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Effects of Stochastic Analysis on Development and Maturation of an Ecological System

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Abstract- The focus of this research is on the role that mathematics has played over the past century in facilitating the modeling and interpretation of a wide range of biological processes. In exchange, biologists presented mathematicians with a number of challenging tasks. The rise of Biology and Mathematics as a shared multidisciplinary theme owes much to the successful combination of mathematical precision with the biologically nuanced subject matter of biology. As sub-fields of statistical biology, statistical ecology and epidemiology focus on the quantitative analysis of populations of organisms and their physical settings. Statistical modeling is an efficient method for learning about the inner workings and complex behaviors of a wide variety of creatures, including plants, insects, and animals. There has been a lot of enthusiasm for the creation of mathematical models of ecological systems. Theory from the field of biology is used to explain ecological phenomena in a different population setting. Mathematicians have long been interested in population problems, making population biology the most statistically developed bio-science field.

Keywords - Stochastic, Ecological System, Maturation, simulation and interpretation, mathematician

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INTRODUCTION

Communalism, neutrality, predation, and mutualism are all part of an ecological system, as are interactions between organisms, such as harvesting prey and killing predators, and changes in the environment. The primary goal of this chapter is to determine the system dynamics by incorporating random sounds into the model under the assumption of stochastically varying driving forces for species evolution. The existence of fluctuating driving forces in a traditional ecosystem assumes to influence the species' growth: 1,2,3,4 I s I ± time 't' The figure (3.1) is the device where four species are living together with the following assumptions:

(i) The system comprises of a prey (S1), predator (S2), two hosts S3 and S4

(ii) S_1 is prey of S_2 and S_1 is neutral to S_3

(iii) S_2 is predator of S_1 and S_2 is commensal to S_3

(iv) S_3 is host of S_2 and S_3 is mutual to S_4

(v) S_4 is mutual to S_3 and S_4 is neutral to S_2

(vi) Harvesting of S1 and S_2 results in the following stochastic system with 'additive noise'.

In conjunction with its stochastic analysis, the local and global stability of the deterministic model were applied around the coexisting State. The system also derives bionomic harmony. In addition, the results are validated using MATLAB numerical simulations.

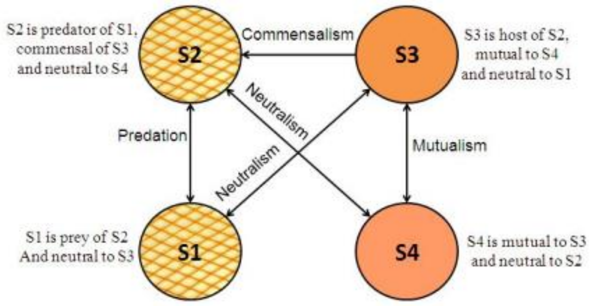

Figure 1 The representation of four species model

Let $x(t), y(t), z(t)$ and $w(t)$ be the population densities of species S_1, S_2, S_3 and S_4 respectively at time instant ' t '. Let a_1, a_2, a_3 and a_4 be the natural growth rates of species S_1, S_2, S_3 and S_4 respectively. Keeping these in view and following, the dynamics of the stochastic system may be governed by the following nonlinear differential equations:

$$\frac{dx}{dt} = a_1 x - a_{11}x^2 - a_{12}xy - q_1E_1x + \beta_1\psi_1(t)$$

$$\frac{dy}{dt} = a_2 y - a_{22}y^2 + a_{21}yx + a_{23}yz - q_2E_2y + \beta_2\psi_2(t)$$

$$\frac{dz}{dt} = a_3 z - a_{33}z^2 + a_{34}zw + \beta_3\psi_3(t)$$

$$\frac{dw}{dt} = a_4 w - a_{44}w^2 + a_{43}wz + \beta_4\psi_4(t)$$

Where a_{11}, a_{22}, a_{33} and a_{44} are self-inhibition coefficients of species S_1, S_2, S_3 and S_4 respectively a_{12} is the interaction coefficient of S_1 due to S_2, a_{21} is the interaction coefficient of S_2 due to S_1, a_{23} is coefficient of commensal for S_2 due to the host S_3, a_{34} is the rate of increase of 3 S due to the interaction with S_4, a_{43} is the rate of increase of S_4 due to the interaction with S_3, K_1, K_2, K_3 and K_4 are the carrying capacities of species S_1, S_2 S_3 and S_4 respectively, where

