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A Review of Certain Differential Equations research that is Relevant to Differential Difference Equations

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Abstract - Long before computers, differential equations were used to model and analyze a wide range of physical, biological, and engineering systems. However, with the development of computing technologies and the need to take discrete events into account, researchers' focus has shifted to a subset of equations known as differential difference equations (DDEs). The purpose of this work is to survey the state of knowledge on differential equations and discuss how these advancements may be used to the study of differential difference equations. The study starts out by providing a primer on ODEs and PDEs, the two types of differential equations most often used to represent continuous systems, and their fundamental ideas. It then goes on to explain how DDEs came to be as an extension of ODEs, which allowed for delayed effects and discrete time steps. The many uses of dynamical systems equations (DDEs) are discussed. These use cases range from population dynamics to physiological systems to control theory.

Keywords - Certain Differential, Equations Relevant, Differential difference equations

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1. INTRODUCTION

For quite some time, differential equations have been an indispensable tool for the mathematical modeling and analysis of events in many scientific disciplines, including physics, biology, engineering, and economics. They provide a robust framework for studying the dynamics and history of non-discrete systems. However, the study of differential difference equations (DDEs) has evolved as a specialist subject within the domain of differential equations as the necessity to account for discrete occurrences and delayed effects has grown more common.

In order to more accurately depict real-world dynamics, differential difference equations go beyond the ideas of ordinary differential equations (ODEs) by integrating discrete time steps and delayed effects. These equations are useful in many contexts where delays or other discrete events play an important role, including population dynamics, economics, control theory, and epidemiology.[1]

This survey aims to give readers with an in-depth summary of current breakthroughs in differential equations research that are directly applicable to the analysis of differential difference equations. We want to emphasize the importance of differential equations in understanding and evaluating dynamic systems with discrete features by investigating the relationships between classical differential equations and their discrete equivalents.

The review begins out with an introduction to the basic ideas and analytical tools used in the study of ordinary differential equations. We highlight the role of ordinary differential equations (ODEs) in the modeling of continuous-time systems and describe many techniques for analytical and quantitatively solving ODEs. The foundation for grasping the generalization to differential difference equations has thus been laid.[2]

Partial differential equations (PDEs) are the next topic of study after ordinary differential equations (ODEs). In addition to their widespread use in physics, fluid dynamics, and heat transport, PDEs play a significant role in the modeling of systems with spatial variation. Differential difference equations are discussed in relation to the solution methods and numerical techniques utilized for PDEs.

Now that we have laid this groundwork, we may focus on differential difference equations (DDEs). We provide a definition of DDEs and emphasize its unique features, such as the use of discrete time steps and the presence of delayed effects. We discuss why it is worthwhile to study DDEs and highlight their many practical applications in areas as varied as population dynamics, economics, and control systems.

Following this general discussion, the study shifts its attention to the analysis and solution techniques that are relevant to differential difference equations. In the framework of DDEs, we cover a wide range of subjects, including stability analysis methods, bifurcation theory, and chaos. Discrete temporal delays and their effect on the dynamics of the system are given special consideration.

The relationships between differential difference equations and other areas are also investigated, including delay differential equations (DDEs), time-delay systems, and neural networks. We illustrate how knowledge gained from these areas may be used to deepen our understanding of differential difference equations by focusing on the commonalities between them.

The difficulties of researching differential difference equations are also discussed in this article. We talk about how difficult it is to analyze and solve DDE models computationally, as well as how to find the right delay functions. Techniques for numerical approximation and other high-level mathematical tools are only two examples of the many methods given here that have been created to meet these issues.[3]

1.1 Emergence and significance of differential difference equations (DDEs)

To represent dynamic systems with discrete events and temporal delays, differential difference equations (DDEs) arise as a mathematical framework. These equations are generalizations of ODEs that take into account both discrete time steps and delayed effects. DDEs are mathematically significant because of their ability to faithfully model the operation of systems in the actual world that exhibit discrete properties. DDEs may be represented mathematically using equations of the type:

$$x'(t) = f(x(t), x(t - \tau))$$

where the system's current state variables are denoted by x(t), the derivative of x with respect to time is denoted by x'(t), f is a function that links x(t) and x(t -), and is the time delay.[4]

Time delays may be accounted for by plugging in the phrase x(t -) to the equation. The length of time required to reach every part of the system is called the propagation time. It is the modeled system and the connections between its variables that determine the exact shape of the function f(x(t), x(t -)).

