



A Parametric Study on Fuzzy Queuing Model with Fuzzy Parameter

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Abstract: The number of servers, self-service queues, and the machine fix model, to give some examples. We will ascertain enduring state probabilities and sitting tight times for the models when conceivable. Regardless of whether you're a business visionary, architect, or supervisor, finding out about queueing theory is an extraordinary method to be increasingly successful. Queueing theory is key to getting great profit for your endeavors. That is on the grounds that the outcomes your systems and groups create are vigorously impacted by how much holding up happens, and holding up is squander. Limiting this waste is critical. It's one of the greatest switches you will discover for enhancing the expense and execution of your groups and systems. As you have seen, there are various types of queueing systems, contingent upon what number of queues and servers there are, and how they're associated together. Different arrangements of queueing systems are ordinarily portrayed with Kendall's Notation, which gives helpful shorthand for labeling classes of systems. These are vital on the grounds that various types of systems have altogether different queueing behavior, which sometimes results in definitely extraordinary hold uptimes and queue lengths. In case you will break down them, you'll should make certain you comprehend what sort of system you're working with, so you can pick the privilege analytical tools. As you have seen, there are various types of queueing systems, contingent upon what number of queues and servers there are, and how they're associated together. Different designs of queueing systems are ordinarily depicted with Kendall's Notation, which gives helpful shorthand for labeling classes of systems.

Keywords: Fuzzy Parameter, Queue Model, Queueing theory, labeling classes

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INTRODUCTION

Queueing models have been turned out to be extremely helpful in numerous reasonable applications in territories, for example, e.g., generation systems, stock systems and correspondence systems. The inconsistency of the arrival and service processes in many applications is fundamental to the performance of the system. Queueing models help us to understand the impact of inconsistencies and to assess them. Kendall had a brief documentation depicting a range of these queueing models. It's a third part a/b/c code. The main letter defines the time distribution of the interarrival and the second the distribution of the time of service. For example, the letter G, M in the exponential distribution (M stands for the Memoryless) and D in deterministic times is used for a general distribution. For example. The number of servers is defined in the third and last letters. M/M/1, M/M/c, M/G/1 and M/D/1 are just a few models. With additional letter to cover other queueing templates the documentation can be reached. For example, the four letter codes M/M/1/N abridge a system with exponential intercom and service times, a single server, and lounge for N customers only (counting the one in service). Models of queueing are also called line models holding up. Here we study what happens when a person or many people go to queues. In our daily life, we are very familiar with queues. Normal cases of queueing models we experience go to a specialist or go to a hairpiece. In essence, a queueing system occurs when elements or people call arrivals who need some kind of service from another element. A service exists and the person connects to that system on a line or a queue.

A stochastic process is a collection of time-listed random variables. One alternative view is that it is a distribution of probabilities across a number of ways: it regularly portrays a random appreciation or system after a while. There is a settled direction (route) in a deterministic process that the process goes but in a stochastic process we don't know how. This should not be regarded as information of the way, as the probability distribution gives the information on the way. This can be likened to a determinist process, for example, given that the probability distribution is given as a probability one. Furthermore, the process usually develops after a while. Nevertheless, from a formal numerical point of view, a higher priority image is that we make them hidden (obscure) and look just at the bottom of the way. A stochastic process has discrete time when the time variable takes positive integer values, and time is uninterrupted when the time variable takes true values. We start by studying discrete processes of stochastic time. These processes can be explicitly communicated and are gradually 'substantial' or 'simple to understand.'

A collection system of customers/things in some place, including people getting the service, is known as queue or line-ups. Queuing theory is the principal part of time patterns, where stochastic exchange is involved. The basic motivation behind the theories is to explore rapidly those queuing issues in companies, transport and business that concern times due to people and machines, as well as to complete the creation of a thing by using tandem, bi-tandem queues, at least two distinct progressive phases of the stochastic process. For researchers who need to carry out basic research on stochastic processes, including numerical models, the trend towards analytical research has been developed and queuing theory has been shown. The theory of probability gives the establishment theory and stochastic model theories of tele-traffic, so it is important for the experts in the understudies to develop the probability ideas for the material they use. We aim to include perusers with some probability foundation adequately in this section. Although the cover is complete here, because it speaks of all the likely ideas and methods used in later parts, it excludes the numerous precedents and activities regularly incorporated into a probability course book, which allows readers to get a better handle on the material. The Queuing models can be used for a wide range of applications, including machine fixes, toll booths, taxi stands, boat stationing and vacuuming, patient bookings in healing centers, PC planning, timing sharing and systems structures, media transmission, and service associations. The traditional theory of queuing expects certain preset probability distributions to be followed by interval time and service times. Ethymological articulations like fast, moderate, moderate, suitable or missing, as opposed to probability distributions, can be all the more practical in numerous practical conditions for arrival and services examples and times. All the qualities of queuing along these lines can be better determined by fuzzy numbers, such as the arrival rate, service and the excursion rate. This gives the extent to which the queues of flouted theory are studied. In areas such as assembly system, media transmission and data processing, fluid queues can be adequately connected.

