Journal of Advances in Science and Technology
Vol. 18, Issue No. 2, September–2021, ISSN 2230-9659

A Study Of Mathematical Models Towards Ecological And Epidemiological Process

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Abstract - As sub-fields of statistical biology, statistical ecology and epidemiology focus on the quantitative analysis of populations of organisms and their physical settings. Population biology, unsurprisingly, is a quantitative field of study. Population decline, growth, extinction, dispersal, immigration effects, emigration, population mixing, age-structure effects, etc. are all factors we will be interested in considering, clarifying, and forecasting as this discussion progresses. After years of intense collaboration between scientists working in different disciplines, the modern field of nonlinear science finally came into being a few decades ago. Different non-linear effects arise from mathematical biology's model equations (e.g. hysteresis, structural instability, dissipatory structures, dynamic disorder, etc.). Over the past two decades, nonlinear dynamics has played a crucial role in the modeling of a wide range of biological and physiological processes. Stability, periodicity, stochasticity bifurcation, fluctuations, and pattern forming are just some of the characteristics of the system that have been studied and determinized, along with stochastic methods developed to address them, because of the critical importance of non-linear dynamic models of complex ecosystems and epidemiological systems. We have also looked at the tools and concepts of thermodynamics and statistical mechanics as they pertain to the investigation of ecologically complex systems.

Keywords - Ecological, Epidemiological Process, Statistical Models, Nonlinear dynamics, stability, periodicity.

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INTRODUCTION

Analysis and modeling of such biologically complicated processes from a nonlinear dynamic perspective are the focus of this research. The time and time can be continuous in the dynamic model. Different types of models, including deterministic, stochastic, thermodynamic, and stochastic ones, are used and analyzed here. Deterministic dynamic models based on differential or differential equations can provide explanations for phenomena like as stability and instability, periodicity, bifurcations, and catastrophic changes in the state of a system. The principles of stability are significant in the investigation of the constructions and operations of complex biological and green systems. The purpose of this paper is to analyze some dynamically complex problems associated with stability and instability, periodicity and bifurcation, and the emergence of complex ecological and epidemiological systems due to spread. Complex behavior emerges in large part due to the dynamic behavior of system probability and stochasticity. Physicochemical, biochemical, social, and technological processes may not always be able to reliably predict the future behavior of the system, as in the case of deterministic dynamical models based on differential or equation. When examining a system, the probability or stochastic definition of the system is the most natural way to approach it when dealing with unexpected, fluctuating occurrences. It is possible for the behavior of a system to be imprevisible because to the many body elements of the system (that is, due to the countless number of components, such as molecular cells, species, etc.) and the impact of a fluctuating random environment on systems.

1.1 RELEVANT MATHEMATICAL BACKGROUND

1. Dynamical System

The idea of a dynamic system is a mathematical formalization of the more broad scientific idea of a deterrent mechanism. Many other physical, chemical, biological, environmental, economic, or social structures can be predicted based on our understanding of their current state and the rules governing their evolution. A system's behavior can be considered fixed at the outset if these laws don't evolve over time. Accordingly, the idea of a dynamic system entails both a set of possible states and a rule for how those states evolve over time. After isolating each component, we formally define the system's dynamic behavior.

State Space: The points of such set X are characterised in all possible system status. This collection is known as the machine state space. Indeed, it should suffice to indicate point x to point X not only for a definition of the system's current "position," but also for a determination of its growth.

Time: The creation of a dynamic system means that the system has changed with time t to T, where T is set. Two types of systems are available: continuous (real) T = < 1, and discrete (integer) time T = Z. The first type of system is known as continuous time systems, while the second type is known as discrete time systems.

Evolution Operator: An evolution law deciding the state of the system at time T is the main component of a dynamic system, given that the initial state x0 is understood. The most common way of determining the creation is to assume that a map μt is defined for the given t by T in the state area X.

$$\phi^t : X \to X$$

which transforms as initial state x0 ∈ X to some state xt ∈ X at time t:

$$x_t = \phi^t x_0$$

The map φ t is often called the evolution operator of the system.

Definition: A dynamical system is a triplet (T, X, φt), where T is a time set, X is a state space, and φ t : X → X is a family of evolution operators parametrized by t ∈ T.

Differential equations are the most common way to describe an ongoing dynamic time system. Assume that the system's state space is X = < n with coordinates (x1, x2, ..., xn). The rule of creation of the system is most often indirectly specified in terms of the speed Únxi as the coordinate function (x1, x2, ..., xn) as

$$\dot{x}_i = f_i(x_1, x_2, ..., x_n), \qquad i = 1, 2, ..., n$$

or in the equivalent vector form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

where the vector-valued function $\mathbf{f} : \Re^n \to \Re^n$ is supposed to be sufficiently differentiable (smooth).

