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A Study of Cordial Labeling of Friendly Index Related Graphs

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Abstract - One of the most interesting problems in the area of Graph Theory is that of labeling of graphs. A labeling or valuation or numbering of a simple graph G is a one-to-one mapping from its vertex set (edge set) into a set of non-negative integers which induces an assignment of labels to the edges (vertices) of G. Labeled graphs serve as useful models for a broad range of applications. They are useful in many coding theory problems, including the design of good radar type codes. Synch-set codes, missile guidance codes and convolution codes with optimal non – standard encodings of integers. Labeled graphs have also been applied in determining ambiguities in X – ray crystallographic analysis, and designing a communication network addressing system in determining optional circuit layouts and radio astronomy problems. Apart from the graceful labeling, some other labeling was also defined and developed. Graham and Sloane introduced harmonious graphs. Some of the other known labelings are Prime labeling, Arithmetic labeling, Edge graceful labeling, Felicitious labeling, Antimagic labeling, Cordial labeling, Prime cordial labeling, Edge cordial labeling, Ek – cordial labeling, Difference cordial labeling, Friendly labeling, Divisor cordial labeling, Skolem graceful labeling, Product cordial labeling.

Keywords - Cordial Labeling, Friendly Index Related Graphs, Graph Theory, interesting problems, non-negative integers, Labeled graphs

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INTRODUCTION

the friendly index set of graphs. Let G be a graph with vertex set V(G) and edge set E(G). A labeling f: V(G) → Z2 induces an edge labeling f*: E(G) → Z2 defined by f*(xy) = f(x) +f(y), for each edge xy Œ E(G). For i ϰ Z2, let vf(i) = number of vertices v ϰ V(G) with f(v) = i and ef(i) = number of edges e ϰ E(G) with f*(e) = i. A labeling f of a graph G is said to be friendly if |vf(0) - vf(1)| ≤ 1. The friendly index set of the graph G, FI(G) is defined as {|ef(0) - ef(1)|: the vertex labeling f is friendly}.

The following works have been done by several authors.

- For any graph with q edges, the friendly index set FI(G) ⊆ {0, 2, 4, ..., q} if q is even, and FI(G) ⊆ {1, 3, ..., q} if q is odd.
- The friendly index set of a cycle, $FI(C_{2n})$ = {0, 4, 8, …, n} if n is even and $FI(C_{2n})$ = {2, 6, 10, ..., 2n} if n is odd.
- The friendly index set of a path, $FI(P_n)$ = {1, 3, 5, ..., n - 1} if n is even and $FI(P_n)$ = {0, 2, 4, ..., (n - 1)} if n is odd
- For n ≥ 2, $FI(C_{2n} \times P_2)$ = {0, 4, 8, ..., 6n - 8, 6n} if n is even and $FI(C_{2n} \times P_2)$ = {2, 6, 10, …, 6n - 8, 6n} if n is odd.
- $F(C_3 \times P_2)$ = {1, 3, 5}
- For n ≥ 2, $FI(C_{2m+1} \times P_2)$ = {6n-1} ∪ {6n-5-2k / where k ≥ 0 and 6n-5-2k ≥ 0}

Friendly index set of some graphs which are not dealt with is proved in this study.

FRIENDLY INDEX SET OF CYCLE RELATED GRAPHS

Theorem 1: The friendly index set of one point union of t copies where t ≥ 5 and t is odd, of cycle C_3, that is $C_3^{(t)}$ is {1, 3, ..., t}.

Proof: Let the common vertex be w and a1, a2, ..., a2t be the vertices of t cycles.

Then the vertex set of the graph $C_3^{(t)}$ is $V(C_3^{(t)}) = \{w\} \cup \{a_i / 1 \le i \le 2t\}$

The edge set of the graph $C_3^{(t)}$ is $E(C_3^{(t)}) = \{wa_i / 1 \le i \le 2t\} \cup \{a_{2i-1}a_{2i} / 1 \le i \le t\}$

It has 3n vertices and 3n - 1 edges.

Make the w label equal to zero. Starting at 0, label the vertices with successively higher numbers. To rephrase: (a1, a2,..., a2t) = (0, 1, 0, 1, ..., 0, 1). In this case, vf(1) - vf(0) = 0 indicates a pleasant vertex labeling.

Without altering any other vertex labels, we rearrange the vertices by changing the labels a4i-2 and a4i-1, where I = 1, to their complement. The vertex labeling is pleasant, and it's clear that vf(0) and vf(1) haven't changed at all.

Similarly, with all the vertex unchanged and changing the vertex label a4i-2 and a4i-1, where i = 2 to its complement and is repeated up to $i = \left[\frac{t}{2}\right]$.

