

A Study of Reliability Models' in Operations Research

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Abstract - The study is to the component and process reliability is the basis of many efficiency evaluations in Operations Management discipline. For example, in the calculation of the Overall Equipment Effectiveness (OEE) introduced it is necessary to estimate a crucial parameter called availability. This is strictly related to reliability. Still as an example, consider how, in the study of service level, it is important to know the availability of machines, which again depends on their reliability and maintainability. Reliability is defined as the probability that a component (or an entire system) will perform its function for a specified period of time, when operating in its design environment. The elements necessary for the definition of reliability are, therefore, an unambiguous criterion for judging whether something is working or not and the exact definition of environmental conditions and usage. Then, reliability can be defined as the time dependent probability of correct operation if we assume that a component is used for its intended function in its design environment and if we clearly define what we mean with "failure". For this definition, any discussion on the reliability basics starts with the coverage of the key concepts of probability.

Keywords - Reliability Models', Operations Research, Equipment Effectiveness, environmental conditions

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INTRODUCTION

Several models are considered, with an emphasis on how fluctuations in the surrounding environment can affect a system's dependability. The failure structure of all components is affected by the varying environmental situations in which complex hardware and software systems function. We will be having an expository discussion, reviewing both completed and ongoing works by the authors. To guide future studies, we will provide an overview of continuous and discrete-time models and the statistical analyses of each.

In this explanatory work, we take into account sophisticated dependability models that function in a dynamic environment whose parameters are subject to random variation. The model's complexity stems from the many ways in which its parts interact with one another as a result of a shared environmental process, in addition to the sheer quantity of these parts. An airplane, for instance, is a complicated device made up of several parts, the failure structure of each of which is highly dependent on the specific environmental circumstances it is exposed to while in flight. Takeoff and landing involve noticeably different vibration, air pressure, temperature, and other conditions. These unpredictable environmental fluctuations affect the lives and depend abilities of components. In addition, because they share a common operating condition, the

parts have interdependent lives. The same may be said for software. As an illustration, a complex system like an airline reservation system may nevertheless have flaws or bugs that cause it to malfunction. In this scenario, the operational profile, or how the user typically interacts with the system, is a crucial factor in determining the software's reliability. The system's reliability and the likelihood of individual module failure are both affected by the system's random operating sequence. The operational profile in this context serves as the software system's unpredictable environment. In this study, "environment" refers to any set of conditions that influences the stochastic nature of the model studied. There are several examples in the literature of the concept of a "environmental" process being employed for different ends.

MEASURES OF SYSTEM PERFORMANCE

Here are a few of the most relevant system performance indicators are:

(a) Reliability:

It is the likelihood that a system will function within acceptable efficiency constraints for some time period. The dependability function of a system can

be mathematically derived from its expected lifetime, denoted by the random variable T at time t is

$$R(t) = P(T \geq 0) = \int_t^{\infty} f(u) du.$$

$$= \begin{cases} 1 - F(t) & t > 0 \\ 1 & t \leq 0 \end{cases}$$

where $f(\cdot)$ and $F(\cdot)$ are pdf and edf of T respectively. There are three important measures connected with reliability.

(i) Point wise availability: Readiness to operate is the likelihood that a system can function within acceptable parameters at a given time. Availability expressed symbolically via points is

$$A(t) = P[X(t) = 1]$$

where $X(t)$ is a binary variable that can take on the values 1 for system operation and 0 for non-operation at time t .

(ii) Interval availability: It is the percentage of a specific time period in which the system is expected to function within the limits. Its limiting value is the inherent availability, which is also known as the system's efficiency.

(iii) Steady-State or Asymptotic availability: It is the likelihood that the system will perform as expected over time. Constant supply is a metaphor for is

$$A(\infty) = \lim_{t \rightarrow \infty} A(t)$$

(b) Interval Reliability:

It is the likelihood that, beginning at some time t , the system will continue to function normally for the next $t + x$. Interval reliability is a symbol of

$$R(t, x) = P[X(u) = 1, \forall u, t \leq u \leq t + x]$$

(c) Instantaneous Failure Rate $[r(t)]$ (sometimes called hazard rate): It is defined as the limit of the failure rate when the interval length approaches to zero i.e.

$$r(t) = \lim_{h \rightarrow 0} \frac{F(t+h) - F(t)}{h[1 - F(t)]}$$

$$= \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)}$$

For small positive h , $r(t)$, $h > 0$ (h) is the likelihood that a component would fail between the provided timestamps ($t, t + h$), assuming that it has functioned as expected up to the supplied timestamps t , i.e.

$$P[t < T \leq t+h | T > t] = \frac{P[t \leq T \leq t+h]}{P[T > t]}$$

$$= \frac{f(t) \cdot h}{R(t)} = r(t) \cdot h$$

The aforementioned relation, often known as $r(t)$ or the intensity function or the force of mortality, can be written in the following ways:

$$F'(t) = f(t) = r(t) \cdot R(t)$$

$$R(t) = \overline{F(t)} = 1 - F(t)$$

The various functions $r(t)$, $R(t)$ and $F(t)$ are related by

$$\int_t^{\infty} f(u) du = \overline{F}(t) = \overline{R}(t) = \exp \left\{ - \int_0^t r(x) dx \right\}$$

$$F(t) = \int_0^t f(u) du = 1 - R(t) = 1 - \exp \left\{ - \int_0^t r(x) dx \right\}$$

$$f(t) = r(t) \exp \left\{ - \int_0^t r(x) dx \right\}$$

The fatigue failure of materials and the lifespan of electrical and mechanical components have been modeled using many different distributions. Negative exponential, gamma, Weibull, lognormal, etc. are some of the most common laws of failure.