2. Equilibrium Points and Periodic Points

Equilibrium Points: If a solution x(t) of continuous dynamical system x˙ = f(x) be such that x(t) = x ∀ t ∈ < i.e. if x(t) is constant, then x is called an equilibrium point. These constants are obviously solutions of the system of n equations

$$f_i(x_1, x_2, ..., x_n) = 0, \qquad i = 1, 2, ..., n$$

Balance points are often called static or stationary resting points or fixed points or critical points. As the rate of change of state variables at stationary points is zero, the system will remain in that state for ever until the system enters the state represented by the stationary point.

Periodic Points: Percentage point/solution of differential equations method x = f(x) at the interval of life for any T > 0 is a non-constant point that satisfies x (t + T) = x(t) = t The minimum T > 0 value is referred to as the solution time.

3. Limit Cycles

Let x(t) be a solution of the continuous dynamical system x˙ = f(x) t > 0 and satisfying the initial condition x(0) = x. A point $y \in \Re^n$ is called ωlimit point of x if there exists a sequences of times {tn} tends to +∞ as n → ∞ such that

$$\lim_{t \to \infty} x(t_n) = y$$

The set of all ω-limit points of x is called the ω-limit set of x denoted by ω(x). A closed orbit γ is said to be limit cycle if γ is a subset of ω(x) for some x that does not lie in γ.

4. Linearisation and Characteristic Equation

Consider an autonomous predator-prey system of the form

$$\frac{dN}{dt} = F(N, P), \qquad \frac{dP}{dt} = G(N, P)$$

where N is the number of prey and P is the number of predator. The equations for the equilibria (N^*, P^*) are found by setting the right hand sides equal to zero,

$$F(N^*, P^*) = 0 = G(N^*, P^*)$$

To determine the stability of equilibrium, we introduce new variables that measure the deviation about the equilibrium,

$$x(t) = N(t) - N^*, \qquad y(t) = P(t) - P^*$$

We then linearise about the equilibrium point,

$$\frac{dx}{dt} = \left[\frac{\partial F}{\partial N}\right]_{(N^*,P^*)} x + \left[\frac{\partial F}{\partial P}\right]_{(N^*,P^*)} y$$
$$\frac{dy}{dt} = \left[\frac{\partial G}{\partial N}\right]_{(N^*,P^*)} x + \left[\frac{\partial G}{\partial P}\right]_{(N^*,P^*)} y$$

This last set of equation can be written as

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = JX$$

Where the different partial derivatives are the aij. Matrix J of Jacobi is called the ecological community-matrix. It captures the power of relationships in a balanced group. The solution is now being pursued.

$$x(t) = x_0 e^{\lambda t}, \qquad y(t) = y_0 e^{\lambda t}$$

With this substitution, the linearised system of equation reduces to

$$\lambda x_0 = a_{11}x_0 + a_{12}y_0, \qquad \lambda y_0 = a_{21}x_0 + a_{22}y_0$$
$$\text{or} \begin{pmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The simplest systematic way for solving the above equation for x0 and y0 is to use Cramer's rule

$$x_0 = \frac{\begin{vmatrix} 0 & a_{12} \\ 0 & a_{22} - \lambda \end{vmatrix}}{\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix}}, \qquad y_0 = \frac{\begin{vmatrix} a_{11} - \lambda & 0 \\ a_{21} & 0 \end{vmatrix}}{\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix}}$$

We've got a problem, however. Each numerator has a determinant of zero. Except that the denominator is identical to zero, we must consider the trivial solution. In order to prevent this, we need it

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$$

By expanding the determinant, we obtain the characteristic equation

$$\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0$$

5, Routh-Hurwitz Stability Criteria

For any m × m matrix A, the characteristic equation for the square matrix A is an mth order polynomial equation

$$| A - \lambda I | = \lambda^m + a_1\lambda^{m-1} + a_2\lambda^{m-2} + \ldots.. + a_m = 0$$