$K_1 = a_1/a_{11}$; $K_2 = a_2/a_{22}$; $K_3 = a_3/a_{33}$, $K_4 = a_4/a_{44}$.

q_1, q_2 are the catch ability coefficients of species S_1, S_2 respectively. E_1, E_2 are the efforts applied to harvest the species S_1, S_2 respectively. $\beta_i \in R, i = 1,2,3,4$ and $\psi(t) = [\psi_1(t), \psi_2(t), \psi_3(t), \psi_4(t)]$ is a four dimensional Gaussian white noise process $E[\psi_i(t)] = 0$, $i = 1,2,3,4$ and $E[\psi_i(t)\psi_j(t')] = \delta_{ij}\delta(t-t')$, $i, j = 1,2,3,4$, where δ_{ij} and δ are Kronecker and Dirac delta functions respectively. In addition to the variables x , y , z , w the model parameters a_1, a_2, a_3, a_4, a_{11}, a_{22}, a_{33}, a_{44}, a_{12}, a_{21}, a_{23}, a_{34}, a_{34} are assumed to be non-negative constants.

STABILITY ANALYSIS OF THE DETERMINISTIC SYSTEM

In the absence of randomly fluctuating driving forces on the growth of the species, the model system reduces to

$$\frac{dx}{dt} = x\left[(a_1 - q_1E_1) - (a_{11}x + a_{12}y)\right]$$

$$\frac{dy}{dt} = y\left[(a_2 - q_2E_2) - (a_{22}y - a_{21}x - a_{23}z)\right]$$

$$\frac{dz}{dt} = z\left(a_3 - a_{33}z + a_{34}w\right)$$

$$\frac{dw}{dt} = w\left(a_4 - a_{44}w + a_{43}z\right)$$

For the analysis, we assume $a_1 - q_1E_1 > 0$ and $a_2 - q_2E_2 > 0$

INTERIOR STEADY STATE

The interior equilibrium point $G(x^*, y^*, z^*, w^*)$ is the solution of $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = \frac{dw}{dt} = 0$.

$$x^* = \frac{\left[a_{22}(a_1 - q_1E_1) - a_{12}(a_2 - q_2E_2)\right]\gamma_2 - a_{12}a_{23}\gamma_3}{\gamma_2\gamma_4};$$

$$y^* = \frac{\left[a_{11}(a_2 - q_2E_2) + a_{21}(a_1 - q_1E_1)\right]\gamma_2 + a_{11}a_{23}\gamma_3}{\gamma_2\gamma_4};$$

$$z^* = \frac{\gamma_3}{\gamma_2};$$

$$w^* = \frac{\gamma_1}{\gamma_2};$$

where $\gamma_1 = a_4 a_{33} + a_3 a_{43}$;

$\gamma_2 = a_{33}a_{44} - a_{34}a_{43}$;

$\gamma_3 = a_3 a_{44} + a_4 a_{34}$;

$\gamma_4 = a_{11}a_{22} + a_{21}a_{12}$;

Further interior equilibrium point $G(x^*, y^*, z^*, w^*)$ exists if the following inequalities hold: $a_{33}a_{44} > a_{34}a_{43}$ and $\frac{a_{22}(a_1 - q_1E_1)}{a_{12}} > \frac{a_{23}\gamma_3}{\gamma_2} + (a_2 - q_2E_2)$.