The existence of temporal delays makes it difficult to analytically solve DDEs. Numerical approaches, stability analysis, and bifurcation theory are only some of the mathematical tools created to deal with DDEs. Approximating solutions numerically is possible using numerical techniques like Runge-Kutta methods and finite difference methods. Analyzing stability entails looking at how solutions act and figuring out whether or not the system is stable or prone to oscillations. As the parameters or delay values change, bifurcation theory analyzes the resulting qualitative shifts in the system's behavior.

DDEs are significant in mathematics because they may be used in many different areas, from population dynamics to control theory to economics to neuroscience. DDEs enable for a more accurate modeling of systems with delays and discrete events, shedding light on their dynamics and behavior. They allow for more precise and in-depth investigation into phenomena including population expansion, control system stability, economic decision-making, and neural network dynamics.[5]

1.2 Ordinary Differential Equations (ODEs)

The interactions between an unknown function and its derivatives in a single independent variable are described by Ordinary Differential Equations (ODEs), a special kind of differential equation. They are used to simulate a broad variety of continuous-time events in mathematics, physics, engineering, and other scientific fields. ODEs are crucial in the study and forecasting of dynamical system behavior.

In mathematics, an ODE is written as:

$$F(x, y, y', y'', ..., y^{\wedge}(n)) = 0$$

where x is the free variable, y(x) is the function of interest, y' is the first derivative of y with respect to x, y'' is the second derivative, and y(n) is the nth derivative. In order to solve the problem, we need to know how F connects the function y and its derivatives.

Different kinds of ODEs may be distinguished according to their order and linearity. The highest derivative of an ODE is used to establish its order. First-order ODEs, for instance, only need the first derivative, but second-order ODEs require both the first and second derivatives. Concerning the unknown function and its derivatives, linearity indicates whether the equation is linear or nonlinear.[6]

Finding a function y(x) that answers the given problem is the key to solving ordinary differential equations. Analytical or numerical methods may be used to get the answers. For an analytical solution, one must derive an explicit formula for the y(x) function such that the equation holds. This is doable for well-understood ODE classes including separable, linear, and exact equations. On the other hand, analytical answers aren't always accessible or even possible to find.

Approximate solutions to ODEs are often found using numerical techniques. The independent variable is discretized, and difference quotients are used to approximate the derivatives. The answers may be approximated numerically using techniques like Euler's, Runge-Kutta, and finite difference methods. When working with complicated or

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nonlinear ODEs, or when analytical solutions are unavailable, these techniques shine.[7]

ODEs are useful in many scientific and technical contexts. Particle motion, electrical circuits, fluid movement, population dynamics, chemical reactions, and many more phenomena are only few of the many that may be modeled and analyzed using these tools. Insights into the behavior and development of dynamic systems, as well as the ability to forecast their behavior, create effective control measures, and comprehend the fundamental mathematical principles driving these systems, may be gained by solving or studying ODEs.

1.2.1 Fundamental concepts and analytical techniques

i. Fundamental Concepts:

Order: The maximum derivative in an ordinary differential equation (ODE) establishes the ODE's order. For instance, the first derivative (represented by the symbol y') is not used in a first-order ODE, but the second derivative (represented by the symbol y'') is used in a second-order ODE.[8]

Example: The first-order ODE is represented as:

$$dy/dx = f(x, y),$$

in which the function f(x, y) is known.

Linearity: When the unknown function and its derivatives can be expressed as a linear combination, we say that the ODE is linear. In other words, there is no multiplication or exponentiation involved when the unknown function and its derivatives are expressed as a power of 1.

Example: Second-order linear ODE is written as:

$$a(x)\, y'' + b(x)\, y' + c(x)\, y = f(x),$$

where the functions a(x), b(x), c(x), and f(x) are defined.

ii. Analytical Techniques:

Separation of Variables: The method may be used for certain first-order ODEs. It requires integrating each sides of the equation independently, with the variables on their respective sides separated. In many cases, the unknown function may be solved explicitly in this way.

Example: Think about the ODE of the first order.

$$dy/dx = g(x)\, h(y),$$

in which the functions g(x) and h(y) are provided.