The parameters Routh-Hurwitz are both formal and general, and restrict the coefficients A1, a2,am which are necessary and adequate for ensuring that all their proper values are found on the left half of the complex plane.. Explicitly Routh-Hurwitz stability conditions for m = 2, 3, 4 and 5 are as follows

$$m = 2, \quad a_1 > 0, \quad a_2 > 0$$
$$m = 3, \quad a_1 > 0, \quad a_3 > 0, \quad a_1a_2 > a_3$$
$$m = 4, \quad a_1 > 0, \quad a_3 > 0, \quad a_4 > 0, \quad a_1a_2a_3 > a_3^2 + a_1^2a_4$$
$$m = 5, \quad a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_4 > 0, \quad a_5 > 0, a_1a_2a_3 > a_3^2 + a_1^2a_4$$
$$(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 - a_1^2a_4) > a_5(a_1a_2 - a_3)^2 + a_1a_5^2$$

6. Bifurcation Theory

A slow change in the input always occurs, that means the output is a continuous function of the input. The result is a small change in the output. This doesn't always happen. Consider the heating process for a water kettle. In the vicinity of the boiling point there may be a slight rise in heat, from liquid to vapour, and this changes in quality. The theory of bifurcation is the analysis of the changes in the consistency of the structure. Consider a device based on the number (µ1, µ2, ..., µn) of parameters. We assume the system to be autonomous and the set of equations describing the system can be written as

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \bar{\mu})$$

Where x and $\bar{\mu}$ are xi and µi column vectors, respectively. Our goal is to evaluate balance states and equilibrium as the $\bar{\mu}$ value is modified. If there is a qualitative change in the solution with those values of $\bar{\mu}$(=$\bar{\mu}$0), then $\bar{\mu}$0 is a point of bifurcation. The balance is determined by the resolution of f(x, $\bar{\mu}$)=0, respectively. The f(x, $\bar{\mu}$) = 0 solution describes the surface of the (x, $\bar{\mu}$) field. A small change in $\bar{\mu}$ leads to a small change in x if the surface is smooth and nothing drastic occurs. If however the surface is collapsed, the collapse will lead to a small change in $\bar{\mu}$, which is precisely the point of bifurcation.

Hopf Bifurcation

Suppose that the system

$$\frac{dx_1}{dt} = f_1(x_1, x_2, \mu), \qquad \frac{dx_2}{dt} = f_2(x_1, x_2, \mu)$$

has an equilibrium state at (0, 0, μ0). The Jacobian matrix evaluated at the equilibrium point,

$$J = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}_{(0,0,\mu_0)}, \quad a_{ij} = \frac{\partial f_i}{\partial x_j}$$

has eigenvalues $\alpha(\mu) \pm i\beta(\mu)$ with $\alpha(\mu_0) = 0,\ \beta(\mu_0) \neq 0$. Further

$$\left[\frac{d\alpha(\mu)}{d\mu}\right]_{(\mu=\mu_0)} \neq 0$$

Then a non-trivial periodic solution exists in the vicinity of (0, 0, μ0). This theorem is known as the Hopf forcation theorem and the Hopf forcation solution.

7. Lyapunov Functions and Stability Criteria

Consider the equation

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}), \quad t \geqslant t_0, \quad \mathbf{x} \in D \subseteq \Re^n$$

and Supposed to be trivia-based solution, i.e., f (t, 0) = 0, t > t0, 0 respectively D. Consider the scalar V (t, x) function, which can constantly be separated into [t0, as opposed to) a function named as standardD. In addition, at x = 0 = D, V (t, 0) = 0. V (t, x) is not in some cases directly dependent on 0 t 0. In short V we can write (x).

Definition: The function $V(\mathbf{x})$ (with $V(t, \mathbf{0}) = 0$) is called positive (negative) definite in D if $V(x) > 0 (< 0)$ for all $\mathbf{x} \in D$ with $\mathbf{x} \neq \mathbf{0}$.
Definition: The function $V(\mathbf{x})$ (with $V(t, \mathbf{0}) = 0$) is called positive (negative) semi-definite in D if $V(x) \geq 0 (\leq 0)$ for all $\mathbf{x} \in D$ with $\mathbf{x} \neq \mathbf{0}$.

If a function V (t, x) depends explicitly on 0 t 0 , these definitions are adjusted as follows:

Definition: The function V (t, x) is called positive (negative) definite in D if there exists a function W(x) with the properties: W(x) is defined and continuous in D, W(0) = 0, 0 < W(x) ≤ V (t, x)(V (t, x) < W(x) ≤ 0) for x ≠0, t ≥ t0.

To define semi-definite functions V (t, x) we replace < (>) by ≤ (≥).

