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A Study of Edge Cordial Graphs on Harmonious Graphs in Graph Theory

Rahul Pratap Singh[1]*, Dr. Birendra Singh Chauhan[2]

[1] Research Scholar, Shri Krishna University, Chhatarpur, M.P.

[2] Associate Professor, Shri Krishna University, Chhatarpur, M.P.

Abstract - This study consists of Special emphasis has been given to Edge cordial graphs, Friendly index set of graphs, Harmonious graphs, Difference cordial graphs and vertex* - graphs. Yilmaz and Cahit introduced a weaker version of edge graceful called edge cordial labeling and also defined a new graph labeling technique called Ek – cordial labeling. This Ek – cordial labeling is the generalization of eadge cordial labeling. In this study it is proved that the following graphs are Ek – cordial graphs. The labeling or valuation or numbering of a simple graph G is a one-to-one mapping from its vertex set (edge set) into a set of non-negative integers which induces an assignment of labels to the edges (vertices) of G. Graph labeling is one of the fascinating areas of graph theory with wide range of applications. Graph labeling problems were formulated in the mid 1960's from a long standing conjecture of Ringel and a paper by Rosa. Even the structure of graceful trees is not completely known till today. Ringel conjectured that all trees are graceful. This conjecture remains unsolved. labeling of a graph G with q edges known as 'β-valuation' which is an injection of the set of its vertices into the set of integers {0, 1, 2, …, q} such that the value of its edges are the numbers from 1 to q, the values of an edge being the positive difference between the numbers assigned to its end vertices. Golomb called such labeling graceful.

Keywords - EDGE Cordial Graphs, Harmonious Graphs, Graph Theory, eadge cordial labeling

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INTRODUCTION

To begin, let's consider a basic finite undirected graph G = (V(G),E(G)) with no isolated vertices, where V(G) and E(G) stand for the vertex set and edge set, respectively, and |V(G)| and |E(G)| stand for the number of vertices and edges. Gross [1] is consulted for all other terms. In this section, we will provide a brief overview of definitions pertinent to this study. Labeling a graph entails assigning numbers to its nodes (vertices) and edges (edges) under specific constraints. Labeling is known as vertex labeling or edge labeling depending on whether the mapped domain is the set of vertices or edges. Gallian [2] provides a detailed literature review on graph labeling. For a graph G, we have the edge labeling function f: E(G) [0,1], and the induced vertex labeling function f :V(G) [0,1] is provided by: f (v) = f(e1)f(e2)f(e3)f(e4)f(e5)f(e6)f(ek) where e1, e2,..., ek are all the edges incident to (ek). To count the vertices and edges labeled with f in graph G, we write vf(i) and ef(i) for I = 0, 1, respectively. If |vf(0) vf(1)| 1 and |ef(0)ef(1)| 1, then f is an edge product cordial labeling of graph G. If G can be labeled using edge products, then it is said to be a cordial graph.

Cahit [3] first instituted cordial labeling in 1987. Product cheerful labeling was first presented by Sundaram et al. [4] in 2004. An edge analogue of product cordial labeling, edge product cordial labeling was introduced by Vaidya and Barasara [5] in 2012, and they have since determined that the following graphs are edge product cordial: Crown Cn KK1; armed crowns Cm KPn; helms; closed helms; webs; flowers; gears Gn and shells Sn for odd n. Trees of greater than two orders. Unicyclic graphs of odd orders. In addition, they demonstrated that the following graphs are not friendly with edge products: Cn = wheels = shells = n even if n is odd. Edge product cordial labeling for several snake-related graphs was discussed by Vaidya and Barasara [6]. Product and edge product cordial labelings of degree splitting graphs of routes, shells, bistars, and gear graphs are explored by Vaidya and Barasara in [7]. Some cycle-related graphs are edge-product cordially labeled, as Prajapati and Patel [8] described.

BRIEF SURVEY ON LABELINGS

The assignment of labels to the edges (vertices) of a simple graph G is induced by a mapping from the vertex set (edge set) to a set of non-negative integers. One of the most interesting subfields in graph theory, graph labeling has numerous practical applications. Ringel's long-standing conjecture [50] and a publication by Rosa [51] in the mid-1960s provided the inspiration for the formulation of graph labeling problems. Not until recently was it understood in full detail how elegant trees are built.

The gracefulness of trees was postulated by Ringel [50]. This hypothesis has not been answered. In 1967, Rosa [51] introduced a labeling of a graph G with q edges called "-valuation," which is an injection of the set of its vertices into the set of integers "0," "1, "2, "..., "q," such that the value of its edges are the numbers from 1 to q, the value of an edge being the positive difference between the numbers assigned to its end vertices. The labeling you used was graceful, according to Golomb [23]. Magic labeling of a graph G(V, E) was first defined by Kotzig and Rosa [32] in 1970 as a bijection f from V» E to 1, 2,..., V »E such that f(x) + f(y) + f(x,y) is constant for all edges (x,y).

