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A Study of Realistic Approach of (M/M/S):(∞/FCFS) Model Over (M/M/1):(∞ /FCFS) Model for Bank ATM System

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Abstract – In this paper we use (M/M/1) : (∞/FCFS) model and (M/M/S): (∞/FCFS) model for the study of waiting lines in Bank ATM having single ATM machine. Banks usually provides one ATM machine in every branch of a particular area of a city. But, one ATM would not be sufficient to serve a long queue. Now a day's people don't have enough time to spend in a long queue. They have their own jobs to do. Furthermore, if ATM machine run out of service due to some technical problem, then, it as well creates a big problem for customers. In this paper, we will evaluate different performance measures of above-mentioned models and compare them. This will help us in study of realistic approach of (M/M/S):(∞/FCFS) model over (M/M/1):(∞ /FCFS) model.

Keywords – Bank ATM, (M/M/1): (∞/FCFS) model, (M/M/S): (∞/FCFS) model, Queue, Performance measures.

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(I) INTRODUCTION:

Queuing Theory is a branch of Operation research, which involves the mathematical study of queues or waiting lines [4].Queues are not only for human beings seeking for service. It also includes airport seeking to land at busy airport, Cars waiting in Traffic light to turn green, Ships to be unloaded, machine parts to be assembled etc. In our daily life, we see a queue at Bank ATM's, queue in Schools colleges fee window, in Hospitals, Queue at medical shops, at cinema windows, at Petrol pumps etc. We know that When Current demand for any service exceeds the current capacity to provide that service then queues forms.

In Bank ATM, Customers arrive in random manner and the time taken i.e. service time (for Transaction, balance inquiry, etc.) by them is also random. Let there is only one ATM in any branch of Bank of a particular area, which means the server is exactly one. Furthermore, ATM is an example of infinite queue length.

So, we can apply here (M/M/1):(∞ /FCFS) model;

It is a Probabilistic Queuing model .The first three characters were introduced by D.G .Kendall in 1953[1].Later, A. Lee. in 1966 [2] added fourth & fifth character.

Here:-

1) First M denotes exponential distribution of arrival time or Poisson's distribution of arrivals [3].

2) Second M denotes exponential service time distribution.

Here letter M is used to represent Markovian property of the exponential process [3].

3) 1 represent single server/service station [3].

4) ∞: Infinite calling Population [3].

5) FCFS: The service discipline is first come first serve.

Now, let in any random day of a week,

II(a). If λ=Average rate of arrival of customer in Bank ATM=30 Customer/hour i.e $\lambda = \frac{1}{2}$ Customer/minute, μ=mean service rate of customer in Bank ATM = $\frac{3}{2}$ Customer/minute.

Then,

1. Probability of ATM machine being busy=Traffic intensity $=\rho=\frac{\lambda}{\mu}=\frac{1}{3}=0.333333$ approx., where $\frac{\lambda}{\mu}<1$.

2. Expected number of Customers in system (waiting + being served) = $L_s=\frac{\lambda}{\mu-\lambda}=\frac{1}{2}=0.5$ Customers.

3. Expected number of Customers in Queue $=L_q=\frac{\rho^2}{1-\rho}=\frac{1}{6}=0.1666666$ customer.

4. P_0=Probability of zero customer in ATM $=1-\rho=\frac{2}{3}=0.666666$ approx.

5. Expected waiting time for a Customer in queue $=W_q=\frac{L_q}{\lambda}=\frac{1}{3}=0.333333$ minute approx.

6. Average waiting time of a customer in the system (including waiting + service time) $=W_s=\frac{L_s}{\lambda}=1$ minute.

7. Average waiting time in queue for those who actually wait = $(W/W>0)=\frac{1}{\mu-\lambda}=1$ minute.

8. Expected length of non-empty queue $=(L/L>0)=\frac{1}{1-\rho}=\frac{3}{2}=1.5$

9. Probability of having n customer in Bank ATM $=P_n=(1-\rho)\rho^n$.

So,

If we fix n=10.Then,

$$P_n=0.00760718158 \text{ approx}$$

II(b). Now we apply (M/M/S):(∞/FCFS) model for same λ & μ but now we take s=2.

