# **An Analysis of Fuzzy Environment with Queuing Model**

**Haneef Mohammad<sup>1</sup> \*, Dr. Birendra Singh Chauhan<sup>2</sup>**

<sup>1</sup> Research Scholar, Shri Krishna University, Chhatarpur M.P.

<sup>2</sup> Associate Professor, Shri Krishna University, Chhatarpur M.P.

*Abstract - Queuing theory can be recognized in four phases of various lengths and distinctive characters, the principal organize is the time of concentrated movement, while the second time frame denotes the scientific modeling of stochastic process being developed of the queuing theory might be known as the time of the development. The third stage demonstrates a time of further broad advancement of queuing theory can be named as a time of development, introducing the utilizations of queuing theory in different territories of genuine circumstances as in PC and correspondence system, Telecommunication system, fabricating, traffic control, medicinal services the board and diverse circles of life and specialized perspectives. The fourth stage or current period accentuation is on development uses of queuing theory ramifications of fuzzy rationale in queuing system for example analytical study of fuzzy queue models. Queuing theory is a part of mathematics that studies and models the demonstration of holding up in lines. This study will investigate the formulation of queuing theory alongside instances of the models and utilizations of their utilization. The objective of the study is to furnish the peruser with enough foundation so as to appropriately model a fundamental queuing system into one of the classes we will take a gander at, when conceivable. Likewise, the peruser should start to comprehend the essential thoughts of how to decide helpful information, for example, normal holding up times from a specific queuing system. The Fuzzy queue with multiple servers is turned into a family of fierce queues with multiple servers by using TV-cut approach. The results are more fluid and helpful for systems designers, as the system characteristics are expressed by the Membership Function. Multiple server queuing models are efficient methods for computer, telecommunications and manufacturing performance analyses. If the usual crisp batch-arrival queues can be extended to fluffy batch-arriving queues with multiple servers, queuing models would have even broader applications. Possibly more useful and realistic are the furious queues than usual clean queues.*

*Keywords - Fuzzy Environment Parameters, Queuing Model Tandem, Queuing theory, stochastic process*

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## **INTRODUCTION**

We discuss serial queues (or tandem), which is a simple one with two stations, to allow a unit to reach the first station from outside. It receive service and goes on to the second station. Many researchers such as Boxma, Daduna, Dallery, Gershwin [10] etc. have studied the model analysis for queuing with overtaking networks. In job stores, different products can follow different routes and some of the machines can operate on different types of products, an important practical issue. However, statistical information may be obtained subjectively in many practical applications, i.e., the arrival pattern and service pattern are described more appropriately in terms of language such as quick, slow and modest rather than probability. The response time or the time of a client is defined in open networks as the time from entry into the network until he leaves the network.

Here we consider the two stage tandem network. The system consists of two nodes with respective service rates  $\mu$ 0 and  $\mu$ 1 which are fuzzy numbers. The external arrival rate is  $\lambda$  which a fuzzy number is also. After the departure time is exponentially distributed, it is routed to node 2 with probability 1-q and returned to node 1 with the probability q where q is regarded as a fuzzy number. In this model we are trying to fluctuate the mean and difference of the time spent in the queuing network.

A mathematical model of the traffic offered to the telecommunication systems is needed in order to obtain analytical solutions to telecommunications problems. Erlang determined that the level of service is, if traffic is offered, the loss chance of a lost called system that has N trunks. The loss system is a system that customers must go to when space is complete due to limited waiting space. The formulas do not take into account the form of time

distribution for the service. This is called robustness. The waiting lines are the conditions we encounter in the store but packets waiting for trunk can also form in the world of telecommunications. The theory of queuing is a set of formulas which describes the behavior of the waiting line and can be used in this telecommunications situation and it's like. Service activities based on telecommunications can also be random and independent. For example, the length of a phone call or data packet size affects telephone switch and route service times. Efficient methods for the analysis of the queuing system were developed where their parameters, like arrival rate and service rates, are precisely known. However, it is not possible to present these parameters precisely because of uncontrollable factors. In particular, statistical data can be subjectively gathered in several practical applications, i.e. arrival and rate of services can be described more appropriately by linguistic terms like fast, medium or slow distribution of probability, based on statistical theory. Such information will accurately determine the measurement of system performance. Fuzzy set theory is a known concept of mental phenomena modeling imprecision or uncertainty. Multiple researchers specifically discussed fuzzy queues. A multi-channel queuing system with endless or endless capacity of waiting and population calling. In accordance with the zadeh extension principle, Negi and Lee formulated both -cut and two variable simulation approaches for the analysis of fluid queues. Li and Lee suggested a foggy queues analysis approach.