A graph G with q edges is said to be harmonious if there exists an injection f from the vertex set of G to 0, 1, 2,..., q-1 such that each edge (xy) is assigned the label (f(x) + f(y))(mod q), the resultant edge labels are different, as proposed by Graham and Sloane [24] in 1980. In the case where G is a tree, no more than one vertex label can be shared by two vertices. Sequentially additive numbering was first proposed by Bange, Barkausker, and Slater [5] in 1983. For a given positive integer k, a k-sequentially additive numbering of a (p, q) graph is the assignment of unique numbers k, k + 1,...., k + p + q - 1 to the p + q elements of G, where the numbers assigned to the vertices u and v are added together to form the numbers assigned to the edges uv. Lo [39] proposed the idea of "edge elegant labels" in 1985. If there exists a bijection f from E to 1, 2,...., E such that the induced mapping from V to 0, 1, 2,...., V - 1 given by f + (x) = (f(xy)/(xy) E)(mod V) is a bijection, then the graph G(V, E) is said to be edge gracious.

One subtype of sequential labeling, called set sequential labeling, was identified by Acharya and Hedge [1] in 1985. From the sequential labeling, Acharya and Hedge [2] generalized a new labeling in 1990 termed the (k, d) arithmetic labeling. If each vertex in an arithmetic graph can be assigned a unique nonnegative integer, and the edge labels can be calculated by adding the values of the incident vertices, then the graph is said to be arithmetic. Similarly, the authors of that study introduced the concept of sequential numbering that is additive in nature (k + d).

E_K - cordial labeling of cycle related graphs

Theorem 1: The double crown $C_n \odot K_2$ is E_k – cordial for n is odd, $n \geq 3$ with k = n.

Proof: Let the vertices of cycle C_n be $v_1, v_2, \ldots, v_n$. Let K2 be the complete graph on two vertices. Now attach K2 to each vertex of the cycle Cn. The resultant graph is denoted by $C_n \odot K_2$.

The graph $C_n \odot K_2$ have 3n vertices and 4n edges. Let ai, bi, i = 1, 2, ..., n be the vertices adjacent to the rim vertices of Cn

Let the vertex set be $V(G) = \{a_i / 1 \leq i \leq n\} \cup \{b_i / 1 \leq i \leq n\} \cup \{v_i / 1 \leq i \leq n\}$

Let the edge set be $E(G) = \{(v_iv_{i+1}) / 1 \leq i \leq n-1\} \cup \{(v_nv_1) \cup \{(a_ib_i) / 1 \leq i \leq n\} \cup \{(v_ib_i) / 1 \leq i \leq n\} \cup \{(v_ia_i) / 1 \leq i \leq n\}$

Define f: $E(G) \to \{0, 1, 2, \ldots, k-1\}$ where k = n as follows.

$$f(v_iv_{i+1}) = i-1, 1 \leq i \leq k \text{ where } v_{k+1} = v_1$$
$$f(v_ia_i) = i-1, 1 \leq i \leq k$$
$$f(a_ib_i) = i-1, 1 \leq i \leq k$$
$$f(v_ib_i) = i, 1 \leq i \leq k-1$$
$$f(v_kb_k) = 0$$

Now $e_f(0) = e_f(1) = \ldots = e_f(k-1) = 4$

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The induced function f^* is given by $f^*(v) = \left\{\sum_u f(uv) / uv \in E(G)\right\} (\text{mod } k)$

= Sum of the edges incident with v (mod k)

Then the induced vertex labels are as follows

$$f^*(v_1) = 0$$
$$f^*(v_i) = [f^*(v_{i-1}) + 4)] \bmod k, 2 \leq i \leq k$$
$$f^*(a_1) = 0$$
$$f^*(a_i) = [f^*(a_{i-1}) + 2)] \bmod k, 2 \leq i \leq k$$
$$f^*(b_1) = 1$$
$$f^*(b_i) = [f^*(b_{i-1}) + 2)] \bmod k, 2 \leq i \leq k$$

Then $v_f(0) = v_f(1) = \ldots = v_f(k-1) = 3$

Theorem 2: A graph obtained by attaching a triangle at each pendant vertex of a crown $C_n \odot K_1$ is E_k – cordial, $k \not\equiv 0 (\text{mod } 3)$ and k = n.