Definition: The orbital derivative Lt of the function V (t, x) in the direction of the vector field f(t, x), is given by

$$L_t V = \frac{\partial V}{\partial t} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(t, \mathbf{x})$$

where $\mathbf{x} = (x_1, x_2, ..., x_n)$ and $\mathbf{f} = (f_1, f_2, ..., f_n)$ and $\mathbf{x}$ is a solution of the equation

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}), \quad t \geqslant t_0, \quad x \in D \subseteq \Re^n$$

Theorem: Consider the equation x˙ = f(t, x) and if it is possible to find a function V (t, x), defined in a neighbourhood of x = 0 and positive definite for t ≥ t0 with the orbital derivative negative semi-definite, the solution x = 0 is stable in Lyapunov sense.

Theorem: Consider the equation x˙ = f(t, x) and if it is possible to find a function V (t, x), defined in a neighbourhood of x = 0 and positive definite for t ≥ t0 with the orbital derivative negative definite, the solution x = 0 is asymptotically stable.

8. Diffusive Instability

The Morphogenesis work of Turing started in 1951. The creation of the organism's shape or structure in the history of the individual is morphogenesis. His evolutionary idea was that passive diffusion could interfere with chemical reactions in a way which could destabilise symmetry solutions, even if the reaction itself does have no symmetry breach capability. "Can diffusion destabilise a spacially uniform and stable state? The obvious question arises? "Where this is the case, this is known as instability induced by diffusion or turing instability.

We consider the system of *m* reaction diffusion equations given by

$$\frac{\partial \bar{\mathbf{u}}}{\partial \tau} = \mathbf{f}(\bar{\mathbf{u}}) + D \nabla^2 \bar{\mathbf{u}}$$

in a domain $\bar{\Omega} \in \Re^N$, where Ω requires separating from the spatial variable x¯. In order to work with a problem on a standard domain, it is often useful to rescale space variables, and it turns out that rescaling the time variable also simplifies the effect. So we define $\mathbf{x} = \gamma \bar{\mathbf{x}}, t = \gamma^2 \tau$ and $\mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\bar{\mathbf{x}}, t)$, to obtain

$$\frac{\partial \mathbf{u}}{\partial t} = \gamma^2 \mathbf{f}(\mathbf{u}) + D \nabla^2 \mathbf{u} = \alpha \mathbf{f}(\mathbf{u}) + D \nabla^2 \mathbf{u}$$

say, on $\Omega \in \Re^N$. In this equation γ is a measure of the linear dimensions of the original domain, so that increasing γ is equivalent to increasing domain size. Let us assume that this has a spatially uniform steady state solution u∗, so that $\mathbf{f}(\mathbf{u}^*) = \mathbf{0}$, and take homogeneous Neumann (zero-flux) boundary conditions on ∂Ω. Let uˆ be the perturbation from the steady state, uˆ =u − u∗. The linearisation of the above equation about u∗ is given by

$$\frac{\partial \mathbf{v}}{\partial t} = \alpha J^* \mathbf{v} + D \nabla^2 \mathbf{v}$$

in Ω with homogeneous Neumann boundary conditions on ∂Ω, where v is the linearised approximation to uˆ and J ∗ is the Jacobian matrix

$$J^* = \left[\frac{\partial f_i}{\partial u_j}\right]_{\mathbf{u}=\mathbf{u}_*}$$

The method of separation of the variables is a common way of finding the linear system solution with a constant coefficient. First of all, let's assume that we know the F(x) function that fulfils – all2F = ŚF in all the same conditions on the Neumann boundary on all the same. (F is an Eigen function of − für 2 on − for − for the border and ć for its own value). A V function of a V form (t, x) = cF(x)exp(µt) is considered. It satisfies the linearised equation and the boundary conditions if

$$\sigma \mathbf{c} = \alpha J^* \mathbf{c} - \lambda D \mathbf{c} = A \mathbf{c}$$

So¨and c are the proprietary value and matrix A = αJ all the corresponding propvector −˙D. Let us describe the spatial modes to be the original Fn(x) of −∇2 on Ω and the corresponding own values should be the Spatial Eigen functions. We know from Fourier's study that every function on Álvarez can be written in linear spatial modes, so that v may be written as

$$\mathbf{v}(\mathbf{x}, t) = \sum_{n=0}^{\infty} F_n(\mathbf{x}) \mathbf{G}_n(t)$$