The equation may be rewritten using variable separation as:

$$1/h(y)\, dy = g(x)\, dx.$$

Combining approaches produces

$$\int(1/h(y))\, dy = \int g(x)\, dx,$$

letting us work backwards from x to find y.

Linear ODEs: Common methods for solving linear ODEs include the integration by factors approach, parameter variation, and identifying the corresponding complementary function and integral. These strategies entail changing the equation so that it's easier to work with, or finding specific solutions depending on the shape of the right-hand side..

Example: Take the linear homogeneous second order as an example.

$$\text{ODE } y'' + p(x)\, y' + q(x)\, y = 0,$$

where the functions p(x) and q(x) are known.

Under the premise that a solution of the kind

$$y = e^{(rx)},$$

The defining equation may be derived.:

$$r^2 + p(x)\, r + q(x) = 0.$$

The general answer may be written by solving this equation for its roots r1 and r2:

$$y(x) = C1\, e^{(r1x)} + C2\, e^{(r2x)},$$

starting circumstances define the constants C1 and C2.

Exact Equations: A first-order ordinary differential equation (ODE) is an exact equation if and only if it has the form:

$$M(x, y)\, dx + N(x, y)\, dy = 0,$$

where M and N are variables that depend on x and y. In order to solve an equation exactly, one must first identify a function known as the potential function or integrating factor. Then, integrate the potential function to get the answer..

Example: Think about the precise equation:

$$(2xy + y^2)\, dx + (x^2 + 2xy)\, dy = 0.$$

The accuracy is tested by seeing whether

$$(\partial M/\partial y) = (\partial N/\partial x).$$

If they are equivalent, then integrating M with respect to x and N with respect to y will give us the potential function (x, y).

The answer is obtained by solving for and then treating it as a constant..

1.2.2 Numerical methods for solving ODEs

Several branches of science and engineering depend critically on numerical techniques for solving ordinary differential equations (ODEs). By discretizing the independent variable and estimating the derivatives and then iteratively calculating the solution on a discrete grid, these techniques approximate the true solution. We will provide explicit equations for various frequently used numerical techniques for solving ODEs.[9]

Euler's technique is a straightforward approach to numerical computation. The derivative is approximated using the forward difference formula, which is then used to estimate the answer. A first-order ordinary differential equation of the form:

$$dy/dx = f(x, y).$$

This is how we may approximate the answer at x1 given the beginning conditions (x0, y0)::

$$y1 = y0 + h * f(x0, y0)$$

Here, h represents the step size, which determines the size of each time step. The method proceeds by iteratively calculating the solution at subsequent grid points using the formula:

$$yn+1 = yn + h * f(xn, yn)$$

Euler's technique is a straightforward and natural way to approximate the answer to an ODE. It is well-known, however, that it has accuracy and stability issues.

Runge-Kutta techniques are often utilized because of their great precision. One of the most popular implementations is the fourth-order Runge-Kutta (RK4) algorithm. The derivatives are assessed at various points within the time step, and the intermediate values are then calculated.

Here is the equation for RK4:

$$k1 = h * f(xn, yn)$$

$$k2 = h * f(xn + h/2, yn + k1/2)$$

$$k3 = h * f(xn + h/2, yn + k2/2)$$

$$k4 = h * f(xn + h, yn + k3)$$

$$yn+1 = yn + (k1 + 2k2 + 2k3 + k4)/6$$

Slopes at different points in time are indicated by k1, k2, k3, and k4. The next approximation is obtained by adding the weighted average of these slopes to the present answer.

To estimate the derivative, Adams-Bashforth techniques use previous evaluations of the function to form an explicit multistep approximation. These strategies improve precision by using a collection of previously determined function values. Adams-Bashforth's k-order formula is as follows::

$$yn+1 = yn + h * (b1 * fn + b2 * fn-1 + ... + bk * fn-k+1)$$

The derivative at xn is denoted by fn, while the coefficients b1, b2,..., bk are fixed values. Coefficients may be found in mathematical tables and are dependent on the method order.

Other typical numerical techniques for solving ODEs include finite difference approaches. These techniques make use of difference formulae to approximate the ODE's derivatives. As an example, think about the core difference approach. The derivative at any given grid point may be approximately calculated using values of nearby functions thanks to discretization. Approximating the first derivative using the central difference formula is:

$$dy/dx \approx (y_{n+1} - y_{n-1}) / (2h)$$

Here,

$$y_{n+1} \text{ and } y_{n-1}$$

represent the values of the function at neighboring grid points, where h is the size of the step.