LOCAL STABILITY

The community matrix of the system is

$$J = \begin{bmatrix} a_1 - 2a_{11}x - a_{12}y - q_1E_1 & -a_{12}x & 0 & 0 \\ a_{21}y & a_2 - 2a_{22}y + a_{21}x + a_{23}z - q_2E_2 & a_{23}y & 0 \\ 0 & 0 & a_3 - 2a_{33}z + a_{34}y & a_{34}z \\ 0 & 0 & a_{43}w & a_4 - 2a_{44}w + a_{43}z \end{bmatrix}$$

At the interior equilibrium point $G(x^*, y^*, z^*, w^*)$,

$a_1 - q_1E_1 = a_{11}x + a_{12}y$;

$a_2 - q_2E_2 = a_{22}y - a_{21}x - a_{23}z$;

$a_3 = a_{33}z - a_{34}y$;

$a_4 = a_{44}w - a_{43}z$

The community matrix evaluated at $G(x^*, y^*, z^*, w^*)$ is

$$J = \begin{bmatrix} -a_{11}x & -a_{12}x & 0 & 0 \\ a_{21}y & -a_{22}y & a_{23}y & 0 \\ 0 & 0 & -a_{33}z & a_{34}z \\ 0 & 0 & a_{43}w & -a_{44}w \end{bmatrix}$$

The characteristic equation of

$$\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D = 0,$$

where, $A = a_{11}x + a_{22}y + a_{33}z + a_{44}w > 0;$

$$B = \gamma_4 xy + \gamma_2 zw + (a_{11}x + a_{22}y)(a_{33}z + a_{44}w);$$

$$C = (a_{11}x + a_{22}y)zw\gamma_2 + (a_{33}z + a_{44}w)xy\gamma_4 > 0;$$

$$D = xyzw\gamma_2\gamma_4 > 0;$$

$$AB - C = m\gamma_4 xy + mn(m+n) + n\gamma_2 zw > 0;$$

$$ABC - C^2 - A^2D = m^3 n\gamma_2 zw + m^2n^2(\gamma_4 xy + \gamma_2 zw) + mn^3\gamma_4 xy > 0;$$

where $m = a_{11}x + a_{22}y; n = a_{33}z + a_{44}w$.

Now using the Routh- Hurwitz criteria the following theorem is stated:

Theorem 1: The interior equilibrium point $G(x^*, y^*, z^*, w^*)$ is locally asymptotically stable when $A > 0;\quad C > 0;\quad D > 0;\quad AB - C > 0;$ $C(AB - C) - A^2D > 0;\ \mathrm{D}(ABC - C^2 - A^2D) > 0$.

GLOBAL STABILITY ANALYSIS

Theorem 1: The interior equilibrium point $G(x^*, y^*, z^*, w^*)$ is globally asymptotically stable if

$$4a_{21}a_{22}a_{33}a_{44} > a_{12}a_{44}a_{23}^2 + a_{21}a_{22}(a_{34} + a_{43})^2.$$

Proof: To find the conditions for global stability at $G(x^*, y^*, z^*, w^*)$, the Lyapunov function is constructed:

$$V(x,y,z,w) = \left[(x-x^*) - x^*\ln(x/x^*)\right] + l_1\left[(y-y^*) - y^*\ln(y/y^*)\right] + l_2\left[(z-z^*) - z^*\ln(z/z^*)\right] + l_3\left[(w-w^*) - w^*\ln(w/w^*)\right]$$

where l_1, l_2 and l_3 are positive constants.

$$(dV/dt) = \left[(x-x^*)/x\right](dx/dt) + l_1\left[(y-y^*)/y\right](dy/dt) + l_2\left[(z-z^*)/z\right](dz/dt) + l_3\left[(w-w^*)/w\right](dw/dt)$$

$$\begin{aligned}(dV/dt) = {} & (x-x^*)\left[-a_{11}(x-x^*) - a_{12}(y-y^*)\right] \\ & + l_1(y-y^*)\left[-a_{22}(y-y^*) + a_{21}(x-x^*) + a_{23}(z-z^*)\right] \\ & + l_2(z-z^*)\left[-a_{33}(z-z^*) + a_{34}(w-w^*)\right] \\ & + l_3(w-w^*)\left[-a_{44}(w-w^*) + a_{43}(z-z^*)\right]\end{aligned}$$

