

# A Comparative Analysis on Gaussian Isoperimetric Inequality, and Its Similar Concentration Trend

Shrikrishna Kakade<sup>1\*</sup> Dr. Bhausaheb Sontakke<sup>2</sup>

<sup>1</sup> PhD Student, Kalinga University, Raipur

<sup>2</sup> PhD Guide, Kalinga University, Raipur

**Abstract** – The Gaussian isoperimetric inequality, and its related concentration phenomenon, is one of the most important properties of Gaussian measures. These notes aim to present, in a concise and self-contained form, the fundamental results on Gaussian processes and measures based on the isoperimetric tool. In particular, our exposition will include, from this modern point of view, some of the by now classical aspects such as inerrability and tail behavior of Gaussian semi norms, large deviations or regularity of Gaussian sample paths. We will also concentrate on some of the more recent aspects of the theory which deal with small ball probabilities.

Actually, the Gaussian concentration inequality will be the opportunity to develop some functional analytic ideas around the concentration of measure phenomenon. In particular, we will see how simple semigroup tools and the geometry of abstract Markov generator may be used to study concentration and isoperimetric inequalities. We investigate in this context some of the deep connections between isoperimetric inequalities and functional inequalities of Sobolev type. We also survey recent work on concentration inequalities in product spaces. Actually, although the main theme is Gaussian isoperimetry and analysis, many ideas and results have a much broader range of applications. We will try to indicate some of the related fields of interest.

## INTRODUCTION

The Gaussian isoperimetric inequality was developed extensively in the study of the functional analytic aspects of probability theory. Here, we only present it to illustrate its power in Information theory. We begin with a definition of a measure. In 1,2 and 3 dimensional Euclidean space, there exists simple and intuitive concepts of measure. We speak of the length of a line segment, area under a curve and the volume of a sphere for instance. The concepts of length, area and volume are familiar concepts. It is only natural that there exists a generalization of the principal concept of measure in higher dimensions.

Theorem 1 : Among all sets in  $\mathbb{R}^n$  with prescribed Gaussian measure, half spaces have minimal Gaussian perimeter.

Definition 1.1 A set of sets  $S$  is said to be pairwise disjoint iff:  $\forall X, Y \in S :$

$$X \neq Y \implies X \cap Y = \emptyset$$

It has been mentioned that measure is a generalisation of volume. Without an explicit definition of a measurable space, we can further explain what a measure is. A measure is a function which takes an element  $X$  (of a measurable space  $\Gamma$ ) and returns a non-negative number which we will refer to as the "measure of  $X$ ". We denote the measure by  $\mu : \Gamma \rightarrow \mathbb{R}_{\geq 0} \cup \{-\infty, \infty\}$ . Further, in addition to non-negativity, a measure satisfies the following properties.

1.  $\mu(\emptyset) = 0$

2. Countable Additivity Let  $\{S_n\} \subset \Gamma$  be a sequence of pairwise disjoint sets. Then,  $\mu(\cup_{n=1}^{\infty} S_n) = \sum_{n=1}^{\infty} \mu(S_n)$

A Gaussian measure is probability measure. This means that is a special case of a measure. It satisfies non-negativity and countable additivity properties. However, it has an additional property that it is normalized, that is,  $\forall A \in \Gamma : 0 \leq \mu(A) \leq 1$

One can split  $n$  dimensional space with an-1 dimensional hyperplane. A halfspace is simply the space which remains after the section of the space which falls on one side of the hyperplane is removed.