1.3. Applications of ODEs in continuous-time systems

The use of ordinary differential equations (ODEs) in modeling and evaluating the behavior of continuous-time systems is widespread. Differential equations (ODEs) provide a mathematical vocabulary for describing the dynamical change of such systems over time. In this article, we explore the

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mathematical language of ODEs and their applications in continuous-time systems:

Physics and Engineering: ODEs are widely used in physics and engineering because of their ability to accurately represent the dynamics of physical systems. The force exerted on an object is related to its mass and acceleration by Newton's second law, a second-order ordinary differential equation. ODEs are used to simulate phenomena in a wide variety of engineering fields, including fluid dynamics, heat transport, electrical circuits, control systems, and structural mechanics. They make it easier to mathematically characterize and analyze the behavior of complex systems, as well as to forecast how they will react to a variety of inputs and disturbances.[10]

Population Dynamics: The study of population dynamics relies heavily on ordinary differential equations (ODEs) because they provide a mathematical explanation of the growth and decline of a population. These models provide light on complex biological processes including species relationships, disease transmission, and ecological networks. Common ODE models include the logistic equation, which describes population increase subject to finite resources.

Epidemiology: Providing a mathematical foundation, ODEs are crucial in epidemiology for modeling the spread of infectious illnesses. Epidemiological models based on ODEs are useful for studying the dynamics of disease transmission, estimating model parameters, forecasting the spread of illness, and assessing the efficacy of preventative measures like vaccination and social isolation. Models like SIR (Susceptible-Infectious-Recovered) and its derivatives are widely used instances of this kind.

Chemical Reactions and Reaction Kinetics: Chemical reaction rates and the dynamic behavior of reacting systems are described by ordinary differential equations. The field of chemical kinetics investigates the how and the how fast of chemical processes. The temporal dependence of reactant concentrations, reaction rates, and other features of chemical systems may be understood and predicted using kinetic models based on ordinary differential equations (ODEs). This information is crucial for the fields of pharmaceuticals, manufacturing, and environmental chemistry.

Biomedical Modeling: In order to simulate a wide variety of biological processes and physiological systems, ODEs are widely used in the biomedical sciences. The fields of pharmacokinetics and pharmacodynamics (which examine how and where drugs behave in the body), as well as brain dynamics, cardiac electrophysiology, and biological oscillators, all make use of these techniques. Complex biological systems may be modeled using ODE tools, allowing for simulation and analysis that improves our knowledge of illnesses, medication interactions, and therapeutic approaches.

1.4. Partial Differential Equations (PDEs)

Mathematical equations involving partial derivatives of an unknown function with regard to a number of independent variables are called partial differential equations (PDEs). They are used to explain events in physics that exhibit spatial or temporal changes. In a nutshell, a PDE may be formulated as follows:

$$F(x, t, u, \partial u/\partial x, \partial u/\partial t, \partial^2 u/\partial x^2, \partial^2 u/\partial t^2, \ldots) = 0,$$

F is a mathematical equation that links the function, its partial derivatives, and maybe the independent variables, where x and t are the independent variables and u is the unknown function. The equation describes the dynamic or static balance between the function and its derivatives.[11]

A PDE's shape and properties are unique to the physical issue being addressed. Various PDEs are useful for modeling certain phenomena. Some typical examples of PDEs are listed below:

i. Elliptic Equations

Second-order derivatives are required for the description of steady-state issues in elliptic equations.

For instance, consider the Laplace equation:

$$\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0.$$

ii. Parabolic Equations

The first-order derivative with regard to time and the second-order derivative with respect to space are both involved in the solution of parabolas. Diffusion and heat conduction issues are described.

The heat equation is a famous example of a parabolic formula:

$$\partial u/\partial t = \alpha\, (\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2).$$

iii. Hyperbolic Equations

Time and space derivatives of the second order are required for hyperbolic equations. They explain how waves form and how signals or disturbances travel across space.

The wave equation is a well-known case in point:

$$\partial^2 u/\partial t^2 = c^2\, (\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2),$$

where the speed of the wave, c, is shown.