By choosing $l_1 = \frac{a_{12}}{a_{21}}; l_2 = 1; l_3 = 1$

$$(dV/dt) = -\begin{bmatrix} a_{11}(x-x^*)^2 + (a_{12}a_{22}/a_{21})(y-y^*)^2 + a_{33}(z-z^*)^2 + a_{44}(w-w^*)^2 \\ -(a_{12}a_{23}/a_{21})(y-y^*)(z-z^*) - (a_{34}+a_{43})(y-y^*)(w-w^*) \end{bmatrix}$$

$$= -X^T AX$$

where $X = \begin{bmatrix} x-x^* \\ y-y^* \\ z-z^* \\ w-w^* \end{bmatrix}; A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & \frac{a_{12}a_{22}}{a_{21}} & -\frac{a_{12}a_{23}}{2a_{21}} & 0 \\ 0 & -\frac{a_{12}a_{23}}{2a_{21}} & a_{33} & -\frac{(a_{34}+a_{43})}{2} \\ 0 & 0 & -\frac{(a_{34}+a_{43})}{2} & a_{44} \end{bmatrix}$

The system is globally stable if the derivative of Lyapunav's function V is negative definite, that is if the matrix A is positive definite, that is if the principal minors of A (say) , 1,2,3,4 M i i = are positive. The principle minors are positive if $4a_{21}a_{22}a_{33}a_{44} > a_{12}a_{44}a_{23}^2 + a_{21}a_{22}(a_{34} + a_{43})^2$.

Hence, the system is globally stable in the above parametric space.

BIONOMIC EQUILIBRIUM

It is the combination of biological balance and economic balance. In the biological balance is given by $\dot{x} = \dot{y} = \dot{z} = \dot{w} = 0$. If the net benefit from the sale of the yielded biomass is equivalent to the total cost, then the bionomic equilibrium is reached. The constant harvest cost of species S1 by unit effort should be c1, and of species S2 by unit effort, constant harvest cost. Let p1 be the price per unit biomass for the species S1 and p2 is the price per unit biomass for the same species S2.

The revenue at any time is given by $A(x, y, z, w, E_1, E_2) = (p_1q_1x - c_1)E_1 + (p_2q_2z - c_2)E_2$.

Now if $c_1 > p_1q_1x$ and $c_2 > p_2q_2y$, then the economic rent obtained from the system becomes negative and the system will be closed. Hence for the existence of bionomic equilibrium, it is assumed that $c_1 < p_1q_1x$ and $c_2 < p_2q_2y$.

The bionomic equilibrium $((x)_\infty, (y)_\infty, (z)_\infty, (w)_\infty, (E_1)_\infty, (E_2)_\infty)$ is the positive solution of $\dot{x} = \dot{y} = \dot{z} = \dot{w} = A = 0$.

$$(x)_\infty = c_1/(p_1q_1);$$

$$(y)_\infty = c_2/(p_2q_2);$$

$$(z)_\infty = \gamma_3/\gamma_2;$$

$$(w)_\infty = \gamma_1/\gamma_2;$$

$$(E_1)_\infty = (1/q_1)\left[a_1 - (a_{11}c_1/p_1q_1) - (a_{12}c_2/p_2q_2)\right]$$

$$(E_2)_\infty = (1/q_2)\left[a_2 - (a_{22}c_2/p_2q_2) + (a_{21}c_1/p_1q_1) + (a_{23}\gamma_3/\gamma_2)\right]$$

$(E_1)_\infty > 0$ when $a_1 > a_{11}(x)_\infty + a_{12}(y)_\infty$.

$(E_2)_\infty > 0$ when $a_2 + a_{21}(x)_\infty + a_{23}(z)_\infty > a_{22}(y)_\infty$.

If $(E_1) > (E_1)_\infty$ and $(E_2) > (E_2)_\infty$, then the total cost utilized in harvesting the species population would exceed the total revenues obtained from the ecological system. Hence $(E_1) > (E_1)_\infty$ and $(E_2) > (E_2)_\infty$ cannot be maintained indefinitely. If $(E_1) < (E_1)_\infty$ and $(E_2) < (E_2)_\infty$, then the ecological system is more profitable, and hence in an open access system. This will have an increasing effect on the yielding effort. Hence $(E_1) < (E_1)_\infty$ and $(E_2) < (E_2)_\infty$ cannot be continued indefinitely.