Definition 1.2 Let  $\mu : (\mathbb{R}^n, \Gamma) \rightarrow [0, 1]$  be a probability measure on the given measurable space. We say that  $\mu$  is a Gaussian measure if it is defined by

$$\mu(X) = \frac{1}{\sqrt{2\pi}^n} \int_X e^{-\frac{1}{2}\|x\|^2} d\mu(x)$$

where  $\|\cdot\|$  denotes the length of vectors in  $\mathbb{R}^n$

In order to gather intuition on the Gaussian isoperimetric inequality, we begin by restating the classical isoperimetric inequality. Among all compact sets  $A$  in  $\mathbb{R}^n$  with a smooth boundary  $\partial A$  and with a fixed volume, Euclidean balls are the ones with the minimal surface measure. The surface measure we were concerned with was the perimeter. The notion behind the Gaussian isoperimetric inequality is similar. The measure, however, is different as we now focus on the Gaussian measure. Also there exists a Steiner symmetrization equivalent in the context of the Gaussian isoperimetric inequality. It is known as Gaussian symmetrization. A formal presentation of the Gaussian isoperimetric inequality is presented by both Rosand Ledoux.

The isoperimetric inequality for the standard Gaussian measure  $\gamma_n$  on  $\mathbb{R}^n$  with density  $\exp(-|x|^2/2)/\sqrt{2\pi}$ ,  $x \in \mathbb{R}^n$  can be stated as follows: for every measurable set  $A \subset \mathbb{R}^n$ , we have

$$\gamma_n^+(A) \geq I(\gamma_n(A)) \tag{1}$$

In other words, if we define  $a \in \mathbb{R}$  by the equation  $\gamma_n(A) = \int_{-\infty}^a d\gamma_1$ , then  $\gamma_n^+(A) \geq \exp(-a^2/2)/\sqrt{2\pi}$ . Clearly, this inequality is an equality for affine half-spaces in  $\mathbb{R}^n$

Recently, Bobkov proved a functional version of the Gaussian isoperimetry: for every locally Lipschitz function  $f : \mathbb{R}^n \rightarrow [0, 1]$ , one has

$$I\left(\int_{\mathbb{R}^n} f d\gamma_n\right) \leq \int_{\mathbb{R}^n} \sqrt{I^2(f) + |\nabla f|^2} d\gamma_n. \tag{2}$$

It is easy to see that this inequality implies (1). Bobkov deduces (2) from the following "two-point" isoperimetric inequality: for all  $a, b \in [0, 1]$ ,

$$I\left(\frac{a+b}{2}\right) \leq \frac{1}{2}\sqrt{I^2(a) + \left(\frac{b-a}{2}\right)^2} + \frac{1}{2}\sqrt{I^2(b) + \left(\frac{b-a}{2}\right)^2}. \tag{3}$$

Using the remarkable tensorisation properties of this inequality and the central limit theorem, Bobkov shows that (3) implies (2). Inequality (2) for  $\mathbb{R}^n$  can also be proved from (1) for  $\mathbb{R}^{n+1}$  by choosing  $A \subset \mathbb{R}^n \times \mathbb{R}$  to be the subgraph of  $\Phi^{-1} \circ f$ .

### Gaussian Isoperimetry

For a Borel set  $A$  in  $\mathbb{R}^n$  and  $t > 0$  let  $A_t = A + tB_2^n = \{x \in \mathbb{R}^n : |x - a| < t \text{ for some } a \in A\}$  be the open  $t$ -enlargement of  $A$ , where  $B^\wedge$  denotes the open unit Euclidean ball in  $\mathbb{R}^n$ . The classical isoperimetric inequality for the Lebesgue measure states that if  $\text{vol}_n(A) = \text{vol}_n(rB_2^n)$  then  $\text{vol}_n(A_t) \geq \text{vol}_n((r+t)B_2^n)$  for  $t > 0$ . In the early 70's C. Borell and V.N. Sudakov and B.S. Tsirel'son proved independently the isoperimetric property of Gaussian measures.

