, 3, …, n]

Subject to the constraints:

(i) $\sum_{i=1}^{n} \tilde{x}_{ij}$ = 1; $j$ = 1, 2, …, n. i.e. $i^{th}$ person will do only one work.

(ii) $\sum_{j=1}^{n} \tilde{x}_{ij}$ = 1; $i$= 1, 2, …, n. i.e. $j^{th}$ person will be done only one person.

Where $\tilde{x}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3)$ = the assignment of facility $i$ to job $j$ such that $\tilde{x}_{ij}$ = 1;

if $i^{th}$ person is assigned $j^{th}$ work

0; if $i^{th}$ person is not assigned the $j^{th}$ work

$\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3)$ = the cost of assignment of resources $i$ to activity $j$.

$\tilde{z}$ = $(z_1, z_2, z_3)$ = min./max. The total cost of the matrix.

**Algorithm**

The fuzzy assignment method proceeds as follows:-

**Step-1** if the number of rows are not equal to the number of columns, the as required a dummy row or dummy column must be added. The cost elements in dummy cells are always zero.

**Step-2 Row reduction**: Subtract each row entries of the fuzzy assignment table from the corresponding row minimum.

**Step-3 Column reduction**: After completion of step-2, subtract each column entries of the fuzzy assignment table from the corresponding column minimum.

**Step-4** Remember that each row and each column now have at least one zero.

**Step-5 Zero assignment:** In the modified matrix obtained in the step-3, search for the optimal assignment as follows:

(a) Examine the rows successively until a row with a single zero is found. Enrectangle this row (□) and cross off (×) all other zeros in its column. Continue in this manner until all rows have been taken care of.

(b) Repeat the procedure for each column of the reduced matrix. If a row and/or column have two or more zeros and one can not be chosen by inspection then assign arbitrary any one of the zero and cross off all other zeros of that row / column.

(c) Repeat (a) through (c) above successively until the assigning (□) or cross (×) ends.

**Step-6** If the number of assignment (□) is equal to n (the order of the cost matrix), an optimal solution is reached.

If the number of assignment is less than n (the order of the matrix), go to the next step.

**Step-7** Draw the minimum number of lines to cover zero's

In order to cover all the zero's at least onceyou may use the following procedure.

(i) Marks (✓) to all rows in which the assignment has not be done.

(ii) See the position of zero in marked (✓) row and then mark (✓) to the corresponding column.

(iii) See the marked (✓) column and find the position of assigned zero's and then mark (✓) to the corresponding rows which are not marked till now.

(iv) Repeat the procedure (ii) and (iii) till the completion of marking.

(v) Draw the lines through unmarked rows and marked columns.

**Step-8** Select the smallest element from the uncovered elements.

(a) Subtract this element from all uncovered elements and add the same to all the elements laying at the intersection of any two lines.

**Step-9** Go to step-5 and repeat the procedure until an optimum solution is attained.

**Numerical examples:-**

The proposed method is illustrated by the following examples.

**Example 1.** Let three persons A, B and C are to be assigned three jobs I, II and III. The cost matrix is given as under, find the proper assignment.

| Man/Jobs | A | B | C |
|---|---|---|---|
| I | (1, 2, 3) | (2, 5, 8 ) | (2, 4, 6) |
| II | (2, 6, 10) | (1, 3, 5) | (0, 1, 2) |
| III | (4, 8, 12) | (3, 9, 15) | (1, 2, 3) |

**Solution:**

In order to find the proper assignment, we apply the fuzzy assignment method as follows:

(I) Row reduction

**Table-1**

| Man\Jobs | A | B | C |
|---|---|---|---|
| I | $\tilde{0}$ | (-1, -3, -5 ) | (-1,- 2,-3) |
| II | (-2, -5, -8) | (-1, -2, -3) | $\tilde{0}$ |
| III | (-3, -6, -9) | (-2, -7, -12) | $\tilde{0}$ |

(II) Column reduction

**Table-2**

| Man\ Jobs | A | B | C |
|---|---|---|---|
| I | $\tilde{0}$ | (-1, -4, -7 ) | (-1, -2, -3) |
| II | (-2, -5, -8) | (-1, -5, -9) | $\tilde{0}$ |
| III | (-3, -6, -9) | $\tilde{0}$ | $\tilde{0}$ |

(III) Zero assignment

**Table-3**

| Man\Jobs | A | B | C |
|---|---|---|---|
| I | $\boxed{\tilde{0}}$ | (0, 1, 2 ) | (1, 2,3) |
| II | (2, 5, 8) | (-1, -5, -9) | $\boxed{\tilde{0}}$ |
| III | (3, 6, 9) | $\boxed{\tilde{0}}$ | $\tilde{0}$ X |

In this way all the zero's are either crossed out or assigned. Also total assigned zero's = 3 (i.e., number of rows or columns). Thus, the assignment is optimal.

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