#### **FUZZY QUEUING NETWORK WITH OVERTAKING**

A network of queues is an important generalization of the process of birth death. Such networks can model conflicting issues arising from the sharing of a number of resources. Every resource is represented by a node or service centre. So, we can have a service center associated with one or more servers in a model for computer system performance analysis. Once the service has been completed in one service facility, the job can be moved to another service unit, returned to the same center or left the system.

In computer performance modeling and communication systems, queuing networks have been successfully used. Most of the analytical techniques discussed focused on the assessment of average performance measures, such as performance, use and reaction time. However in real-time situations, it is necessary to have an understanding of the response time or time distribution so that a missing deadline can be calculated. Here we develop a fluffy queuing network model with overtaking, i.e. a second queue FM/FM/1- IBF in series. We consider a fuzzy queuing network in which customers arrive at node according to Poisson

process with fuzzy rate  $\lambda$ . After an exponentially distributed service time at node 1 with service  $H_0$ , customers are routed to node 2 with a fuzzy probability

 $\overline{q}$ . The service time at node 2 is also fuzzy rate  $\overline{H_1}$ .

In this model the arrival rate  $\lambda$  and service rates at node1 and node 2  $\tilde{\mu}_0$  and  $\tilde{\mu}_1$  and also the fuzzy probability  $\overline{q}$  are known and are represented by the following fuzzy set

$$
\tilde{\lambda} = \{x : \mu_{\tilde{\lambda}}(x)/x \in X\}
$$
\n
$$
\tilde{\mu} = \{y : \mu_{\tilde{\mu}}(y)/y \in Y\}
$$
\n
$$
\tilde{q} = \{q : \mu_{\tilde{q}}(u)/u \in U\}
$$

Where the universal arrival rate, service rate and probability are crisp in X, Y and U. Let P(x, y, u) indicate an interest measure of system performance. Clearly, the system's performance measure is also fluid if the arrival rate is fluid. This model uses mathematical program techniques to approach the problem.A pair of parametric non-linear programs is

developed to find the  $\alpha$ -cuts of P( $\lambda$ ,  $\tilde{\mu}$ ,  $\tilde{q}$ ) based on Zadeh's Extension Principle [8]. The membership function performance measure is

$$
\mu_{P(\tilde{\lambda},\,\tilde{\mu},\,\tilde{q})}(z)\quad =\quad \underset{x\in X,\ y\in Y, z\in Z}{\text{Sup}}\;\left\{\mu_{\tilde{\lambda}(x),\,\tilde{\mu}(y),\,\tilde{q}(u)}/z\, = P(x,\,y,\,z)\right\}
$$

The variance sojourn time of a queuing network for a crisp queuing system is

$$
N = \frac{1}{(\mu_1(1-q)-\lambda)} \frac{\mu_1(q^2-1)-\lambda q}{\mu_1(q^2-1)+\lambda q} + \frac{1}{(\mu_2-\lambda)^2}
$$

The membership function for  $\tilde{N}$  is

$$
\mu_{\hat{N}}(z) \quad = \quad \underset{x \in X, \, y \in Y, z \in Z}{\text{Supp}} \, \left\{ \mu_{\hat{\lambda}(x), \, \hat{\mu}(y), \, \hat{\eta}(u)} / z \, \text{and} \, \frac{1}{\left(y, (1-q) - x\right)^2} \, \frac{y, (q^2 - 1) - xq}{y, (q^2 - 1) + xq} + \frac{1}{\left(y_2 - x\right)^2} \right\}
$$