Theorem 2 : Let  $A$  be a Borel set in  $\mathbb{R}^n$  and let  $H$  be an affine halfspace such that  $\gamma_n(A) = \gamma_n(H) = \Phi(a)$  for some  $a \in \mathbb{R}$ . Then

$$\gamma_n(A_t) \geq \gamma_n(H_t) = \Phi(a + t) \text{ for all } t \geq 0.$$

Theorem has an equivalent differential analog. To state it let us define for a measure  $\mu$  on  $\mathbb{R}^n$  and any Borel set  $A$  the boundary  $\mu$ -measure of  $A$  by the formula  $\mu^+(A) = \liminf_{t \rightarrow 0^+} \frac{\mu(A_t) - \mu(A)}{t}$ .

Moreover let  $\varphi(x) = \Phi'(x) = (2\pi)^{-1/2} \exp(-x^2/2)$  and let  $I(t) = \varphi \circ \Phi^{-1}(t)$ ,  $t \in [0, 1]$  be the Gaussian isoperimetric function.

The equivalent form of Theorem is that for all Borel sets  $A$  in  $\mathbb{R}^n$   $\gamma_n^+(A) \geq I(\gamma_n(A))$ .

The equality in above equation holds for any affine halfspace.

For a probability measure  $\mu$  on  $\mathbb{R}^n$  we may define the isoperimetric function of  $\mu$  by

$$Is(\mu)(p) = \inf\{\mu^+(A) : \mu(A) = p\}, \quad 0 \leq p \leq 1.$$

Only few cases are known when one can determine exactly  $Is(\mu)$ . For Gaussian measures (2) states that  $Is(\gamma_n) = I$

Let us finish section by an example of application of equation.

Corollary 1 Let  $X$  be a centered Gaussian random vector in a separable Banach space  $(F, \|\cdot\|)$ . Then for any  $t > 0$

$$P(\|\|X\| - \text{Med}(\|X\|)\| \geq t) \leq 2(1 - \Phi(\frac{t}{\sigma})) \leq e^{-t^2/2\sigma^2},$$

where

$$\sigma = \sup\{\sqrt{E(x^*(X))^2} : x^* \in F^*, \|x^*\| \leq 1\}.$$

### SOME ISOPERIMETRIC INEQUALITIES

In this study, we present the basic isoperimetric inequalities which form the geometric background of this study. Although we will not directly be concerned with true isoperimetric problems and description of extremal sets later on, these inequalities are at the basis of the concentration inequalities on which most results of these notes will be based. We introduce the isoperimetric ideas with the classical isoperimetric inequality on  $\mathbb{R}^n$  but the main result will actually be the isoperimetric property on spheres and its limit version, the Gaussian isoperimetric inequality. More on isoperimetry may be found.

The classical isoperimetric inequality in, which at least in dimension 2 and for convex sets may be considered as one of the oldest mathematical statements, asserts that among all compact sets  $A$  in  $\mathbb{R}^n$  with smooth boundary  $\partial A$  and with fixed volume, Euclidean balls are the ones with the minimal surface measure. In other words, whenever  $\text{vol}_n(A) = \text{vol}_n(B)$  where  $D$  is a ball (and  $n > 1$ ),

$$\text{vol}_{n-1}(\partial A) \geq \text{vol}_{n-1}(\partial B). \quad (4)$$

There is an equivalent, although less familiar, formulation of this result in terms of isoperimetric neighborhoods or enlargements which in particular avoids surface measures and boundary considerations; namely, if  $A_r$  denotes the (closed) Euclidean neighborhood of  $A$  of order  $r \geq 0$ , and if  $D$  is as before a ball with the same volume as  $A$ . then, for every  $r \geq 0$ ,

$$\text{vol}_n(A_r) \geq \text{vol}_n(B_r). \quad (5)$$

Note that  $A_r$  is simply the Minkowski sum  $A + B(0, r)$  of  $A$  and of the (closed) Euclidean ball  $B(0, r)$  with center the origin and radius  $r$ . The equivalence between (4) and (5) follows from the Minkowski content formula

$$\text{vol}_{n-1}(\partial A) = \liminf_{r \rightarrow 0} \frac{1}{r} [\text{vol}_n(A_r) - \text{vol}_n(A)]$$