It follows after separation of variables and some linear algebra that the general solution of the linearised equation may be put into the form

$$\mathbf{v}(\mathbf{x}, t) = \sum_{n=0}^{\infty} \sum_{i=0}^{m} a_{ni} \mathbf{c}_{ni}(t) F_n(\mathbf{x}) exp(\sigma_{ni} t)$$

where a_{ni} are arbitrary constants. σ_{ni} are the eigenvalues of the matrix $A_n = \alpha J^* - \lambda_n D,$ which we shall refer to as the temporal eigenvalue of the problem when we need to distinguish them from the spatial eigenvalues λn. The eigenvalues σni of An satisfy

$$det(\sigma_n I - A_n) = det(\sigma_n I - \alpha J^* - \lambda_n D) = 0$$

These are polynomials of mth order, so that m eigenvalues ̈σn1,are found in each of them. If ̇0i has a real negative part for all I but òni has a real positive part for some I and n ̇0 then we conclude that it's turing instability. Let's look at the last equation a little more closely, which shows us the relationship between spatial and temporal values. First we can evaluate the equation if the spatial value of the space is not negative.

$$det(\sigma I - A) = det(\sigma I - \alpha J^* - \lambda D) = 0$$

For each λ this is a polynomial of degree m solutions σi . Let

$$\rho(\lambda) = max_{1 \le i \le m} Re \sigma_i(\lambda)$$

with the most tangible part of the immovable part. A relation of time and space value such as this is usually referred to as a dispersion relationship between ρ and λ.

9. Stochastic process and random function

A random feature is a random family of variables Xt = Xt(ω) that take values in a certain SP space and depends on a certain T-parameter. When the parameter set T is part of the true line: T ⊆ R; when we interpret t as time; and if we interpret Xt as a movement of a random point in the space SP, then random function Xt is called a stochastic operation. The parameter set T

Stochastic differential equation

A stochastic differential equation is a differential equation where one or more terms are a stochastic process, resulting in a stochastic solution.

Langevin equation

Langevin equation is an additive white noise linear stochastic differential equation. Langevin invented it to explain Brown's motion in 1908. The definition refers to the noisy differential equation, in which you divide the movement into two parts, a structural part which changes slowly and an altering part which differs quickly. For the moisture of a spontaneously forced particle

$$m\dot{v} = -m\gamma v + \xi(t), \qquad \dot{x} = v$$

with $\langle \xi(t) \xi'(t) \rangle = \frac{2\gamma kT}{m} \delta(t - t')$

Spectral density and Auto-covariance function

The auto-covariance of the random function X(t) is defined by

$$C_X(\tau) = \langle X(t) \;\; X(t+\tau) \rangle = lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} X(t) X(t+\tau) dt$$

The spectral density of the random function X(t) is defined as the Fourier transform of the auto-covariance function CX(τ) (taking T −→ ∞)

$$S_X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_X(\tau) e^{i\omega\tau} d\tau$$

The inverse of the spectral density is the auto-covariance

$$C_X(\tau) = \int_{-\infty}^{\infty} S_x(\omega) e^{i\omega\tau} d\omega$$

CONCLUSION

A key concept in ecosystem analysis is that of stability. Many instances of ecological interaction with the word "instability" are possible. As a statistically formalized version of the more general scientific concept of a deterministic process, the idea of the dynamical system is extremely important. Mathematical principles had a major impact on the modeling and interpretation of many biological events. In exchange, biology has helped mathematicians with a number of challenging issues. Statistical biology, often known as bio-mathematics, is a growing field thanks to the connections it has made between mathematics and biological dynamics. To quantify how populations of organisms are affected by their physical surroundings, statisticians have developed the field of statistical ecology and epidemiology. Discovering and comprehending the myriad fundamental processes and intricate behaviors of a wide range of flora, fauna, and microorganisms has been greatly aided by statistical modeling. Statistical models of ecological systems have attracted a lot of attention. Ecological theory in biological sciences, to put it another way. Statistics-minded mathematicians have a long history of involvement in the field of population biology, making it the most statistically advanced subfield of biology. Population biology is fundamentally a quantitative discipline. As a result, we'd like to think about, clarify, and predict things like the effects of immigration, emigration, population mixing, and age systems on the population, among other things. Strong interactions have emerged in recent years between the many branches of Nonlinear Science as a result of several significant theoretical, computational, and experimental developments. Mathematical biology's model equations also contain a wide variety of non-linear effects (e.g. hysteresis, structural instability, dissipative structures, dynamic chaos etc.). In the last twenty years, nonlinear dynamics has been increasingly useful in simulating a wide range of biological and physiological processes. Because of its critical importance, we have devised both deterministic and stochastic methods, taking into account non-linear dynamic models of complex ecosystems or epidemiologic processes. The normal functioning of structural functions, as well as their stability, periodicity, stochastic bifurcation, fluctuations, and pattern formation, have all been investigated. We also talked about how thermodynamics and statistical mechanics can be used to the study of ecological systems with a high degree of complexity.

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