Proof: Let the vertices of the cycle C_n be $u_1, u_2, \ldots, u_n$. Let vi be the vertex which is adjacent to u_i, $1 \leq i \leq n$. Then the graph obtained is $C_n \odot K_1$ Let the vertices of the ith copy of C_3 be x_i, y_i, z_i. Identify zi with vi. The resultant graph is denoted by G.

Then the vertex set of G is $V(G) = \{x_i / 1 \leq i \leq n\} \cup \{y_i / 1 \leq i \leq n\} \cup \{u_i / 1 \leq i \leq n\} \cup \{v_i / 1 \leq i \leq n\}$

The edge set of G is $E(G) = \{(u_iu_{i+1}) / 1 \leq i \leq n-1\} \cup \{(u_nu_1)\} \cup \{(u_iv_i, v_ix_i, v_iy_i, x_iy_i / 1 \leq i \leq n\}$

Clearly it has 4n vertices and 5n edges.

Define f: $E(G) \to \{0, 1, 2, \ldots, k-1\}$ when k = n as follows.

$$f(u_iu_{i+1}) = i-1, 1 \leq i \leq k, \text{ where } u_{n+1} = u_1$$

$f(u_iv_i) = i - 1, 1 \leq i \leq k$

$f(x_iv_i) = i - 1, 1 \leq i \leq k$

$f(x_iy_i) = i - 1, 1 \leq i \leq k$

$f(y_iv_i) = i, 1 \leq i \leq k - 1$

$f(y_kv_k) = 0$

Now $e_f(0) = e_f(1) = ... = e_f(k-1) = 5$

The induced vertex labels $f^*(v) = \left\{\sum_u f(uv) / uv \in E(G)\right\} (\bmod k)$ are as follows.

$f^*(u_1) = k - 1$

$f^*(u_i) = [f^*(u_{i-1}) + 3] \bmod k, 2 \leq i \leq k$

$f^*(v_1) = 1$

$f^*(v_i) = [f^*(v_{i-1}) + 3] \bmod k, 2 \leq i \leq k$

$f^*(x_1) = 0$

$f^*(x_i) = [f^*(x_{i-1}) + 2] \bmod k, 2 \leq i \leq k$

$f^*(y_i) = 1$

$f^*(y_i) = [f^*(y_{i-1}) + 2] \bmod k, 2 \leq i \leq k$

then $v_f(0) = v_f(1) = ... = v_f(k - 1) = 4$

In both the cases

$$|e_f(i) - e_f(j)| \leq 1, \forall i, j \text{ and } |v_f(i) - v_f(j)| \leq 1, \forall i, j$$

Hence the graph G obtained by attaching a triangle at each pendant vertex of a crown

$C_n \odot K_1$ is E_k – cordial, $k \not\equiv 0 (\bmod 3)$ and $k = n$.

HARMONIOUS PATH GRAPHS LABELING

The term "Harmonic labeling" was proposed in 1980 by Graham and Sloane [24]. Assume G is a graph with q edges. If the induced function f*: E(G) 0, 1, 2,..., q - 1 defined as f*(e = uv) = (f(u) + f(v)) (mod q) is bijective, then the function f is said to be a harmonic labeling of the graph G. The term "Harmonic graph" is used to describe a graph that can be properly labeled as a harmonic sequence.

The following works have been done by several authors.

v Wheel graph Wn is Harmonious, □n

v Petersen graph is Harmonious

v Fan graph Fn = Pn + K1 is Harmonious

v Graham and Sloane Conjectured that every tree is Harmonious.

v The cycle Cn(n ≥ 3) is harmonious iff n is odd.

v Friendship graph is Harmonious except n ي 2 (mod 4)

v The graph B2

(n, n) is Harmonious, □n

v Caterpillars are harmonious

v All ladders are Harmonious, except L2

v Webs are Harmonious

Now we shall prove the following graphs are harmonious graphs.

HARMONIOUS LABELING OF PATH RELATED GRAPHS

Theorem 1: The graph G obtained by joint sum of two copies of fans Fn = Pn + K1 is harmonious, for all n.

Proof: Let v, v1, v2, ..., vn and v1', v2', ..., vn', v' be the vertices of Fn and Fn' respectively, where v and v' are apex vertices. G be the graph obtained by joining the apex vertices by an edge.

The vertex set of G is $V(G) = \{v, v', v_i, v_i' / 1 \leq i \leq n\}$

The edge set of G is $E(G) = \{v_iv_{i+1} / 1 \leq i \leq n - 1\} \cup \{vv_i / 1 \leq i \leq n\} \cup \{v_i'v_{i+1}' / 1 \leq i \leq n - 1\} \cup \{v'v_i' / 1 \leq i \leq n\} \cup \{vv'\}$.

Note that G has 2(n + 1) vertices and 4n - 1 edges.