Membership function for the other performance measures is

Mean sojourn  $(i)$ 

$$
\mu_{\tilde{L}}(z) \ = \ \ \underset{\kappa = X, \, y \in Y, z \in Z}{\text{Supp}} \ \left\{ \mu_{\tilde{\lambda}(x)} \mu_{\tilde{\mu}(y)} \mu_{\tilde{q}(u)} \big/ z \ = \ \frac{1}{\big( y_1 (1-q) - x \big)} + \frac{1}{y_2 - x} \right\}
$$

 $(ii)$ Number of customers in the system

$$
\mu_{\tilde{L}_x}(z) \;\; = \;\; \underset{x \in X, y \in Y, z \in Z}{\text{Supp}} \; \left\{ \mu_{\lambda(x)} \mu_{\mu(y)} \mu_{\tilde{q}(u)} \big/ z \; = \; \frac{x}{\left(y_{_1} - x\right)} + \frac{x}{y_{_2} - x} \right\}
$$

Expected waiting time in the system

$$
\mu_{W_q}(z) = \sup_{x \in X, y \in Y, z \in Z} \left\{ \mu_{\lambda(x)} \mu_{\mu(y)} \mu_{q(u)} / z = \frac{1}{R} \frac{x}{(y_1 - x)} + \frac{x}{y_2 - x} \right\}
$$

where R is the response time.

Although the membership functions are theoretically correct, they are not in the usual forms for practical use and they are very difficult to imagine their shapes.

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# **MATHEMATICAL PROGRAMMING APPROACH**

One approach to construct the membership function 
$$
\mu_{P(\tilde{\lambda}, \tilde{\mu}, \tilde{q})}
$$
 is to derive the  $\alpha$ -cuts of  $\mu_{P(\tilde{\lambda}, \tilde{\mu}, \tilde{q})}$ . They  $\alpha$ -cuts are defined as

$$
\lambda(\alpha) = \{x \in X/\mu_{\lambda}(x) \geq \alpha\}
$$
  

$$
\mu(\alpha) = \{y \in Y/\mu_{\mu}(y) \geq \alpha\}
$$
  

$$
q(\alpha) = \{u \in U/\mu_{\hat{q}}(y) \geq \alpha\}
$$

Note that  $\lambda(\alpha)$ ,  $\mu(\alpha)$  and  $q(\alpha)$  are crisp sets rather than fuzzy sets. Therefore the  $\alpha$ -level sets of  $\tilde{\lambda}$ ,  $\tilde{\mu}$ ,  $\tilde{q}$  are crisp intervals which can be expressed in the following forms.

$$
\begin{array}{rcl} \lambda(\alpha) & = & \left[ \begin{array}{c} x_a^1, & x_a^n \end{array} \right] \\ & = & \left[ \begin{array}{c} \min\limits_{x \in X} \left\{ x / \mu_{\lambda}(x) \geq \alpha \right\}, & \max\limits_{x \in X} \left\{ x / \mu_{\lambda}(x) \geq \alpha \right\} \right] \\ \\ \mu(\alpha) & = & \left[ \begin{array}{c} y_a^1, & y_a^n \end{array} \right] \\ & = & \left[ \begin{array}{c} \min\limits_{y \in Y} \left\{ y \in Y / \mu_{\mu}(y) \geq \alpha \right\}, & \max\limits_{y \in Y} \left\{ y \in Y / \mu_{\mu}(y) \geq \alpha \right\} \right] \\ \\ \alpha(\alpha) & = & \left\{ u_a^1, & u_a^n \right\} \\ & = & \left[ \begin{array}{c} \min\limits_{y \in Y} \left\{ u \in U / \mu_{\bar{\alpha}}(u) \geq \alpha \right\}, & \max\limits_{y \in Y} \left\{ u \in U / \mu_{\bar{\alpha}}(u) \geq \alpha \right\} \end{array} \right] \end{array}
$$

These intervals show where the constant arrival rate, service rate and probability are at the possibility level respectively. With the idea of -cuts, the Markov imbedded chain in FM/FM/1-IBF can be broken up into a family of ordinary Markov chains with different parameterized matrices for the probability of transition. The rate of arrival, service rate and probability can also be indicated by different confidence levels. The FM/FM/1-IBF therefore has different levels **Therefore** 