In this model, customers also arrive randomly in Poisson's manner. Only difference is that there are fixed number of servers/service stations arranged in parallel, and customer is liberated to go to any of the free stations for his service. The service time at each station is identical and follows same exponential distribution law.

Since, $\lambda=30$ Customer/hour i.e. $\lambda=\frac{1}{2}$ Customer/minute, $\mu=\frac{3}{2}$ Customer/minute and s=2.Then,

1. Probability of ATM machine being busy=Traffic intensity $=\rho=\frac{\lambda}{s\mu}=\frac{1}{6}=0.166666$ approx.

2. P_0=Probability of zero customer in ATM $=\left\{\sum_{n=0}^{s-1}\frac{(s\rho)^n}{n!}+\frac{s^s}{s!}\left(\frac{\rho^s}{1-\rho}\right)\right\}^{-1}=\frac{5}{7}=0.714285714$

3. Expected number of Customers in Queue $=L_q=\frac{\rho}{(1-\rho)^2}\left[\frac{(s\rho)^s}{s!}\right]P_0=$0.00952380952 customers.

4. Expected number of Customers in system (waiting + being served) $=L_s=L_q+\frac{\lambda}{\mu}=0.342857143$ Customers.

5. Expected waiting time for a Customer in queue = $W_q=\frac{L_q}{\lambda}=0.019047619$ minute approx.

6. Average waiting time of a customer in the system (including waiting + service time) $=W_s=W_q+\frac{1}{\mu}=0.685714286$ minute approx.

7. Average waiting time in queue for those who actually wait = $(W/W>0)=\frac{1}{s\mu-\lambda}=\frac{2}{5}=0.4$ minute.

8. Expected length of non-empty queue = $(L/L>0)=\frac{1}{1-\rho}=\frac{6}{5}=1.2$

9. Probability of having n customer in

$$ATM=P_n=\begin{cases}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0, n=0,1,2,\ldots\ldots\ldots s-1\\ \frac{1}{s!}\frac{1}{s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0, if\ n=s,s+1,s+2,\ldots\ldots\ldots\end{cases}$$

If we fix n=10.Then,

$$P_{10}=0.00079256265 \text{ approx}$$

(II) (c) Comparison Table:

S. No.	Performance measures	(M/M/1):(∞ /FCFS) model Here S=1, $\lambda=\frac{1}{2}$ Customer/minute, $\mu=\frac{3}{2}$ Customer/minute.	(M/M/S):(∞/FCFS) model. Here S=2, $\lambda=\frac{1}{2}$ Customer/minute, $\mu=\frac{3}{2}$ Customer/minute.
1.	ρ	0.333333333 approx.	0.166666666 approx.
2.	L_s	0.5 Customer approx.	0.342857143 Customer approx.
3.	L_q	0.166666666 Customer approx.	0.00952380952 Customer approx.
4.	P_0	0.666666 approx.	0.714285714 approx.
5.	W_q	0.333333 minute approx.	0.019047619 minute approx.
6.	W_s	1 minute	0.685714286 minute approx.
7.	W/W>0	1 minute	0.4 minute approx.
8.	L/L>0	1.5 approx.	1.2 approx.
9.	P_n(for fix n=10)	0.00760718158 approx.	0.00079256265 approx.

(III) CONCLUSION:

Above comparison shows the realistic approach of (M/M/S):(∞/FCFS) model over (M/M/1):(∞ /FCFS) model. So, if Bank uses (M/M/S) :(∞/FCFS) model instead of(M/M/1):(∞ /FCFS) model then they will get rid of Problem of losing their customer due to long wait in queue. So banks need to improve their service facility/service time by introducing at least two ATM machines in every branch of the bank .The number of ATM machines could be increased as per the population of the particular area in which Bank ATM is present.

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