STOCHASTIC ANALYSIS

A natural phenomenon is hard to explain as a deterministic model, and particularly the aquatic ecosystem, which still has the unpredictable environmental fluctuations. The stochastic analysis lets you gain an insight into every ecosystem's dynamics. The deterministic model with random ambient noise effect leads to a stochastic model in which the system parameters oscillate over their mean values. The point of balance that is supposed to be constant therefore now bounces around the average state. The random noise incorporated in the form of additive Gaussian white noise to the model and then any parameter ' ' p of the system reduces to ' $p + \beta\psi(t)$ ' , where $\beta \in R$ is the amplitude of the noise and $\psi(t)$ is the Gaussian white noise process. This analysis is mainly to depict the dynamics of the system around the equilibrium point; therefore we linearize the model using the following perturbation method:

Let $x(t) = u_1(t) + S^*$; $y(t) = u_2(t) + P^*$; $z(t) = u_3(t) + T^*$; $w(t) = u_4(t) + U^*$;

$$\frac{dx}{dt} = \frac{du_1(t)}{dt};\ \frac{dy}{dt} = \frac{du_2(t)}{dt};\ \frac{dz}{dt} = \frac{du_3(t)}{dt};\ \frac{dw}{dt} = \frac{du_4(t)}{dt};$$

Using the respective linear system is identified as

$$\frac{du_1(t)}{dt} = -a_{11}u_1(t)S^* - a_{12}u_2(t)S^* + \beta_1\psi_1(t)$$

$$\frac{du_2(t)}{dt} = -a_{22}u_2(t)P^* + a_{21}u_1(t)P^* + a_{23}u_3(t)P^* + \beta_2\psi_2(t)$$

$$\frac{du_3(t)}{dt} = -a_{33}u_3(t)T^* + a_{34}u_4(t)T^* + \beta_3\psi_3(t)$$

$$\frac{du_4(t)}{dt} = -a_{44}u_4(t)U^* + a_{43}u_3(t)U^* + \beta_4\psi_4(t)$$

Using Fourier transform methods on the linear system

$$A(\omega)\tilde{u}(\omega) = \tilde{\psi}(\omega)$$

Where, $A(\omega) = \begin{pmatrix} A_{11}(\omega) & A_{12}(\omega) & A_{13}(\omega) & A_{14}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) & A_{23}(\omega) & A_{24}(\omega) \\ A_{31}(\omega) & A_{32}(\omega) & A_{33}(\omega) & A_{34}(\omega) \\ A_{41}(\omega) & A_{42}(\omega) & A_{43}(\omega) & A_{44}(\omega) \end{pmatrix}$;

$$\tilde{u}(\omega) = \begin{bmatrix} \tilde{u}_1(\omega) \\ \tilde{u}_2(\omega) \\ \tilde{u}_3(\omega) \\ \tilde{u}_4(\omega) \end{bmatrix};\ \tilde{\psi}(\omega) = \begin{bmatrix} \beta_1\tilde{\psi}_1(\omega) \\ \beta_2\tilde{\psi}_2(\omega) \\ \beta_3\tilde{\psi}_3(\omega) \\ \beta_4\tilde{\psi}_4(\omega) \end{bmatrix};$$

$A_{11}(\omega) = (i\omega + a_{11}S^*)$; $A_{12}(\omega) = a_{12}S^*$; $A_{13}(\omega) = 0$; $A_{14}(\omega) = 0$;
$A_{21}(\omega) = -a_{21}P^*$; $A_{22}(\omega) = (i\omega + a_{22}P^*)$; $A_{23}(\omega) = -a_{23}P^*$; $A_{24}(\omega) = 0$;
$A_{31}(\omega) = 0$; $A_{32}(\omega) = 0$; $A_{33}(\omega) = (i\omega + a_{33}T^*)$; $A_{34}(\omega) = -a_{34}T^*$;
$A_{41}(\omega) = 0$; $A_{42}(\omega) = 0$; $A_{43}(\omega) = -a_{43}U^*$; $A_{44}(\omega) = (i\omega + a_{44}U^*)$;

Equation (3.17) can also be written as $\tilde{u}(\omega) = B(\omega)\tilde{\psi}(\omega)$