(whenever the boundary  $\partial A$  of  $A$  is regular enough). Actually, if we take the latter as the definition of  $\text{vol}_{n-1}(\partial A)$ , it is not too difficult to see that (4) and (5) are equivalent for every Borel set  $A$ . The simplest proof of this isoperimetric inequality goes through the Brunn-Minkowski inequality which states that if  $A$  and  $D$  are two compact sets in  $\mathbb{R}^n$ , then

$$\text{vol}_n(A + B)^{1/n} \geq \text{vol}_n(A)^{1/n} + \text{vol}_n(B)^{1/n}. \quad (6)$$

To deduce the isoperimetric inequality (5) from the Brunn-Minkowski inequality (6), let  $r_0 > 0$  be such that  $\text{vol}_n(A) = \text{vol}_n(B(0, r_0))$ . Then, by (6),

$$\begin{aligned} \text{vol}_n(A_r)^{1/n} &= \text{vol}_n(A + B(0, r))^{1/n} \geq \text{vol}_n(A)^{1/n} + \text{vol}_n(B(0, r))^{1/n} \\ &= \text{vol}_n(B(0, r_0))^{1/n} + \text{vol}_n(B(0, r))^{1/n} \\ &= (r_0 + r) \text{vol}_n(B(0, 1))^{1/n} \\ &= \text{vol}_n(B(0, r_0 + r))^{1/n} = \text{vol}_n(B(0, r_0)_r)^{1/n} \end{aligned}$$

As an illustration of the methods, let us briefly sketch the proof of the Brunn-Minkowski inequality (6). By a simple approximation procedure, we may assume that each of  $A$  and  $D$  is a union of finitely many disjoint sets, each of which is a product of intervals with edges parallel to the coordinate axes. The proof is by induction on the total number  $p$  of these rectangular boxes in  $A$  and  $D$ . If  $p = 2$ , that is if  $A$  and  $D$  are products of intervals with sides of length  $(a_i)_{1 \leq i \leq n}$  and  $(b_i)_{1 \leq i \leq n}$  respectively, then

$$\begin{aligned} \frac{\text{vol}_n(A)^{1/n} + \text{vol}_n(B)^{1/n}}{\text{vol}_n(A + B)^{1/n}} &= \prod_{i=1}^n \left( \frac{a_i}{a_i + b_i} \right)^{1/n} + \prod_{i=1}^n \left( \frac{b_i}{a_i + b_i} \right)^{1/n} \\ &\leq \frac{1}{n} \sum_{i=1}^n \frac{a_i}{a_i + b_i} + \frac{1}{n} \sum_{i=1}^n \frac{b_i}{a_i + b_i} = 1 \end{aligned}$$

where we have used the inequality between geometric and arithmetic means. Now, assume that  $A$  and  $D$  consist of a total of  $p > 2$  products of intervals and that (6) holds for all sets  $A'$  and  $D'$  which are composed of a total of at most  $p - 1$  rectangular boxes. We may and do assume that the number of rectangular boxes in  $A$  is at least 2. Parallel shifts of  $A$



and  $D$  do not change the volume of  $A$ ,  $D$  or  $A + D$ . Take then a shift of  $A$  with the property that one of the coordinate hyperplanes divides  $A$  in such a way that there is at least one rectangular box in  $A$  on each side of this hyperplane. Therefore  $A$  is the union of  $A'$  and  $A''$  where  $A'$  and  $A''$  are disjoint unions of a number of rectangular boxes strictly smaller than the number in  $A$ . Now shift  $D$  parallel to the coordinate axes in such a manner that the same hyperplane divides  $D$  into  $B'$  and

$$B'' \text{ with } \frac{\text{vol}_n(B')}{\text{vol}_n(B)} = \frac{\text{vol}_n(A')}{\text{vol}_n(A)} = \lambda.$$

