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Linear Algebra Linear Transformations

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Abstract – A linear transformation is an important concept in mathematics because many real world phenomena can be approximated by linear models. A linear transformation operates on both vectors and numbers, as opposed to a linear function. A linear mathematical transformation, with the aid of a formula of a certain format, to turn a geometric figure (or matrix or vector) into another. A linear combination in which the original components are present must be the format. A transition between two vector spaces in linear algebra is a rule that assigns a vector in one space to a vector in the other. Linear transformations are transformations which satisfy a specific property for addition and scaffolding. The present lecture discusses the fundamental notation of transformations, what 'pic' and what is meant by 'range,' and what distinguishes a linear transformation from other transformations.

Key Words: Linear Algebra, Linear Transformations, Linear Models, Linear Function

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INTRODUCTION

The interrelationships between the theory of linear transformations which preserve the invariant matrix and various branches of mathematics are examined. These methods and motives are given the expectations for studying these changes which are derived from general algebra.

A *linear transformation*, T:U→VT:U→V, is a function that carries elements of the vector space UU (called the *domain*) to the vector space VV (called the *codomain*), and which has two additional properties

1. $T(\mathbf{u}_1+\mathbf{u}_2)=T(\mathbf{u}_1)+T(\mathbf{u}_2)$ for all $\mathbf{u}_1,\ \mathbf{u}_2\in U$
2. $T(\alpha\mathbf{u})=\alpha T(\mathbf{u})$ for all $\mathbf{u}\in U$ and all $\alpha\in\mathbb{C}$

In the concept of a linear transformation, the two defining conditions "feel linear" anyway. In the other hand, each of these conditions can be taken as exactly what linearity means. -- property of a linear transformation comes from these two defining properties, as each vector space characteristic comes from vector addition and scalar multiplication. While these conditions remember how we measure subspaces, they are actually very different, so do not confuse them.

Two diagrams display the basic characteristics of the two distinguishing characteristics of a linear transformation. Start in the upper left corner in each case and follow the arrows at the lower right corner around the rectangle, follow two routes and perform the specified operations marked on the armbands. There are two conclusions. These two expressions are exactly the same for a linear transformation.

$$\begin{tikzcd}
\mathbf{u}_1,\ \mathbf{u}_2 \arrow[r, "T"] \arrow[d, "+"] & T(\mathbf{u}_1),\ T(\mathbf{u}_2) \arrow[d, "+"] \\
\mathbf{u}_1+\mathbf{u}_2 \arrow[r, "T"] & T(\mathbf{u}_1+\mathbf{u}_2)=T(\mathbf{u}_1)+T(\mathbf{u}_2)
\end{tikzcd}$$

Diagram DLTA Definition of Linear Transformation, Additive

$$\begin{tikzcd}
\mathbf{u} \arrow[r, "T"] \arrow[d, "\alpha"] & T(\mathbf{u}) \arrow[d, "\alpha"] \\
\alpha\mathbf{u} \arrow[r, "T"] & T(\alpha\mathbf{u})=\alpha T(\mathbf{u})
\end{tikzcd}$$

Diagram DLTM Definition of Linear Transformation, Multiplicative

A couple of words about notation. TT is the *name* of the linear transformation, and should be used when we want to discuss the function as a whole. T(u)T(u) is how we talk about the output of the function, it is a vector in the vector space VV. When we write T(x+y)=T(x)+T(y)T(x+y)=T(x)+T(y), the plus sign on the left is the operation of vector addition in the vector space UU, since xx and yy are elements of UU. The plus sign on the right is the operation of vector addition in the vector space VV, since T(x)T(x) and T(y)T(y) are elements of the vector space VV. These two instances of vector addition might be wildly different.

Linear Transformation and Linear combination

It is the relationship between linear and linear transformations that is at the core of many important linear algebra theorems. This is the basis of the next theorem. The evidence is not conclusive, the outcome is not surprising, but it is sometimes stated. This theorem says that we can "throw" linear transformations "down" on "linear" combinations, or "pull" linear transformations "up" onto linear combinations. We've already passed it by for a time in the evidence of theorem MLTCV. We will be able to push and pull them.