 $\{\lambda(\alpha)/0 \leq \alpha \mid \pi\}$ ,  $\{\mu(\alpha)/0 \leq \alpha \mid \pi\}$  and  $\{q(\alpha)/0 \leq \alpha \mid \pi\}$  1. . These sets represent sets of movable boundaries forming nested structures for expressing the relationship between ordinary sets and fuzzy sets. By the convexity of a fuzzy number the bounds of these intervals are functions of and can be obtained<br>as  $x'_a = \min \mu_{\lambda}^{-1}(\alpha)$ ,  $x_a^u = \max \mu_{\lambda}^{-1}(\alpha)$ ,  $y_a^l = \min \mu_{\Omega}^{-1}(\alpha)$ ,  $y_a^u = \max \mu_{\Omega}^{-1}(\alpha)$  and

 $q_n^1$  = min  $\mu_n^{-1}(\alpha)$ ,  $q_n^0$  = max  $\mu_n^{-1}(\alpha)$  respectively.

Assume that the performance measure of interest is N, ie. $P(x,y,u) = N$ . From the membership functions stated above which is not in the usual form and is very difficult to imagine its shape  $\mu_{\tilde{N}}(z)$  is the minimum of  $\mu_{\tilde{x}}(x), \mu_{\tilde{u}}(y)$ and  $\mu_{\mathbf{\hat{q}}}(\mathsf{u})$ . To take from the membership value we need atleast one of the following cases held such that z satisfies  $\mu_{\tilde{N}}(z) = \alpha$ .

\nCase (i) : \n
$$
\{\mu_{\bar{\lambda}}(x) = \alpha, \mu_{\bar{\mu}}(y) \geq \alpha \text{ and } \mu_{\bar{\alpha}}(u) \geq \alpha\}
$$
\n

\n\nCase (ii) : \n $\{\mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\mu}}(y) = \alpha \text{ and } \mu_{\bar{\alpha}}(u) \geq \alpha\}$ \n

\n\nCase (i) : \n $\{\mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\mu}}(y) \geq \alpha \text{ and } \mu_{\bar{\alpha}}(u) = \alpha\}$ \n

This can be accomplished via parametric non-linear programming techniques. For the former case the corresponding parametric non-linear programs for finding the lower and upper bounds of the  $\alpha$  cuts of  $H_{\rm N}$ 

 $N\mu_{\tilde{M}}(Z)$ . are

$$
N_{\alpha}^{I_1} = min \left\{ \frac{1}{(y_1(1-q)-x)^2} \frac{y_1(q^2-1) - xq}{y_1(q^2-1) + xq} + \frac{1}{(y_2-x)^2} \right\}
$$

such that  $x_{\alpha}^{\perp} \le x \le x_{\alpha}^{\perp}$ ;  $y \in \mu(\alpha)$ ,  $u \in q(\alpha)$ .

$$
N_{\alpha}^{\upsilon_{1}} = max \left\{ \frac{1}{\left(y_{1}(1-q)-x\right)^{2}} \frac{y_{1}(q^{2}-1) - xq}{y_{1}(q^{2}-1) + xq} + \frac{1}{\left(y_{2}-x\right)^{2}} \right\}
$$

such that  $x_{\alpha}^1 \le x \le x_{\alpha}^0$ ;  $y \in \mu(\alpha)$ ,  $u \in q(\alpha)$ .

For the second case

$$
N_{\alpha}^{I_2} = min \left\{ \frac{1}{(y_1(1-q)-x)^2} \frac{y_1(q^2-1) - xq}{y_1(q^2-1) + xq} + \frac{1}{(y_2-x)^2} \right\}
$$

such that  $y_{\alpha}^{\dagger} \le y \le y_{\alpha}^{\dagger}$ ;  $x \in \lambda(\alpha)$ ,  $u \in q(\alpha)$ .

$$
N_{\alpha}^{\,u_2} \quad = \max \Bigg\{ \frac{1}{(y_{_1}(1-q)-x)^2} \frac{y_{_1}(q^2-1)-xq}{y_{_1}(q^2-1)+xq} + \frac{1}{(y_{_2}-x)^2} \Bigg\}
$$

such that  $y_{\alpha}^{\dagger} \le y \le y_{\alpha}^{\dagger}$ ;  $y \in \lambda(\alpha)$ ,  $u \in q(\alpha)$ .