Where, $B(\omega) = [A(\omega)]^{-1} = \frac{Adj\,A(\omega)}{|A(\omega)|}$

$|A(\omega)| = R(\omega) + iI(\omega)$

$$R(\omega) = \omega^4 - \omega^2 a_{33}a_{44}T^*U^* + \omega^2 a_{34}a_{43}T^*U^* - \omega^2 a_{22}a_{44}P^*U^* - \omega^2 a_{22}a_{33}P^*T^*$$
$$-\omega^2 a_{11}a_{44}S^*U^* - \omega^2 a_{11}a_{33}S^*T^* - \omega^2 a_{11}a_{22}S^*P^* + a_{11}a_{22}a_{33}a_{44}S^*P^*T^*U^*$$
$$-a_{11}a_{22}a_{34}a_{43}S^*P^*T^*U^* - \omega^2 a_{12}a_{21}S^*P^* + a_{12}a_{21}a_{33}a_{44}S^*P^*T^*U^* - a_{12}a_{21}a_{34}a_{43}S^*P^*T^*U^*$$

$$I(\omega) = -\omega^3 a_{11}S^* - \omega^3 a_{22}P^* - \omega^3 a_{33}T^* - \omega^3 a_{44}U^* + \omega a_{22}a_{33}a_{44}P^*T^*U^*$$
$$-\omega a_{22}a_{34}a_{44}P^*T^*U^* + \omega a_{11}a_{33}a_{44}S^*T^*U^* - \omega a_{11}a_{34}a_{43}S^*T^*U^* + \omega a_{11}a_{22}a_{44}S^*P^*U^*$$
$$+\omega a_{11}a_{22}a_{33}S^*P^*T^* + \omega a_{12}a_{21}a_{33}S^*P^*T^* + \omega a_{12}a_{21}a_{44}S^*P^*U^*$$

$$\tilde{u}_i(\omega) = \sum_{j=1}^{4} B_{ij}(\omega)\tilde{\psi}_j(\omega),\ i = 1,2,3,4$$

The corresponding spectrum is

$$S_{u_i}(\omega) = \sum_{j=1}^{4} \beta_j |B_{ij}(\omega)|^2,\ i = 1,2,3,4$$

The intensities of fluctuations in the variables u_i, $i = 1,2,3,4$ are given by

$$\sigma_{u_i}{}^2 = \frac{1}{2\pi}\sum_{j=1}^{4}\int_{-\infty}^{\infty} \beta_j |B_{ij}(\omega)|^2 d\omega,\ i = 1,2,3,4$$

That is, the variances of u_i, $i=1,2,3,4$ are obtained as

$$\sigma_{u_1}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\beta_1|B_{11}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_2|B_{12}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_3|B_{13}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_4|B_{14}(\omega)|^2\,d\omega\right\};$$

$$\sigma_{u_2}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\beta_1|B_{21}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_2|B_{22}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_3|B_{23}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_4|B_{24}(\omega)|^2\,d\omega\right\};$$

$$\sigma_{u_3}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\beta_1|B_{31}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_2|B_{32}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_3|B_{33}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_4|B_{34}(\omega)|^2\,d\omega\right\};$$

$$\sigma_{u_4}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\beta_1|B_{41}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_2|B_{42}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_3|B_{43}(\omega)|^2\,d\omega+\int_{-\infty}^{\infty}\beta_4|B_{44}(\omega)|^2\,d\omega\right\}$$

where $B_{jk}(\omega)=\dfrac{X_{jk}+iY_{jk}}{R(\omega)+iI(\omega)};\ j,k=1,2,3,4$

$$X_{11}=-\omega^2a_{22}P^*-\omega^2a_{33}T^*-\omega^2a_{44}U^*+\omega a_{22}a_{33}a_{44}P^*T^*U^*-\omega a_{22}a_{33}a_{44}P^*T^*U^*;$$

$$Y_{11}=-\omega^3+\omega a_{22}a_{33}P^*T^*+\omega a_{33}a_{44}T^*U^*+\omega a_{22}a_{44}P^*U^*-\omega a_{34}a_{43}P^*T^*U^*;$$
$$X_{12}=a_{12}S^*\left(\omega^2-a_{33}a_{44}T^*U^*+a_{34}a_{43}T^*U^*\right);Y_{12}=-\omega a_{12}a_{33}S^*T^*-\omega a_{12}a_{44}S^*U^*;$$