Here q = 4n - 1

Define $f: V(G) \to \{0, 1, 2, ..., q - 1\}$ as follows.

$f(v) = 0$

$f(v') = 2n + 1$

$f(v_i) = 2i - 1, 1 \leq i \leq n$

$f(v_i') = 2i, 1 \leq i \leq n$

The induced labeling $f^*: E(G) \to \{0, 1, 2, ..., q - 1\}$ defined by

$f^*(uv) = (f(u) + f(v)) (\bmod q)$ as follows

$f^*(vv_i) = 2i - 1, 1 \leq i \leq n$

$= \{1, 3,..., 2n - 1\}$

$f^*(vv') = 2n + 1$

$f^*(v'v_i') = [f(vv') + 2i](\bmod q), 1 \leq i \leq n$

$= [f(v) + f(v') + 2i](\bmod q), 1 \leq i \leq n$

$= [2n + 1 + 2 + 2i](\bmod q), 1 \leq i \leq n$

$$= \{2n + 3, 2n + 5, \ldots, 4n + 1\} (\mathrm{mod}\ q)$$

$$= \{2n + 3, 2n + 5, \ldots, 4n - 3, 0, 2\}$$

$$= \{2n + 3, 2n + 5, \ldots, q - 2, 0, 2\}$$

$$f^*(v_i v_{i+1}) = 4i, 1 \leq i \leq n-1$$

$$= \{4, 8, \ldots, 4n - 4\}$$

$$= \{4, 8, \ldots, q - 3\}$$

$$f^*(v_i' v_{i+1}') = 4i + 2, 1 \leq i \leq n - 1$$

$$= \{6, 10, \ldots, 4n - 2\}$$

$$= \{6, 10, \ldots, q - 1\}$$

Thus the edge labels are {0, 1, 2,…, q - 1}

Thus all the edge labels are distinct.

Hence the graph G is harmonious, for all n.

Illustration: Harmonious labeling of joint sum of two copies of fans F6 = P6 + K1 is given in figure 1

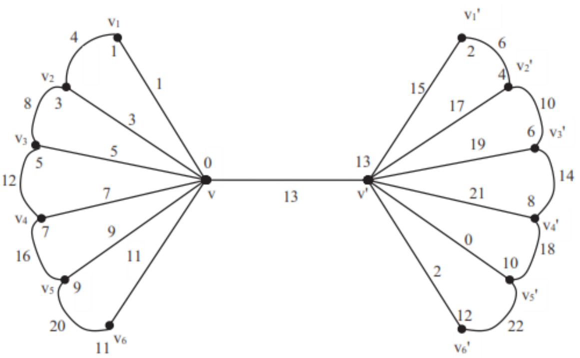

Figure 1. Joint sum of two copies of fans F6 = P6 + K1

Here q = 23

f(v1) = 1, f(v2) = 3, f(v3) = 5, f(v4) = 7, f(v5) = 9, f(v6) = 11, f(v) = 0, f(v ') = 13, f(v1') = 2, f(v2') = 4, f(v3') = 6, f(v4') = 8, f(v5') = 10, f(v6') = 12

Then the induced edge labels are

$$f^*(vv_i) = \{1, 3, 5, 7, 9, 11\}$$

$$f^*(vv') = 13$$

$$f^*(v'v_i') = \{15, 17, 19, 21, 0, 2\}$$

$$f^*(v_i v_{i+1}) = \{4, 8, 12, 16, 20\}$$

$$f^*(v_i' v_{i+1}') = \{6, 10, 14, 18, 22\}$$

Thus the induced edge labels are {0, 1, 2, …, 22}.

Thus all the edge labels are distinct.

Hence the above graph is a Harmonious graph.

CONCLUSION

The study's goal is to label, value, or number a simple graph G by mapping its vertex set (edge set) to a set of non-negative integers, which then causes an assignment of labels to the graph's edges (vertices). One of the most interesting subfields in graph theory, graph labeling has numerous practical applications. Around the middle of the 1960s, a study by Rosa and a long-standing conjecture of Ringel formalized the problem of labeling graphs. We still don't know everything there is to know about the structure of those beautiful trees. As far as Ringel can tell, every tree has a certain gracefulness about it. This hypothesis has not been answered. An edge's value is the positive difference between the numbers assigned to its end vertices, so the '-valuation' of a graph G with q edges is the injection of the set of its vertices into the set of integers '0, 1, 2,..., q' in such a way that the values of its edges are the numbers from 1 to q. Golomb characterized this sort of tagging as elegant.

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Corresponding Author

Rahul Pratap Singh*

Research Scholar, Shri Krishna University, Chhatarpur, M.P.