Each of  $B'$  and  $B''$  has at most the same number of products of intervals as  $B$  has. Now, by the induction hypothesis,

$$\begin{aligned} \text{vol}_n(A+B) &\geq \text{vol}_n(A'+B') + \text{vol}_n(A''+B'') \\ &\geq [\text{vol}_n(A')^{1/n} + \text{vol}_n(B')^{1/n}]^n + [\text{vol}_n(A'')^{1/n} + \text{vol}_n(B'')^{1/n}]^n \\ &= \lambda[\text{vol}_n(A)^{1/n} + \text{vol}_n(B)^{1/n}]^n + (1-\lambda)[\text{vol}_n(A)^{1/n} + \text{vol}_n(B)^{1/n}]^n \\ &= [\text{vol}_n(A)^{1/n} + \text{vol}_n(B)^{1/n}]^n \end{aligned}$$

which is the result. Notethat, by concavity, (6) implies

(is actually equivalent to the fact) that, for every  $\lambda$  in  $[0,1]$ ,

$$\begin{aligned} \text{vol}_n(\lambda A + (1-\lambda)B) &\geq [\lambda \text{vol}_n(A)^{1/n} + \\ (1-\lambda)\text{vol}_n(B)^{1/n}]^n &\geq \text{vol}_n(A)^\lambda \text{vol}_n(B)^{1-\lambda}. \end{aligned}$$

### ISOPERIMETRIC INEQUALITY FOR r-SETS

Isoperimetric problems are classical objects of study in mathematics. In general, they ask for the smallest possible ‘boundary’ of a set of a certain ‘size’. For example, of all shapes in the plane with area 1, which has the smallest perimeter? The ancient Greeks ‘knew’ that the answer was a circle, but it was not until the 19th century that this was proved rigorously.

In the last fifty years, discrete isoperimetric problems have been extensively studied. These deal with notions of boundary in graphs. Here, there are two competing notions of boundary. If  $G = (V, E)$  is a graph, and  $S \subset V$ , the *vertex-boundary* of  $S$  in  $G$  is the set of all vertices in  $V \setminus S$  which have a neighbour in  $S$ . Similarly, the *edge-boundary* of  $S$  in  $G$  is the set of all edges of  $G$  between  $S$  and  $V \setminus S$ . The *vertex-isoperimetric problem* for  $G$  asks for the minimum possible size of the vertex-boundary of a  $k$ -element subset of  $V$ , for each  $k \in \mathbb{N}$ . Similarly, the *edge-isoperimetric problem* for  $G$  asks for the minimum possible size of the edge-boundary of a  $k$ -element subset of  $V$ , for each  $k \in \mathbb{N}$ .

For surveys of discrete isoperimetric inequalities from a combinatorial perspective, for a discussion of the connection with concentration of measure, and for several applications, notably in geometric probability and percolation theory.

A fundamental example arises from taking our graph  $G$  to be the  $n$ -dimensional hypercube  $Q_n$ , the graph on  $\{0, 1\}^n$  where  $x$  and  $y$  are adjacent if they differ in exactly one coordinate. It turns out that the edge-boundary of a  $k$ -element set is minimized by taking the first  $k$  elements of the binary ordering on  $\{0, 1\}^n$ . Hart’s proof uses induction on  $n$ , combined with an inequality concerning the number of 1’s in initial segments of the binary ordering on  $\{0, 1\}^n$ .

The vertex-isoperimetric problem for  $Q_n$  was solved by Harper. To state it, we identify  $\{0, 1\}^n$  with  $\mathcal{P}([n])$ , the power-set of  $[n]$ , by identifying  $v \in \{0, 1\}^n$  with the set  $\{i \in [n] : v_i = 1\}$ . Harper’s Theorem states that the vertex-boundary of a  $k$ -element subset of  $\mathcal{P}([n])$  is minimized by taking the first  $k$  elements of the *simplicial* ordering on  $\mathcal{P}([n])$ . (If  $x, y \subset [n]$ , we say that  $x < y$  in the simplicial ordering if  $|x| < |y|$ , or  $|x| = |y|$  and  $\min(x \Delta y) \in x$ .)