$$X_{13}=-a_{12}a_{23}a_{44}S^*P^*U^*;Y_{13}=-\omega a_{12}a_{23}S^*P^*;X_{14}=-a_{12}a_{23}a_{34}S^*P^*T^*;Y_{14}=0;$$

$$X_{23}=-\omega^2a_{23}P^*+a_{11}a_{23}a_{44}S^*P^*U^*;Y_{23}=\omega a_{23}a_{44}P^*U^*+\omega a_{11}a_{23}S^*P^*;$$
$$X_{24}=a_{11}a_{23}a_{34}S^*P^*T^*;Y_{24}=\omega a_{23}a_{34}P^*T^*;X_{31}=0;Y_{31}=0;X_{32}=0;Y_{32}=0;$$

$$X_{33}=-\omega^2a_{11}S^*-\omega^2a_{22}P^*-\omega^2a_{44}U^*+a_{11}a_{22}a_{44}S^*P^*U^*+a_{12}a_{21}a_{44}S^*P^*U^*;$$

$$Y_{33}=-\omega^3+\omega a_{22}a_{44}P^*U^*+\omega a_{11}a_{44}S^*U^*+\omega a_{11}a_{22}S^*P^*+\omega a_{12}a_{21}S^*P^*;$$
$$X_{34}=a_{34}T^*\left(-\omega^2+a_{11}a_{22}S^*P^*+a_{12}a_{22}S^*P^*\right);Y_{34}=a_{34}T^*\left(\omega a_{22}P^*+\omega a_{11}S^*\right);$$

$$X_{41}=0;Y_{41}=0;X_{42}=0;Y_{42}=0;X_{43}=a_{43}U^*\left(-\omega^2+a_{11}a_{22}S^*P^*+a_{12}a_{21}S^*P^*\right);$$
$$Y_{43}=a_{43}U^*\left(\omega a_{22}P^*+\omega a_{11}S^*\right);$$

$$X_{44}=-\omega^2a_{11}S^*-\omega^2a_{22}P^*-\omega^2a_{33}T^*+a_{11}a_{22}a_{33}S^*P^*T^*+a_{12}a_{21}a_{33}S^*P^*T^*;$$

$$Y_{44}=-\omega^3+\omega a_{22}a_{33}P^*T^*+\omega a_{11}a_{33}S^*T^*+\omega a_{11}a_{22}S^*P^*+\omega a_{12}a_{21}S^*P^*;$$

The expressions of population variances can explicitly be written as

$$\sigma_{u_1}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\begin{bmatrix}\beta_1\left(X_{11}{}^2+Y_{11}{}^2\right)+\beta_2\left(X_{12}{}^2+Y_{12}{}^2\right)+\beta_3\left(X_{13}{}^2+Y_{13}{}^2\right)\\+\beta_4\left(X_{14}{}^2+Y_{14}{}^2\right)\end{bmatrix}d\omega\right\};$$

$$\sigma_{u_2}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\begin{bmatrix}\beta_1\left(X_{21}{}^2+Y_{21}{}^2\right)+\beta_2\left(X_{22}{}^2+Y_{22}{}^2\right)+\beta_3\left(X_{23}{}^2+Y_{23}{}^2\right)\\+\beta_4\left(X_{24}{}^2+Y_{24}{}^2\right)\end{bmatrix}d\omega\right\};$$

$$\sigma_{u_3}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\begin{bmatrix}\beta_1\left(X_{31}{}^2+Y_{31}{}^2\right)+\beta_2\left(X_{32}{}^2+Y_{32}{}^2\right)+\beta_3\left(X_{33}{}^2+Y_{33}{}^2\right)\\+\beta_4\left(X_{34}{}^2+Y_{34}{}^2\right)\end{bmatrix}d\omega\right\};$$

$$\sigma_{u_4}{}^2=\frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\begin{bmatrix}\beta_1\left(X_{41}{}^2+Y_{41}{}^2\right)+\beta_2\left(X_{42}{}^2+Y_{42}{}^2\right)+\beta_3\left(X_{43}{}^2+Y_{43}{}^2\right)\\+\beta_4\left(X_{44}{}^2+Y_{44}{}^2\right)\end{bmatrix}d\omega\right\}$$

The behaviour of the system (3.1)-(3.4) with either $\beta_1=0$ or $\beta_2=0$ or $\beta_3=0$ or $\beta_4=0$, can be analysed to measure the effect of noise on selective species.