T:U→VT:U→V *is a linear transformation,* u1,u2,u3,…,utu1,u2,u3,…,ut *are vectors*
from UU *and* a1,a2,a3,…,ata1,a2,a3,…,at *are scalars from* CC. *Then*

$$T(a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3 + \cdots + a_t\mathbf{u}_t) = a_1T(\mathbf{u}_1) + a_2T(\mathbf{u}_2) + a_3T(\mathbf{u}_3) + \cdots + a_tT(\mathbf{u}_t)$$

Linear Algebra/Linear Transformations

A linear transformation is an important concept in mathematics because many real world phenomena can be approximated by linear models.

Say we have the vector $\begin{pmatrix}1\\0\end{pmatrix}$ in $\mathbb{R}^2$, in, and we rotate it through 90 degrees, to obtain the vector $\begin{pmatrix}0\\1\end{pmatrix}$

Another example instead of rotating a vector, we stretch it, so a vector $\mathbf{v}$ becomes , $2\mathbf{v}$ for example.

$$\begin{pmatrix}2\\3\end{pmatrix} \text{ becomes } \begin{pmatrix}4\\6\end{pmatrix}$$

Or, if we look at the *projection* of one vector onto the *x* axis - extracting its *x* component - , e.g. from

$$\begin{pmatrix}2\\3\end{pmatrix} \text{ we get } \begin{pmatrix}2\\0\end{pmatrix}$$

These examples are all an example of a *mapping* between two vectors, and are all linear transformations. If the rule transforming the matrix is called T, , we often write $T\mathbf{v}$ for the mapping of the vector $\mathbf{v}$ by the rule T. T is often called the transformation.

Linear Operators

If you have a field K, and let x be a field element. Let O be a feature that takes values from K where O(x) is a J field variable. Define O to be a linear form if and only if:

O(x+y)=O(x)+O(y)

O(λx)=λO(x)

Linear Forms

Assume that one has a space vector V, and that x is part of this vector space. Let F be a V values function where F(x) is a K field unit. Define F to be a linear form if and only if:

F(x+y)=F(x)+F(y)

F(λx)=λF(x)

Linear Transformation

Let us consider this time functions from one vector space to another vector space instead of a field. *T* can be a value-taking function from one space vector *V* where *L*(*V*) is an element of another space vector. Define *L* to be a linear transformation when it:

preserves scalar multiplication: T(λx) = λTx

preserves addition: T(x+y) = Tx + Ty

Remember, not everyone's linear transformations. Many simple transformations which are also non-linear in the real world. Your research is harder and will not be carried out here. The transformation, for example *S* (whose input and output are both vectors in $\mathbf{R}^2$) defined by

$$S\mathbf{x} = S\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}xy\\\cos(y)\end{pmatrix}$$

We can learn about nonlinear transformations by studying easier, linear ones.

We often describe a transformation T in the following way

$$T : V \to W$$

This means that T, whatever transformation it may be, maps vectors in the vector space V to a vector in the vector space W.

The actual transformation could be written, for instance, as

$$T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x+y\\x-y\end{pmatrix}$$

Examples and proofs

Some examples of such linear transformations are given here. Let's simultaneously explore how we can show that a transition we find is or cannot be linear..

Projection

Let us take the projection of vectors in $\mathbf{R}^2$ to vectors on the *x*-axis. Let's call this transformation T.

We know that T maps vectors from $\mathbf{R}^2$ to $\mathbf{R}^2$, so we can say

$$T:\mathbb{R}^2 \to \mathbb{R}^2$$

and we can then write the transformation itself as

$$T\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ 0 \end{pmatrix}$$

Clearly this is linear. (Can you see why, without looking below?)

Let's go through a proof that the conditions in the definitions are established.

Scalar multiplication is preserved

We wish to show that for all vectors v and all scalars λ, T(λv)=λT(v).