(d) Mean Time to Failure (MTTF):

The probability that a random event will occur Mean Time to Failure (MTTF) is another name for Mean Time to Systems Failure (MTSF) or Mean Time to First Failure (MTFF). Mathematically, it is

$$M.T.T.F. = E(T) = \int_0^{\infty} uF(u) du = - \int_0^{\infty} u d\overline{F}(u)$$

$$= \left[-u\overline{F}(u) \right]_0^{\infty} + \int_0^{\infty} \overline{F}(u) du$$

$$= \int_0^{\infty} R(u) du$$

where $f(\cdot)$, $F(\cdot)$ and $R(t)$ are functions representing the probabilities of occurrence, the sum of occurrences, and the predictability of the random variable T at time t .

If $\bar{R}(s)$ is Laplace transform of $R(t)$, then

$$\begin{aligned} \text{M.T.T.F. } \lim_{s \rightarrow 0} \int_0^{\infty} e^{-st} R(t) dt &= \lim_{s \rightarrow 0} \bar{R}(s) \\ &= \lim_{s \rightarrow 0} \frac{1}{s} - \bar{F}(s) = \frac{\lim_{s \rightarrow 0} 1 - s\tilde{F}(s)}{s} \\ &= \left[-\frac{d}{ds} \tilde{F}(s) \right]_{s=0} \end{aligned}$$

where $\bar{F}(s)$ and $\tilde{F}(s)$ denote the Laplace and Laplace-Stieltjes transforms of distribution function $F(\cdot)$.

(e) Mean Sojourn Time:

Mean sojourn time or mean survival time is the average amount of time a system spends in a given state before changing states. The average Sojourn Time in any given state, S_i , is T_i , where S_i is the state in question.

$$\mu_1 = \int_0^{\infty} P(T > t) dt$$

(f) Expected Number of Repairs by Repair Facility:

Let $V(t)$ be a random variable reflecting the number of repairs by repair facility in the interval $(0,t)$. If we assume that $V(t)$ is independent and normally distributed, then the expected number of repairs in $(0,t)$ is $EV(t)$, and the expected number of repairs per unit time in the long run is given by

$$E(V) = \lim_{t \rightarrow 0} \frac{E\{V(t)\}}{t}$$

CONTINUOUS TIME MODELS WITH INTRINSIC AGING

In their fascinating model of stochastic component reliance, create a shared environment that fluctuates at random, thereby introducing stochastic dependence on all system components. The basic premise of this model is that the environment in which a given component operates has a significant impact on the rate at which that component ages or degrades. They propose building an intrinsic clock, which would tick in a slightly different way depending on the surrounding conditions, in order to determine the device's true age. The environment is modeled as a semi-Markov jump process, and intrinsic age is modeled as the time-varying cumulative hazard encountered by the device while it operates in the unpredictable environment. This is a sleek option since it assumes an exponential distribution with parameter 1 for the intrinsic lifetime of any given device. Of course, there are a variety of

additional ways to build an intrinsic clock to determine the intrinsic age. Studying reliability and maintenance models for devices with multiple interconnected parts is another application of the random environment model. Due to being exposed to the same conditions, the individual parts of such complicated devices have unpredictable lifespans.

Intrinsic Aging in a Fixed Environment

Gaver and Arjas also make use of the idea of random hazard functions. The intrinsic aging model proposed by C nlar and Ozekici is examined in further depth by C nlar et al. to characterize the multivariate increasing failure rate (IFR) and new better than used (NBU) life distributions, as well as the factors that lead to associated component lifetimes. Additional models with multicomponent replacement policies were discussed in Shaked and Shanthikumar. The consequences of a random environment on the reliability of a system made up of components that all operate in the same environment are discussed by Lindley and Singpurwalla. They postulate that components have exponential life distributions in each environment, and that the environment remains stable throughout time despite its unpredictable beginning. Lefevre and Malice investigate this model further by determining, for various partial orderings of the probability distribution on the environmental state, the orderings of the number of working components and the dependability of k-out-of-n systems. Lefevre and Milhaud address the relationship between the lives of components exposed to a stochastically changing environment. Multivariate distributions in models where the failure rates of components are affected by a changing environment are also discussed by Singpurwalla and Youngren. C nlar and Ozekici describe intrinsic aging for a large model with m components using a simple connection.

CONCLUSION

The reliability is a vital factor in the efficiency of equipment, keeping all these facts in view, a necessity arose to develop reliability models with a view to help designers to predict reliability at the design stage. As a matter of fact, in the field of defence, the entire character of military operations has progressively changed through electronics, which plays a dominant role in practically every sphere of weapons and equipment's with ever increasing sophistication. High degree of reliability under operating conditions is the urgent need of military equipment's. This in turn demands high, often extreme degree of quality and reliability of piece part, components and circuit design to ensure that a given order of reliability of the system as a whole is maintained for a long period. However, the theory of reliability is not applied in present day practice on such a wide scale as it unquestionably deserves. The diversity of practical problems demands various methods of calculating the

reliability of various complex systems. Prediction of highly reliable equipment's requires the development of new methods of designing such equipment's and new technology for producing it, as well as scientific methods of its operation with appearance of complicated automatic systems there arose the need to consider reliability as a technical parameter and to measure it numerically. All this has led to the situation that reliability has become one of the most important technical problems and reliability theory has become an independent scientific discipline. In the fast developing countries, like India, recent technologies in the electrical and electronic equipment's are being used on mass scale in many industries. Industries are trying to introduce more and more automation in their industrial processes in order to meet the ever-increasing demands of society.

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