For the third case

$$
N_{\alpha}^{I_3} = min \left\{ \frac{1}{(y_1(1-q)-x)^2} \frac{y_1(q^2-1)-xq}{y_1(q^2-1)+xq} + \frac{1}{(y_2-x)^2} \right\}
$$

such that  $q_{\alpha}^{\dagger} \leq q \leq q_{\alpha}^{\alpha}$ ;  $x \in \lambda(\alpha)$ ,  $y \in \mu(\alpha)$ .

$$
N_{\alpha}^{\upsilon_{3}} = \max \left\{ \frac{1}{(y_{1}(1-q)-x)^{2}} \frac{y_{1}(q^{2}-1) - xq}{y_{1}(q^{2}-1) + xq} + \frac{1}{(y_{2}-x)^{2}} \right\}
$$

such that  $q_a^i \le q \le q_a^u$ ;  $x \in \lambda(\alpha)$ ,  $y \in \mu(\alpha)$ .

## **FUZZY BATCH QUEUE WITH MULTIPLE SERVERS**

We consider a batch arrival queuing system with C servers where the customers arrive in batches to occur according to a compound Poisson process with batch-arrival rate  $\lambda$ . Let  $A_{\kappa}$  denote the number of customers belonging to the  $K^{th}$  arrival batch, where  $A_K$ ,  $K = 1, 2, 3, \dots$  are with a common distribution

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$$
P_r[A_K = n] = a_n, n = 1, 2, 3, ...
$$
 and  $E[A] = \sum_{n=1}^{\infty} n a_n$ .

Let Ws be the waiting time in the system. Through a Markov process, we can easily obtain Ws in terms of system parameters

$$
W_S = \frac{\lambda (2 E[A] + E[A(A-1)]) + 2 \mu \sum_{n=0}^{C-1} n(c-n) P_n(\lambda \cdot \mu)}{2 \lambda E[A](c \mu - \lambda E[A])}
$$
(1)

Where  $\frac{1}{n}$   $\sqrt{n}$ ,  $\frac{1}{n}$  represents the probability that there are n customers in the system and the probability depends on  $\lambda$  and  $\mu_{\text{th}}$  steady –state, it is necessary<br>  $0 < \frac{\lambda E[A]}{c\mu} < 1.$ 

that we have

Suppose the batch-arrival rate  $\lambda$  and service rate  $\mu$  are fuzzy sets, we then have

$$
\widetilde{\lambda} = \{ (x, \phi_{\widetilde{\lambda}}(x)) / x \in X \}
$$
\n
$$
\widetilde{\mu} = \{ (y, \phi_{\widetilde{\mu}}(y)) / y \in Y \}
$$
\n
$$
(2)
$$

Where X and Y are the crisp universal sets of the batch-arrival and service rates and  $\phi_{\tilde{\lambda}}(x)$  and  $\phi_{\tilde{\mu}}(y)$ are membership functions of  $\widetilde{\lambda}$  and  $\widetilde{\mu}$ .

Let f(x, y) denote the system characteristic of interest. Since  $\tilde{\lambda}$  and  $\tilde{\mu}$  are fuzzy numbers,  $f(\tilde{\lambda}, \tilde{\mu})$  is also a fuzzy number. By Zadeh's extension principle the membership function of the system characteristic  $f(\widetilde{\lambda}, \widetilde{\mu})$  is defined as

$$
\phi_{\tilde{q}(\overline{\lambda},\overline{\mu})}\left(z\right)=\underset{x\in X,\;y\in Y,\;0< x}{\text{Sup}}\;\min\;\{\phi_{\overline{\lambda}}(x),\,\phi_{\overline{\mu}}(y)\;\;/\;\;Z=f(x,\;y)\} \tag{4}
$$

Assume that the system characteristic of interest is the waiting time in the system. It follows from (1) that the waiting time in the system is:

$$
f(x, y) = \frac{x(2E[A] + E[A(A-1)]) + 2y\sum_{n=0}^{C-1} n(c-n) P_n(x, y)}{2xE[A](c\mu - x E[A])}
$$
(5)