Theorem 2: Union of the cycle C3 and a path Pn sharing a vertex in common has the friendly index set {1, 3, 5, …, n}, if n is odd and {0, 2, 4, …, n}, if n is even.

Proof: Let u1, u2, u3 be the vertices of C3. Let w1, w2,…, wn be the vertices of Pn. Let w1 is identified with u1. Then the graph obtained is union of C3 and Pn sharing a vertex in common. Let it be G.

The vertex set of G is $V(G) = \{w_i \,/\, 1 \le i \le n-1\} \cup \{u_2, u_3\}$

The edge set of G is $E(G) = \{w_1u_2, u_2u_3, w_1u_3\} \cup \{w_iw_{i+1} \,/\, 1 \le i \le n-1\}$

Note that G has (n + 2) edges and (n + 2) vertices

Case (i): n is odd

Fix $u_2 = 1$ and $u_3 = 0$

Label the vertices $w_1, w_2, \ldots, w_n$ of P_n by alternatively 0's and 1's starting with zero. Then $v_f(0) - v_f(1) = 1$. Therefore $|e_f(0) - e_f(1)| = n$

Then keep the labels of w1 and w2 unchange and change all the remaining labels of the vertices to their complement. Then the vertex labeling is vf(1) - vf(0) = 1 and $|e_f(0) - e_f(1)| = n - 2$

Then keep the labels of w1, w2, w3, w4 remain unchanged and change all the remaining vertices to its complement. Then the vertex labeling is friendly with $v_f(0) - v_f(1) = 1$ and $|e_f(0) - e_f(1)| = n - 4$

Continuing this to the end of the path, we get one step That is, $|e_f(0) - e_f(1)| = 3$ Finally change the label of wn to its complement and the vertex labeling is friendly and $|e_f(0) - e_f(1)| = 1$

Then the friendly index set is {1, 3, 5, ..., n}

FRIENDLY INDEX SET OF PATH RELATED GRAPHS

Theorem 1: FI(P2 + mk1) = {1, 3}, if m is even and FI(P2 + mk1) = {1, 5}, if m is odd.

Proof : Consider a path P2 with two vertices v1, v2. Let y1, y2, ..., ym be the m isolated vertices. Join v1, v2 with yi, 1 ≤ i ≤ m. The graph obtained is P2 + mk1. The vertex set of G is V(G) = {v1, v2, y1, y2, ..., ym}.

The edge set of G is E(G) = {(v1v2)} » {(v1yi) | I ≤ i ≤ m} » {(v2yi) | I ≤ i ≤ m}. Note that the graph has m + 2 vertices and 2m + 1 edges

Case (i): m is even

Label v_1 and v_2 as 0 and $y_{\frac{m}{2}}, y_{\frac{m}{2}+1}, \ldots, y_m$ as 1 and the remaining vertices as 0. Then the vertex labeling is friendly with $v_f(1) - v_f(0) = 0$ and $|e_f(1) - e_f(0)| = 3$.

Then label v1 as 0 and v2 as 1 and label the vertices of yi's as alternatively 0 and 1. This vertex labeling is also friendly with $v_f(1) - v_f(0) = 0$. Then $|e_f(0) - e_f(1)| = 1$.

Therefore the friendly index set is {1, 3}.

Theorem 2: Union of a path Pn and a star K1,n sharing a vertex in common has the friendly index set { 1, 3, 5, …, 2n - 5}

Proof: Let the n vertices of path Pn be u1, u2, ..., un. Let the n spokes of star K1,n be v1, v2, …, vn and v0 be the centre vertex of the star. Identify u1 and v0. Let the graph so obtained is G.

The vertex set of G is $VG) = \{u_i \,/\, 1 \le i \le n-1\} \cup \{v_i \,/\, 1 \le i \le n\}$

The edge set of G is $E(G) = \{u_iu_{i+1} \,/\, 1 \le i \le n-1\} \cup \{u_1v_i \,/\, 1 \le i \le n\}$

Clearly G has 2n vertices and 2n – 1 edges.

First label $(u_1, u_2, \ldots, u_n) = (1, 0, 0, \ldots, 0)$ and $(v_1, v_2, \ldots, v_n) = (0, 1, 1, \ldots, 1)$.

The vertex labeling is friendly and $|e_f(0) - e_f(1)| = 2n - 5$

Now we have a friendly labeling by keeping all the vertex labels the same except for u2 and v2, which have been given their complements. Then $|e_f(0) - e_f(1)| = 2n - 3$.

Now, a pleasant labeling is accomplished by keeping all the vertex labels the same except for the fact that vertices u3 and v3 have been swapped. Then $|e_f(0) - e_f(1)| = 2n - 1$.