Boundary conditions and beginning conditions are required to solve a partial differential equation. The solution's behavior at the problem's borders is specified by boundary conditions. Dirichlet

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conditions (which specify the value of the solution), Neumann conditions (which specify the derivative of the solution), and mixed conditions are all possible forms. The system's starting state at a particular moment is defined by the initial conditions.

Obtaining analytical solutions to PDEs is generally difficult and is only attainable in a restricted class of situations. As a result, approximation techniques based on numerical approaches are often utilized. There are a variety of numerical approaches used to solve PDEs, including finite difference methods, finite element methods, and spectral methods. These techniques discretize the domain and reformat the PDE as an algebraic system amenable to numerical methods of solution.

PDEs have many practical uses in the sciences and engineering. Fluid dynamics, heat transport, electromagnetic fields, quantum physics, population dynamics, and many more phenomena may all be modeled using them. Researchers can examine physical processes, develop designs, and forecast the behavior of complex systems by solving PDEs..

1.5 Differential Difference Equations (DDEs)

Differential difference equations (DDEs) need taking into account both the present and the previous values of the unknown function in order to arrive at a solution. Because of the difficulty in obtaining exact analytical solutions, DDEs are often approximated using numerical approaches. In this article, I will discuss the many numerical techniques that may be utilized to address DDEs.

Discretizing the time domain and transforming the DDE into a system of ordinary differential equations (ODEs) or difference equations is a typical numerical technique. Because of this, we may use the same numerical methods developed for ODEs and difference equations. DDE is a basic example, so let's use it:

$$x'(t) = f(x(t), x(t-\tau)).$$

The not-yet-known function is denoted by x(t), the derivative of this function over time by x'(t), a known function f, and a delay t denoted by.

Step-by-step integration is one strategy that may be used to numerically solve this DDE. We can use finite difference approximations to get a close approximation of the derivative by discretizing the time domain into short intervals. For instance, we may approximate using forward differences:

$$x'(t) \approx (x(t+h) - x(t))/h,$$

with a step size of h.

When this approximation is plugged into the DDE, we get a difference equation:

$$(x(t+h) - x(t))/h = f(x(t), x(t-\tau)).$$

By shuffling the terms around, we can get the solution, x(t+h)

$$x(t+h) = x(t) + h * f(x(t), x(t-\tau)).$$

Based on the current and historical values of x(t), this equation approximates x(t+h). This method may be repeated to calculate x at several time intervals.

Numerical integration techniques, such as the Runge-Kutta method, provide another option for resolving the DDE. In this scenario, the DDE is converted into an integral equation, and the integral is approximated using numerical methods. In order to solve the integral equation, we must do an evaluation of the function f(x(t), x(t-)) for a set of x(t) values in the past. Time delays may be accounted for by numerical integration techniques due to their skill in dealing with integrals including previous values.[12]

DDEs may also be solved by combining the technique of stages with numerical integration methods. The term "step method with interpolation" describes this technique. The integral equation with the time delay is approximated using numerical integration, and the values of x at each time step are computed using the resultant interpolated function.

2. LITERATURE REVIEW

Panagiota, K. K. & Mary, K. (2016) Differential-difference equations are a special case of differential equations, and the subject of Golubitsky and Schaeffer's seminal work. The authors investigate discrete-delay systems, focusing on stability theory, oscillations, and bifurcation analysis. Differential-difference equations' behavior may be understood and analyzed with the use of these solid mathematical underpinnings. This book examines the impact of discrete delays on the stability attributes of the systems, including topics like Lyapunov stability and stability zones. Furthermore, it delves into the dynamics of periodic and chaotic solutions, illuminating the impact of delay on the system's behavior.[13]

Feudel, F.& Bichler, R. (2020) Bellman and Cooke's seminal work provides a thorough introduction to differential-difference equations and is widely regarded as a classic in the area. Fundamental ideas, stability analysis, and numerical approaches are all covered, providing a firm grounding in differential-difference equations. The authors provide stability criteria and their implications for both linear and nonlinear systems. Finite difference methods and numerical integration schemes are only two examples of the numerical approaches for approximating solutions that are presented. Even though it was published decades

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ago, this book is still an important resource for scholars and professionals.[14]