If $\beta_1=\beta_2=\beta_3=0$, then

$$\sigma_{u_1}{}^2=\frac{\beta_4}{2\pi}\int_{-\infty}^{\infty}\frac{X_{14}{}^2}{R^2(\omega)+I^2(\omega)}d\omega;\ \sigma_{u_2}{}^2=\frac{\beta_4}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{24}{}^2+Y_{24}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;$$

$$\sigma_{u_3}{}^2=\frac{\beta_4}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{34}{}^2+Y_{34}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;\ \sigma_{u_4}{}^2=\frac{\beta_4}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{44}{}^2+Y_{44}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega.$$

If $\beta_1=\beta_2=\beta_4=0$, then

$$\sigma_{u_1}{}^2=\frac{\beta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{13}{}^2+Y_{13}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;\ \sigma_{u_2}{}^2=\frac{\beta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{23}{}^2+Y_{23}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;$$

$$\sigma_{u_3}{}^2=\frac{\beta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{33}{}^2+Y_{33}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;\ \sigma_{u_4}{}^2=\frac{\beta_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{43}{}^2+Y_{43}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega.$$

If $\beta_1=\beta_3=\beta_4=0$, then

$$\sigma_{u_1}{}^2=\frac{\beta_2}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{12}{}^2+Y_{12}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;\ \sigma_{u_2}{}^2=\frac{\beta_2}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{22}{}^2+Y_{22}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;$$
$$\sigma_{u_3}{}^2=\frac{\beta_2}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{32}{}^2+Y_{32}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega=0;\ \sigma_{u_4}{}^2=\frac{\beta_2}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{42}{}^2+Y_{42}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega=0.$$

If $\beta_2=\beta_3=\beta_4=0$, then

$$\sigma_{u_1}{}^2=\frac{\beta_1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{11}{}^2+Y_{11}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;\ \sigma_{u_2}{}^2=\frac{\beta_1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{21}{}^2+Y_{21}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;$$

$$\sigma_{u_3}{}^2=\frac{\beta_1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{31}{}^2+Y_{31}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega=0;\ \sigma_{u_4}{}^2=\frac{\beta_1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_{41}{}^2+Y_{41}{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega=0.$$

Analytical evaluation of the population variance is difficult, but it can be evaluated numerically for different set of values of parameters.

CONCLUSION

Mathematics modeling is a powerful tool in the fields of ecology and epidemiology. Appropriate mathematical models can be used to study a wide range of ecological phenomena, such as prey-predator interactions, intraspecific competition, the transmission of infectious diseases, and their subsequent vaccination, treatment, and other controls. Multispecies ecosystems are complex dynamical systems because they feature many different species or components that interact with one another in a wide variety of ways. The dynamic behaviors of several plant, insect, and animal species have contributed greatly to the advancement of dynamic models of complex ecosystems (both deterministic and stochastic). Deterministic dynamic models are used to describe system properties such as stability, instability, bifurcations, and catastrophic state shifts. The study examines the dynamics of change through time, the perseverance of stable states, and the transformation of stable states by parametric change (bifurcation), among other topics. A key concept in ecosystem analysis is that of stability. Many instances of ecological interaction with the word "instability" are possible. As a statistically formalized version of the more general scientific concept of a deterministic process, the idea of the dynamical system is extremely important. The present and the laws governing the evolution of many physical, chemical, biological, ecological, economic, and even social systems allow for predictions of their future states. Assuming these rules do not evolve over time, the initial conditions would fully characterize the system's behavior. Because of this, we can define a dynamical system as something that has both a collection of possible states and a law governing the evolution of that state through time. We then define a dynamical system by combining our previous definitions of its constituent parts.

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