Note that if  $k$  is of the form  $\sum_{i=0}^d \binom{n}{i}$ , then the first  $k$  elements of the simplicial ordering are simply all the subsets of  $[n]$  with size at most  $d$ , i.e. the Hamming ball with centre  $\emptyset$  and radius  $d$ .

Both theorems can be proved using compressions. However, this technique relies upon the fact that the extremal examples are ‘nested’; isoperimetric problems without this property require other techniques, and tend to be harder.

### GAUSSIAN ISOPERIMETRY FOR MULTIPLE SETS

Classical isoperimetry can be traced to ancient times, though its full understanding still remains incomplete. Generally speaking, we look for an object with least perimeter among all objects of fixed volume. And we expect that the smallest perimeter object has a simple structure. For example, if our notions of volume and perimeter have some symmetry, we expect the smallest perimeter object of fixed volume to also exhibit some symmetries. Since the Euclidean volume and Euclidean surface area are highly symmetric, we expect that the set of fixed Euclidean volume and least Euclidean perimeter will be highly symmetric. However, symmetry considerations are often insufficient to find a set of least perimeter.

The classical isoperimetric problem asks for a Euclidean set of fixed Euclidean measure and

minimal surface area. The solution of this problem, the Euclidean ball, has been known since antiquity. Steiner nearly proved this fact in 1838 using symmetrization. Steiner showed that, starting from a fixed Euclidean set  $A$ , it is possible to create a more symmetric set from  $A$  with the same volume and smaller or equal surface area. This argument was made rigorous by Weierstrass. We can think of symmetrization as a gradient descent method. That is, starting from any set, there exists a way to decrease the perimeter of this set, such that we are now closer to a least perimeter object, with respect to some metric.

In this way, the classical isoperimetric problem can be intuitively understood as maximizing a concave function over a convex set. The standard reference for symmetrization methods, where several previous approaches are unified.

Hurwitz solved the isoperimetric problem in the plane using the rather striking method of Fourier series in 1902. Given a non-intersecting closed rectifiable curve in the plane, the curve can be represented as a periodic function of one real variable with convergent Fourier series. Then, Fourier analysis shows that the perimeter is smallest when all higher order Fourier coefficients are zero. Surprisingly, it is still an open problem to try to extend his proof to all Euclidean spaces. Yet, for convex bodies, the essence of his proof can be used, since in that case, one can express a convex body as a function on the sphere, and then expand this function into spherical harmonics.

In the mid-1900s, mathematicians began to solve isoperimetric problems in spaces other than Euclidean space. In the sphere, the set of fixed volume and smallest boundary is a cap, or geodesic ball, as shown by Levy. A nice proof of this fact by symmetrization appears. Levy's result was generalized by Gromov in the 1980s. Gromov's result says that in a smooth manifold with Ricci curvature bounded below by a positive number  $R$ , the boundary of any set is greater than the boundary of a geodesic ball on a sphere with Ricci curvature  $R$ .

Gromov used the methods of comparison geometry, and he realized that a crucial result of Almgren gave the existence and regularity for Gromov's isoperimetric problem.

We should mention that existence and regularity of isoperimetric problems has been an important issue. In particular, Steiner's avoidance of this issue led to the error in his proof. In the mid-1970s, Almgren achieved a breakthrough in regularity theory which would change the entire direction of isoperimetric theory. As we just mentioned, this result allowed Gromov's general isoperimetric inequality. However, Almgren's result also gave the existence and regularity for a wide class of isoperimetric problems, including problems involving multiple sets. The two set isoperimetric problem or double bubble problem asks for two

Euclidean sets of fixed volume and smallest combined surface area. That is, if we have two sets  $A_1, A_2$  of fixed Euclidean volume, how can we minimize volume of  $(\partial A_1) \cup (\partial A_2)$ ?