Let,

$$\mathbf{v} = \begin{pmatrix} v_0 \\ v_1 \end{pmatrix}.$$

Then

$$\lambda\mathbf{v} = \begin{pmatrix} \lambda v_0 \\ \lambda v_1 \end{pmatrix}$$

Now

$$T(\lambda\mathbf{v}) = T\begin{pmatrix} \lambda v_0 \\ \lambda v_1 \end{pmatrix} =$$
$$\begin{pmatrix} \lambda v_0 \\ 0 \end{pmatrix}$$

If we work out λT(**v**) and find it is the same vector, we have proved our result.

$$\lambda T\mathbf{v} = \lambda\begin{pmatrix} v_0 \\ 0 \end{pmatrix} =$$
$$\begin{pmatrix} \lambda v_0 \\ 0 \end{pmatrix}$$

This is the same vector as above, so under the transformation T, *scalar multiplication is preserved*.

Addition is preserved

We wish to show for all vectors x and y, T(x+y)=Tx+Ty.

Let

$$\mathbf{x} = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}.$$

and

$$\mathbf{y} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}.$$

Now

$$T(\mathbf{x}+\mathbf{y}) = T\left(\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} + \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}\right) =$$
$$T\begin{pmatrix} x_0+y_0 \\ x_1+y_1 \end{pmatrix} =$$
$$\begin{pmatrix} x_0+y_0 \\ 0 \end{pmatrix}$$

Now if we can show T**x**+T**y** is this vector above, we have proved this result. Proceed, then,

$$T\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} + T\begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ 0 \end{pmatrix} + \begin{pmatrix} y_0 \\ 0 \end{pmatrix} =$$
$$\begin{pmatrix} x_0+y_0 \\ 0 \end{pmatrix}$$

So we have that the transformation T preserves addition.

Zero vector is preserved

Clearly we have

$$T\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Disproof of linearity

In order to refute linearity-in other words to show that a transformation is not linear, only a counter-example is required.

If we can find only one case that does not sustain addition, scale-based multiplication or zero vectors in the transformation, we can infer that the transformation is not linear.

For example, consider the transformation

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^3 \\ y^2 \end{pmatrix}$$

We suspect it is not linear. To prove it is not linear, take the vector

$$\mathbf{v} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

then

$$T(2\mathbf{v}) = \begin{pmatrix} 64 \\ 16 \end{pmatrix}$$

but

$$2T(\mathbf{v}) = \begin{pmatrix} 16 \\ 8 \end{pmatrix}$$

so we can immediately say T is not linear because it doesn't preserve scalar multiplication.

Problem set

Given the above, determine whether the following transformations are in fact linear or not. Write down each transformation in the form T:V -> W, and identify V and W. (Answers follow to even-numbered questions):

1. $T\begin{pmatrix} v_0 \\ v_1 \end{pmatrix} = \begin{pmatrix} v_0^2 + v_1 \\ v_1 \end{pmatrix}$
2. $T\begin{pmatrix} v_0 \\ v_1 \end{pmatrix} = \begin{pmatrix} 1 \\ v_0 \end{pmatrix}$
3. $T\begin{pmatrix} v_0 \\ v_1 \end{pmatrix} = \mathbf{0}$
4. $T\begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_0 - v_2 \\ v_1 \end{pmatrix}$

CONCLUSION

A linear transformation is a function that respects each vector space 's underlying (linear) structure. Often known as a linear operator or map is a linear transformation. The range of the transformation may be the same as that of the domain and, if it occurs, it is known as a finalomorphism, or automatic Orphism if invertible. The same field must be used to support the two vector spaces. Linear transformations are beneficial because they retain a vector space structure. So, under some circumstances several qualitative evaluations of a vector space which are the domain of a linear transformation will automatically keep a linear transformation image. For example, the structure immediately indicates that the kernel and image are both subspaces of the linear transformation spectrum (not just subsets). The majority of linear functions can be interpreted in the correct setting as linear transitions. Linear and most geometric transformations of the shift of base forms are, including rotations, reflections, and contractions / dilations. Even more effectively, lineary algebra techniques may use either approximation by linear functions or reinterpretation as linear functions in uncommon vector spaces in some rather not linear functions. An analysis of linear changes shows many relations between mathematical fields.

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