FUZZY LOGIC

Fuzzy logic is best defined as a mathematical logic in which true values between 0 and 11 can be assumed. It is known as bivalent logic that each statement must either be true or false. The central concept of fluid logic is that each proposition can be partially or partially false, as well as true or false. In addition, fugitive logic allows a given statement to be partially true and at the same time partially false. It is a system that mathematically expresses partial truth. Fuzzy's logic is understood by examples, as are many theories. "Georgia Tech is a great school," take the statement. This statement can either be true or false by means of

the bivalent logic. Fuzzy Logic says that this is 100% true if student registration is 10,000 or higher, but only 50% true if registration is three,000 and 0% true if the registration is fifty per cent. The first introduction of the Fuzzy Logic theory in a 1965 paper titled "Fuzzy Sets" was by Lotfi Zadeh, a professor at Berkeley University California. There can be four major contributions to the philosophical basis of the Fuzzy logic. In 200 B.C., Aristotle suggested the "Law of the Excluded Middle," which held that all statements must be false or true. Plato was one of the first to suggest the dichotomy did not describe reality fully. He theorized that between true and false there was a state. Logician Jan Lukaisiewicz suggested mathematics in 1920 for a tri-valued logic that incorporated the concept of fractional reality. This third logic value was referred to as "possible," beyond true and false. The ideas of Lukaisiewicz formed the basis of a wide range of research on what became known as highly valued logic.

FUZZY NUMBERS

Many fuzzy linguistic designs, like 'low,' 'medium,' 'high,' etc., are used to define a variable status. The relevancy of a fluid variable is that it facilitates gradual transitions between states and therefore has a neutral ability to express and manage uncertainties. Computing involves the handling of numbers and symbols in the traditional sense. But humans, by contrast, use mainly words in computing, reasoning, and natural language, or mental perceptions. One key aspect of word computation is that natural languages are merged and calculated with fluctuating variables. In computing words, the concept of a granule plays an essential part. "Study plays a key role in human cognition," Zadeh says. This is a way to achieve comparison of data for humans. Fuzzy sets, defined in the set R of real numbers, have a particular significance. Membership functions $\mu : R \rightarrow [0,1]$ clearly possess a quantitative meaning and may be viewed as fuzzy numbers provided they satisfy certain conditions. Initiative concepts of approximate numbers or intervals such as near 5' numbers or numbers around the actual numbers. Such concepts are essential to characterize state of fluctuating variables. In many applications, these fugitive numbers play an important role, including fugitive control, decision-making, approximate reasoning and optimization. A fuzzy number is the fuzzy subset of the real line, where a given real number is the highest membership values. The membership function on both sides of the key values is monotonous for a fuzzy number. The following thesis defines a fuzzy number that is generally accepted in literature.

Definition: Fuzzy Number

A fuzzy subset A of the real line R with membership function $\mu_A : R \rightarrow [0, 1]$ is called a fuzzy number if

- (i) A is normal, i.e., there exists an element $x_0 \in A$ such that $\mu_A(x_0) = 1$.
- (ii) A is fuzzy convex, i.e., $\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \{\mu_A(x_1) \wedge \mu_A(x_2)\} \quad \forall x_1, x_2 \in R$ and $\forall \lambda \in [0, 1]$.
- (iii) μ_A is upper semi continuous and
- (iv) $\text{Supp } A$ is bounded where $\text{supp } A = \{x \in R : \mu_A(x) > 0\}$

Types of Fuzzy Numbers

Triangular Fuzzy Number

A triangular fuzzy number $A \sim$ is a fuzzy number specified by 3-tuples (a_1, a_2, a_3) such that $a_1 \leq a_2 \leq a_3$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ (x - a_1)/(a_2 - a_1) & \text{if } a_1 \leq x \leq a_2 \\ (x - a_3)/(a_2 - a_3) & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{if } x \geq a_3 \end{cases}$$