Thanks to the rigorous foundation given to this ancient problem, the double bubble problem was finally solved in 2002. Essentially nothing is known when try to solve this problem for three or more sets. That is, if we are given three sets  $A_1, A_2, A_3$  of fixed Euclidean volume, we cannot find the minimum volume of  $(\partial A_1) \cup (\partial A_2) \cup (\partial A_3)$ . The three set case of this problem in the plane was solved in 2004 by Wichiramala by a variational argument which considers over fifty separate cases. So, the proof does not seem to generalize to higher dimensions.

Concurrent with Almgren's existence and regularity result, even more isoperimetric inequalities were investigated. In particular, the isoperimetric problem with respect to the Gaussian measure was investigated. This measure could be considered natural in light of the Central Limit Theorem, or the fact that it is the unique rotation invariant product probability measure on Euclidean space. We now need to introduce some notation to state the Gaussian isoperimetric problem.

$$\text{Let } n \geq 1, n \in \mathbb{Z}, \text{ let } x = (x_1, \dots, x_n) \in \mathbb{R}^n, \\ \text{define } \|x\|_2^2 := x_1^2 + \dots + x_n^2, \text{ and define} \\ \gamma_n(x) := (2\pi)^{-n/2} \cdot e^{-\|x\|_2^2/2}. \quad (7)$$

Let  $A \subseteq \mathbb{R}^n$  be a set with smooth boundary and let  $H \subseteq \mathbb{R}^n$  be a half-space.

That is,  $H$  is the region that lies on one side of a hyperplane. Denote the Gaussian perimeter of  $\partial A$  by

$$\gamma_{n-1}(\partial A) := \liminf_{\varepsilon \rightarrow 0} (2\varepsilon)^{-1} \gamma_n(x \in \mathbb{R}^n : \exists y \in \partial A, \|x - y\|_2 < \varepsilon).$$

The Gaussian Isoperimetric Inequality says that the half-space  $H$  has the smallest Gaussian boundary among all sets of fixed Gaussian volume.

## REFERENCES

- C. Borell (2008). "Inequalities of the Brunn-Minkowski type for Gaussian measures," *Probab. Theory Related Fields* 140, No. 1–2, 195–205.
- C. Borell (1975). "The Brunn-Minkowski inequality in Gauss space," *Invent. Math.* 30, No. 2, pp. 207–216.
- F. Barthe, B. Maurey (2000). Some remarks on isoperimetry of Gaussian type, *Ann. Inst. H. Poincaré Probab. Statist.* 36, pp. 419–434.

- Hilbert spaces (2005). Technical report, University of Ottawa.
- M. Gromov Paul (2000). L\_ey's isoperimetric inequality. Preprint I.H.E.S..
- M. Ledoux (1998). "A short proof of the Gaussian isoperimetric inequality. High dimensional probability" In: Oberwolfach, 1996, pp. 229–232, *Progr. Probab.* 43, Birkh1auser, Basel.
- M. Ledoux (2006). *Isoperimetry and Gaussian analysis*. In *Lectures on Probability Theory and Statistics*, Lecture Notes in Mathematics, Vol. 1648. Springer Berlin Heidelberg. S. pp. 165–294.
- R. Osserman (1998). The isoperimetric inequality. *Bull. Amer. Math. Soc.* 84, pp. 1182-1238.
- The Gaussian Isoperimetric Inequality and Decoding Error Probabilities for the Gaussian Channel, Jean-Pierre Tillich and Gilles Zmor, IEEE, Faculty of Computer Science and System Engineering, Okayama Prefectural University, Okayama, pp. 719-1197, Japan, pg328
- X. Fernique. Gaussian random vectors and their reproducing kernel

---

### Corresponding Author

**Shrikrishna Kakade\***

PhD Student, Kalinga University, Raipur

[kakdeshrikrishna83@gmail.com](mailto:kakdeshrikrishna83@gmail.com)