The membership function for the waiting time in the system is



#### **PARAMETRIC NONLINEAR PROGRAMMING**

By using Zadeh's approach, the  $\alpha$ -cuts of  $\widetilde{\lambda}$  and  $\widetilde{\mu}$  are crisp intervals and can be expressed as

$$
\lambda(\alpha) = [x_{\alpha}^{\ L}, x_{\alpha}^{\ U}] = \left[\min_{x \in X} \{x \phi_{\tilde{\lambda}}(x) \ge \alpha\} \max_{x \in X} \{x \phi_{\tilde{\lambda}}(x) \ge \alpha\}\right]
$$
  

$$
\mu(\alpha) = [y_{\alpha}^{\ L}, y_{\alpha}^{\ U}] = \left[\min_{y \in Y} \{y/\phi_{\tilde{\mu}}(y) \ge \alpha\} \max_{y \in Y} \{y/\phi_{\tilde{\mu}}(y) \ge \alpha\}\right]
$$
(7)

The bounds of these intervals are functions of  $\alpha$  and can be obtained as

 $x_{\alpha}^{L} = \min \phi_{\overline{i}}^{-1}(\alpha), x_{\alpha}^{U} = \max \phi_{\overline{i}}^{-1}(\alpha), y_{\alpha}^{L} = \min \phi_{\overline{i}}^{-1}(\alpha), y_{\alpha}^{U} = \max \phi_{\overline{i}}^{-1}(\alpha)$ 

(8)

By using Zadeh's principle,  $\phi_{\tilde{W}_c}(z)$  is the minimum of  $\phi_{\tilde{\chi}}(x)$  and  $\phi_{\tilde{\mu}}(y)$ .

To derive the membership function of  $\phi_{\hat{W}_S}(z)$ , we need at least one of the

following cases to hold such that

$$
Z = \frac{x(2E[A] + E[A(A-1)]) + 2y \sum_{n=0}^{C-1} n(c-n) P_n(x, y)}{2x E[A](cy - x E[A])}
$$
 satisfies  $\phi_{\tilde{W}_S}(z) = \alpha$   
Case (i) :  $(\phi_{\tilde{Z}}(x) = \alpha, \phi_{\tilde{\mu}}(y) \ge \alpha)$   
Case (ii) :  $(\phi_{\tilde{Z}}(x) \ge \alpha, \phi_{\tilde{\mu}}(y) = \alpha)$ 

This can be accomplished using parametric NLP techniques. The NLP to find the lower and upper bounds of the  $\alpha$ -cut off for cases (i) are:

$$
(W_S)_{\alpha}^{L_1} = \min \frac{x(2E[A] + E[A(A-1)]) + 2y \sum_{n=0}^{C-1} n(c - n) P_n(x, y)}{2x E[A] (cy - xE[A])}
$$
  
(9)

$$
(W_S)^{U_1}_{\alpha} = \max \frac{x(2E[A] + E[A(A-1)]) + 2y \sum_{n=0}^{C-1} n(c-n) P_n(x, y)}{2x E[A] (cy - xE[A])}
$$
\n(10)

and for case (ii) are

$$
(W_S)_{\alpha}^{L_2} = \min \frac{x(2E[A] + E[A(A-1)]) + 2y \sum_{n=0}^{C-1} n(c-n) P_n(x, y))}{2x E[A] (cy - x E[A])}
$$
\n(11)

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$$
(W_S)_\alpha^{U_2} = \max \frac{x(2E[A] + E[A(A-1)] + 2y \sum_{n=0}^{C-1} n(c-n) P_n(x, y))}{2x E[A](cy - x E[A])}
$$
\n(12)