This is represented diagrammatically as

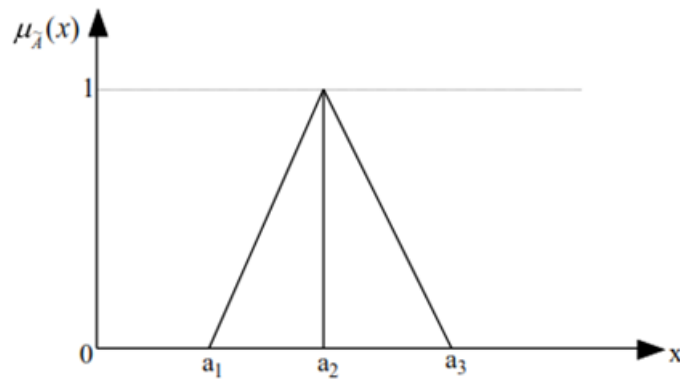


Figure 1. Membership Function of Triangular Fuzzy Number A

Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $A \sim$ is a fuzzy number fully specified by 4-tuples (a_1, a_2, a_3, a_4) such that $a_1 \leq a_2 \leq a_3 \leq a_4$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ (x - a_4)/(a_3 - a_4) & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

This is represented diagrammatically as

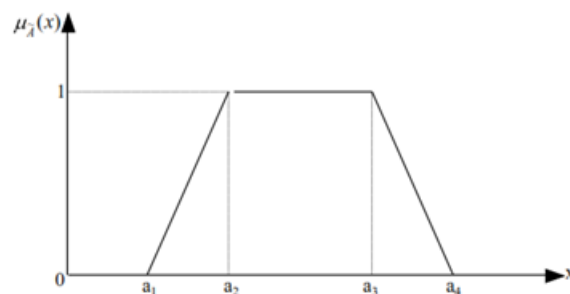


Figure 2. Membership function of a Trapezoidal fuzzy number A

Piecewise Quadratic Fuzzy Number

A piecewise – quadratic fuzzy number (PQFN) is defined by the membership function as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{-(x-a_1)^2 + 1}{2(a_2-a_1)^2} & \text{for } a_1 \leq x \leq a_2 \\ \frac{-(x-a_3)^2 + 1}{2(a_3-a_2)^2} & \text{for } a_2 \leq x \leq a_3 \\ \frac{-(x-a_3)^2 + 1}{2(a_4-a_3)^2} & \text{for } a_3 \leq x \leq a_4 \\ \frac{-(x-a_5)^2}{2(a_5-a_4)^2} & \text{for } a_4 \leq x \leq a_5 \\ 0 & \text{otherwise} \end{cases}$$

The PQFN is a bell shaped function and symmetric about the line $x = a_3$, has a supporting interval $[a_1, a_5]$. Moreover, and α -cut at level between the points $(a_2, 1/2)$ and $(a_4, 1/2)$ called cross over points. Also the interval of confidence at level α is

This is diagrammatically

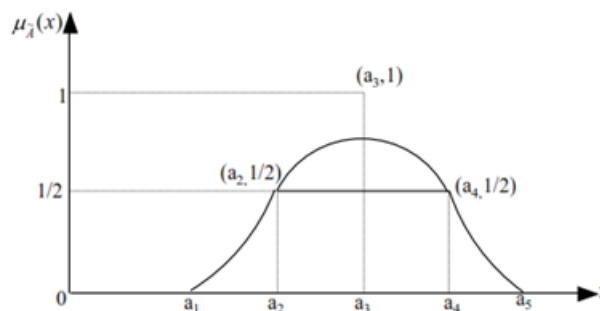


Figure 3. Membership Function of a Piecewise Quadratic Fuzzy Number

Function Principle

Function principle was introduced by Chen [12] to treat the fuzzy diametrical operations with triangular, trapezoidal and piecewise quadratic fuzzy number.

PARAMETRIC STUDY ON FUZZY QUEUEING MODEL

Queuing models are broader for both the services organization and the manufacturing companies, where different clients are serviced according to the specific queue discipline through different types of servers. The intervals and the service times of certain distributions must be followed within the context of traditional queuing theory. In practice, service rate is often described as fast, slow or moderate by linguistic terms, which can best be described by the fluency sets.

Li and Lee had analysis findings based upon a Zadeh extension principle for two fuzzy queuing systems. As Negi and Lee commented, their approach is however rather complicated and not usually computational.

Therefore, Negi and Lee propose two approaches: the rest of the formulations and two variable. Sadly, their approach only provides possible numbers rather than intervals; in other words, the performance measurements' membership functions are not fully described. This model follows the cut-off approach of breaking a fluffy queue into a family of crisp queues. The parametric programming technique for describing the crisp queues family is used when T-varies. The solution derives the membership of the crisp queues from the parametric programs. In order for a fluid queue FM/FM/1 to show the validity of the proposed approach, FM indicates fluid exponential time.

PROBLEM FORMULATION

Consider a general queuing system with one server. The inter arrival time and service time are approximately known and are represented by the following fuzzy sets.