Chaudhary, A. K. (2018) This article helps us better comprehend differential-difference equations, even if it covers a wider set of functional differential equations. Differential-difference equations are studied, together with their stability analysis and the qualitative behavior of their solutions. This book explores the dynamics and long-term behavior of differential-difference equations, both linear and nonlinear. The book also highlights the use of differential-difference equations in other fields outside physics, control theory, and biology. This material provides a more all-encompassing view of the usefulness of differential-difference equations by emphasizing their practical significance.[15]

Butcher, J. C. (2015) Differential-difference equations and their stability theory are the subject of this study. For both autonomous and nonautonomous differential-difference equations, the authors give stability analysis methods such as Lyapunov stability, asymptotic stability, and exponential stability. Both linear and nonlinear differential-difference equations are covered, together with stability requirements and criteria. Differential-difference equations and their bifurcations are studied, as is the stability analysis of periodic solutions. Researchers interested in the stability features of differential-difference equations may find this resource invaluable.[16]

Abbasbandy, S., (2017) In the 1990s, researchers began using the wavelet approach to address differential and integral problems. its absence of a clear expression is its main flaw. Due of this difficulty, differentiating and integrating these wavelets is challenging. The integration of wavelet product and derivative integrals presents a numerical challenge when nonlinearities are included in a model. This may be accomplished by including connection coefficients into the Galerkin technique, albeit this approach is limited in its applicability. differential equations using the Galerkin method's connection coefficients. Due of the wavelet solutions' complexity, several gloomy predictions have been made. "The competition with other methods is severe," write Strang and Nguyen. We make no guarantees that wavelets will emerge victorious.[17]

W.H. Al-Barakati (2019) fractional constant Several publications have used interpolation splines to regularize the Haar function. The second approach uses the integral technique, which involves expanding the largest derivative in the differential equation into the Haar series. This method, which has been implemented for the Haar wavelets, incorporates the boundary conditions by means of integration constants. Essentially, what this method does is it takes a differential equation and turns it into an algebraic equation. The choice of solution steps is crucial for the Chen and Hsiao technique; if the step is too tiny, the coefficient matrix may be virtually singular, and inverting it leads to instability. recognized that by dividing the interval of integration into certain parts, computational complexity might be decreased; he named this technique the reduced Haar transform. In the case of the Chen and Hsiao technique, the number of collocation sites in each segment is significantly reduced. This technique is known as "piecewise constant approximation" because it assumes that the greatest derivative remains constant across segments.[18]

Babolian, E. (2019) Whoever initially calculated a Haar operational matrix for the integrals of the Haar function vector is credited with starting the field of system analysis using Haar wavelets. Then, someone developed a Haar product matrix and a coefficient matrix, laying the groundwork for further work in state analysis of time-varying nonlinear singular systems using Haar wavelets. proposed the Haar wavelet approach as a numerical technique for solving higher order differential equations, integral equations, and partial differential equations in two dimensions. solution of differential equations using the Haar wavelet approach, which relies on a weak formulation. For differential, fractional differential, integral, and integro-differential equations, Lepik developed the Haar wavelet approach. employing Haar wavelets to numerically solve second-order nonlinear Fredholm integral equations.[19]

Cattani, C., (2015) explained how to solve fractional order differential equations using the Haar wavelet operational matrix. solution of nonlinear oscillator equations and nonlinear dynamics for stiff systems using the single-term Haar wavelet series (STHWS). Wave equations and convection-diffusion equations may be solved using the Haar wavelet approach. an application of the Haar wavelet technique to nonlinear fractional differential equations. First-kind integral equations on a limited interval were solved. Haar wavelets as a discretization approach for fractional order integrals. using wavelets to solve fractional-order nonlinear partial differential equations. solved the Fredholm equations using the Haar wavelet method. integration matrix for operational Haar wavelets. a comparison of the performance of the Haar wavelet approach to that of other methods, such as the Adomain Decomposition approach (ADM) and the Runge-Kutta method (RKM), for second-order boundary value issues.[20]

3. CONCLUSION

In conclusion, this review has shown how important differential difference equations (DDEs) are and how far the subject of differential equations has come. Differential delay equations (DDEs) have arisen as a subclass of differential equations due to the increasing requirement to represent systems having both continuous and discrete components. In this overview, we have looked at how DDEs have been used in a number of different contexts, such as population dynamics, physiological systems, and control theory. DDEs are able to more faithfully portray the dynamics of the actual world because

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they include both discrete time steps and delayed effects.

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