Continuing this process up to changing the vertex un-2 and vn-2 to its complement.

Then the friendly index set is {1, 3, 5, …, 2n - 5}

Theorem 3: The Umbrella graph U(m, n) [1.2.17] where n = 2 has the friendly index set {0, 2, 4, ..., m + 1}, if m is odd and m ≥ 9 and {0, 2, 4, ..., m}, if m is even and m ≥ 4

Proof: Let G = U(m, 2). The vertex set of U(m, 2) is $V(G) = \{x_1, x_2, \ldots, x_m, y_1, y_2\}$

The edge set of U(m, 2) is $E(G) = \{(x_ix_{i+1}) | 1 \le i \le m - 1\} \cup \{(y_1y_2)\}$
$\cup \{(x_iy_1), 1 \le i \le m\}$.

Note that the graph has m + 2 vertices and 2m edges

Case (i) : m is odd First label alternatively with 0's and 1's starting with 0 in xi's and label y1 by 1 and y2 by 0. Then $v_f(0) - v_f(1) = 1$ and $|e_f(0) - e_f(1)| = m + 1$

Next interchange only y1 and y2. Then $v_f(0) - v_f(1) = 1$ and $|e_f(0) - e_f(1)| = m - 1$.

After x1, the following two vertices are both labeled with 1, and the following two vertices are both labeled with 0, and so on. In that case, we can indicate that y1 is 1 and y2 is 0 by using those labels. Then $|v_f(0) - v_f(1)| = 1$ and $|e_f(0) - e_f(1)| = m - 3$.

FRIENDLY INDEX SET OF STAR RELATED GRAPHS

Theorem 1: The friendly index set of spl(k1,n) is {0, 2, 4, ..., n}, if n is even and {1, 3, ..., n}, if n is odd.

Proof: Let v1, v2, ..., vn be the pendant vertices, v be the apex vertex of K1,n and u, u1, u2, .., un are the added vertices corresponding to v, v1, v2, ..., vn in spl (k1,n)

The vertex set is $V(G) = \{v, v_1, v_2, ..., v_n, u, u_1, u_2, ..., u_m\}$

The edge set is $E(G) = \{vv_i / 1 \le i \le n\} \cup \{vu_i / 1 \le i \le n\} \cup \{uv_i / 1 \le i \le n\}$

Thus $spl(k_{1,n})$ has 2n + 2 vertices and 3n edges.

Case (i): n is even

Fix v = 0 and u = 1

First label the vertices of v_i's as (0, 0, ..., 0) and u_i's as (1, 1, ..., 1) in $spl(k_{1,n})$. Then the vertex labeling is friendly and $|e_f(0) - e_f(1)| = n$

In the next step decrease one vertex labeling ie, '0' from v_i's and label with its complement ie, '1'. Also decrease one vertex labeling, ie '1' from u_i's and label with its complement ie, '0'. Then the vertex labeling is friendly and $|e_f(0) - e_f(1)| = n - 2$

The vertices remain unchanged and continue the above step upto $\frac{n}{2}$ vertices having the label 0 in v_i's and $\frac{n}{2}$ vertices having the label 1 in u_i's.

Therefore the friendly index set is {0, 2, 4, ..., n}

DIFFERENCE CORDIAL LABELING OF GRAPHS CYCLE

A concept called difference cordial labeling is presented in [45] by R. Ponraj, S. Sathish Narayanan, and R. Kala in 2013. In this case, we'll pretend that G is a (p, q) graph. Take into account the injective function f: V(G) [-1, +2..., +p]. With this, we obtain the map f* on E(G) with the equation f*(uv) = |f(u) - f(v)| for all u, v. This value is 1 if the difference is 1, and 0 otherwise. With ef(i) equal to the number of edges labeled with I where I = 0, 1, f is a difference cordial labeling if and only if |ef(0) - ef(1)| > 1. Differentially labeled graphs are referred to as "difference cordial graphs." These works are the product of a collaborative effort by multiple authors.

v Any path is a difference cordial graph.

v The star K1,n is difference cordial iff n ≤ 5.

v Kn is difference cordial iff n ≤ 4.

v K2,n is difference cordial iff n ≤ 4

v K3,n is difference cordial iff n ≤ 4

v B1,n is difference cordial iff n ≤ 5

v B2,n is difference cordial iff n ≤ 6

v B3,n is difference cordial iff n ≤ 5

v The gear graph Gn is difference cordial.

v All webs are difference cordial

v Every ladder graph Ln is difference cordial for all n.

v The triangular ladder graph TLn, n ≥ 2 is a difference cordial for all n.

v Every step ladder graph is a difference cordial for all n.

v All Bow graphs are difference cordial.

v All Butterfly graphs are difference cordial

Now we shall prove the following graphs are difference cordial graphs.