Where x and y are the crisp universal sets of the inter arrival time and service time, and μ_x and μ_y are the corresponding membership functions.

Where $A(\alpha)$ and $S(\alpha)$ are crisp sets. Using α -cuts, the inter arrival time and service time can be represented by different levels of confidence intervals. Consequently, a fuzzy queue can be reduced to a family of crisp queues with different α -level sets. These two sets form nested structures for expressing the relationship between ordinary sets and fuzzy sets. Let the confidence intervals of the fuzzy sets are $[a_1, a_2]$ and $[b_1, b_2]$ respectively. When both inter arrival time and service time are fuzzy numbers, based on Zadeh's extension principle, the membership function of the performance measure $P(x, y)$ is defined as

Our approach is to construct the membership function which is based on deriving the α -cuts of μ_x and μ_y . The corresponding, Parametric programming technique for finding lower and upper bounds of the α -cut of μ_x and μ_y are

If both μ_x and μ_y are invertible with respect to α , then a left shape function and a right shape function can be obtained from which the membership function $\mu_{P(x,y)}$ is constructed.

THE (FM/FM/1): (\square /FCFS) QUEUES

This queue adopts a first-come first-served queue discipline and consider an infinite source population where both the inter arrival time and the service time follow exponential distributions with ratio λ and μ respectively, which are fuzzy variables rather than crisp values.

NUMERICAL EXAMPLE

Consider a FM/FM/1 queue, where both the arrival rate and service rate are fuzzy numbers represented by $\lambda = [3, 4, 5, 6]$ and $\mu = [19, 20, 21, 22]$. The interval of confidence at possibility level α as $[3 + \alpha, 6 - \alpha]$ and $[19 + \alpha, 22 - \alpha]$.

The parametric programs to derive the membership functions of LS

(1)

(2)

When x reaches its lower bound and y reaches its upper bound, attains its minimum. Consequently, the optional solution for (1) is . On the contrary, to maximize increases to its upper bound and y decreases to its lower bound. In this case the optimal solution for (2) is

Table 1: The α -cuts of at 11 distinct α Value

α	$\ell_{x(\alpha)}$	$u_{x(\alpha)}$	$\ell_{y(\alpha)}$	$u_{y(\alpha)}$	$\ell_{L_s(\alpha)}$	$u_{L_s(\alpha)}$
0.0	3.0	6.0	19.0	22.0	0.157	0.462
0.1	3.1	5.9	19.1	21.9	0.165	0.447
0.2	3.2	5.8	19.2	21.8	0.172	0.433
0.3	3.3	5.7	19.3	21.7	0.179	0.419
0.4	3.4	5.6	19.4	21.6	0.187	0.406
0.5	3.5	5.5	19.5	21.5	0.194	0.393
0.6	3.6	5.4	19.6	21.4	0.202	0.380
0.7	3.7	5.3	19.7	21.3	0.210	0.368
0.8	3.8	5.2	19.8	21.2	0.218	0.356
0.9	3.9	5.1	19.9	21.1	0.227	0.345
1.0	4.0	5.0	20.0	21.0	0.235	0.333

CONCLUSION

Queueing models can be isolated into two general gatherings. On one hand, those that depict genuine circumstances and, for other, those regularizing that report a remedy of what the genuine circumstance ought to be or, said in another way, the ideal perspective to which ought yearn for. Graphic models give normal values and the probabilities of execution estimates that portray the system when examples of arrivals and services, the number of servers, system limit and queue discipline have all been set. As opposed to these models, the second gathering, which is regularly called queueing choice models (plan and control models), endeavors to compute what the parameters ought to be to improve the models. When there is vulnerability with regard to data incorporation, models considered in this work must be improved. The use of fuzzy subset theory is used to resolve vulnerability. The count of questionable parameters and furious data in the queueing models involves that the capacity to be improved contains furious coefficients. Service actions, for instance the service rate, the number of servers and the queue discipline or a mixture of components, are generally controllable. Some checks can sometimes be applied when customers arrive, in order to increase their decrease, to set them to a server, or even to pay some sort of fee. For example, plan parameters can normally impose constraints on physical space and require specific Parameters which would be wild from the beginning. The study of classical tape models with fluid data that this study tries to use on both previously described routes. In this line, the paper manages different models that show the real circumstances of the queue when one or more parameters are uncertainly known and illuminates plan and control models to streamline fluid queueing systems. Basic queueing systems include composed queues where units are managed by their arrival request. This "holding up control" is often found in queueing models, yet for reasons of productivity and picking order, favorite classes of units with a specific need are characterized by the status of message transmission within a media communication system. Many genuine queueing systems are more careful than any other models that are possible to access this disciplinary requirement model.

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