Since  $\alpha$ -cuts form a nested structure with respect to  $\alpha$ (9) and (10) has the same smallest element and (11) and (12) has the same largest element. To find the membership function  $\phi_{W_s}$  (z) it is enough to find the lower bound  $(W_S)^L_{\alpha}$  and upper bound  $(W_S)^U_{\alpha}$  of the  $\alpha$ -cuts of  $\tilde{W}_s$ , which can be written as:

$$
(Ws)_{\alpha}^{L} = \min \frac{x(2E[A] + E[A(A-1)]) + 2y \sum_{n=0}^{C-1} n(c-n) P_{n}(x, y))}{2x E[A](cy - xE[A])}
$$
\n(13)

such that  $x_a^L \le x \le x_a^U$  and  $y_a^L \le y \le y_a^U$ 

$$
(W_S)^U_u = \max \frac{x(2E[A] + E[A(A-1)]) + 2y\sum_{n=0}^{C-1} n(c-n)P_n(x, y)}{2x E[A](cy - x E[A])}
$$
\n(14)

If both  $\frac{1}{2}$  in (13) are invertible with respect to  $\alpha$ , then a left shape function  $L(z) = [(W_s)_\alpha^L]^{-1}$  and a right shape function  $R(z) = \left[ (W_s)_\alpha^U \right]^{-1}$  can be derived, from which the membership function  $\overline{\varphi}_{\tilde{W}_S}(z)$  is constructed.

$$
\phi_{W_S}\left(z\right)=\begin{cases}L(z)\ ,\quad \left(Ws\right)_{\alpha=0}^{L}\leq z\leq \left(Ws\right)_{\alpha=1}^{L}\\ 1\qquad ,\quad \left(Ws\right)_{\alpha=1}^{L}\leq z\leq \left(Ws\right)_{\alpha=1}^{U}\\ R(z)\ ,\quad \left(Ws\right)_{\alpha=1}^{U}\leq z\leq \left(Ws\right)_{\alpha=0}^{U}\end{cases}
$$

In most cases the values of cannot be derived analytically. Consequently, a closed form membership function for  $\phi_{W_s}(z)$  cannot be obtained. However, the numerical solutions for  $(W_S)^L_{\alpha}$  and  $(W_s)^U_{\alpha}$  at different possibility levels can be collected to approximate the shape of  $L(z)$  and  $R(z)$ . That is the set of intervals  $\{[(W_S)^L_{\alpha}, (W_S)^U_{\alpha}] / \alpha \in [0,1] \}$  shows the shape of  $\phi_{W_S}(z)$  although the exact function is not known explicitly.

#### **CONCLUSION**

The study examines interlaced methods for the furious problems of queuing in particular. We chose to display the vagueness governing fluid queue issues in trapezoidal fuzzy numbers. The Zadeh extension principle and the "alternate" approach are widely used to derive the membership feature from the measurements of various queuing problems when linking problems in the fuzzy environment. Little formulas are commonly used to define the interplay of properties of queuing problems. Models that is more useful in modeling many real-life situations, with several servers and arrivals by batch. The performance measurement of the queuing systems is defined by a pair of parametric NLPs. The batch size is treated mostly as a random variable in previous concepts but it is more accurate to use that the size of the battery is also fluid because it is completely fluid. The performance measures of the bulk arrival queuing model are discussed with a MINLP pair. Models of the arrival queue with a set-up of the server are commonly used before starting the service in practical situations. The measurement of performance of different system features is discussed. Yager index ranking is used for analysis of results using the fuse theory and conventional method. Its features for single and multiple holidays are discussed. In real life, the models are always uncertain in their parameters. Queuing models have wide applications in machine fix, toll stalls, taxi stands, stacking and emptying of boats, booking patients in healing centers, PC field as for program planning, time sharing, system structure, media transmission and service associations, where in various sort of clients are serviced by various kind of servers. In the conventional queuing theory, the inter arrival times and service times are expected to pursue certain preassigned probability distributions. Anyway in numerous handy circumstances the arrival examples and service times can be all the more practically depicted by etymological articulations like quick, moderate, moderate, adequate or lacking as opposed to by probability distributions. Along these lines all the queuing qualities like arrival rate, service rate and excursion rate can be better determined by fuzzy numbers. This gives the extent of studying queues with regards to fuzzy theory. Fuzzy queues can be adequately connected in the fields like assembling system, media transmission and data processing.

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#### **Corresponding Author**

#### **Haneef Mohammad\***

Research Scholar, Shri Krishna University, Chhatarpur M.P.