Theorem 1: A vertex switching of cycle Cn(VSCn) [1.2.22] is difference cordial, for all n ≥ 4

Proof: Let G be a (p, q) graph.

A cycle's vertex switch is indicated by the symbol VSCn. One obtains it by starting with a vertex a1 in Cn, eliminating any edges incident with a1, and then adding edges attaching a1 to every vertices that is not adjacent to a1 in Cn.

Let the vertex set of VSC_n be $V(VSC_n) = \{a_i / 1 \le i \le n\}$ and

Let the edge set of VSC_n be $E(VSC_n) = \{(a_ia_{i+1}) / 2 \le i \le n - 1\} \cup \{(a_1a_j) / 3 \le j \le n - 1\}$

Note that the graph VSC_n has 2n - 5 edges and n vertices.

Define f: $V(G) \to \{1, 2, ..., p\}$ as follows

$$f(a_1) = n$$

$$f(a_i) = i - 1, 2 \le i \le n$$

Then the function f induces the function f* on $E(VSC_n)$ as follows

$$f^*(a_ia_{i+1}) = 1, 2 \le i \le n - 1$$

From this induced edge label, we have the number of edges labeled 1 is n – 2

That is, $e_f(1) = n - 2$

$$f^*(a_1a_{i+2}) = 0, 1 \le i \le n - 3$$

From this induced edge label, we have the number of edges l

That is, $e_f(0) = n - 3$

Thus $e_f(0) = n - 3$ and $e_f(1) = n - 2$

Therefore we get $|e_f(0) - e_f(1)| \leq 1$

Hence VSC_n is Difference cordial, for all $n \geq 4$

Illustration: Difference cordial labeling of VSC9 is given in figure 1

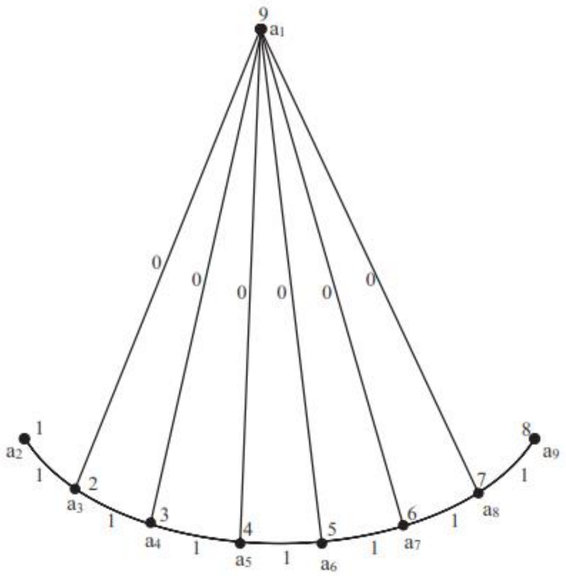

Figure 1. VSC9

G is a (9, 13) graph

Here number of edges labeled 0 is 6

That is, ef(0) = 6

Number of edges labeled 1 is 7

That is, ef(1) = 7

Then |ef(0) - ef(1)| = 1

Hence the above graph is difference cordial.

CONCLUSION

Labeling graphs is one of the most fascinating issues in Graph Theory. The assignment of labels to the edges (vertices) of a simple graph G is induced by a mapping from the vertex set (edge set) to a set of non-negative integers. Labeled graphs can be used as models in many contexts. They have several applications in coding theory, including the creation of high-quality radar-type codes. Best non-standard integer encodings in synch-set codes, missile guidance codes, and convolution codes. Labeled graphs have also been used to solve difficulties in radio astronomy, X-ray crystallography, and the design of a communication network's addressing scheme, among other areas of study. It was G Ringel's conjecture, "All trees are graceful," that sparked interest in the graph labeling problem. The term "-valuation of graph" was coined by Rosa in 1967, and Golomb praised its elegance in labeling. Graceful labeling enabled a great deal of productivity. The families have been elegantly numbered in the thousands. The definition and development of other labels occurred alongside the elegant labeling. It was Graham and Sloane who first used a graph with symmetry and harmony. Other names for this process include: the Prime label, the Arithmetic label, the Edge graceful label, the Felicitous label, the Antimagic label, the Cordial label, the Prime cordial label, the Edge cordial label, the Ek - cordial label, the Difference cordial label, the Friendly label, the Divisor cordial label, the Skolem graceful label, and the Product cordial label.

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