Journal of Advances in Science and Technology
Vol. 20, Issue No. 1, March-2023, ISSN 2230-9659

Developments of the I-Function Several Complex Variables

Priti Pandey[1]*, Dr. Neelam Pandey[2], Dr. Yagya Pratap Sahu[3]

[1] Research Scholar Mathematics, Awadhesh Pratap Singh University, Rewa, Madhya Pradesh.

[2] Professor of Mathematics, Govt. Model Science College, Rewa, Madhya Pradesh

[3] Govt. SGS College Sidhi, Madhya Pradesh.

Abstract - The purpose of this study is to present the I-function of r-variables, a natural extension of the popular, intriguing, and practical Fox H-function. The results are quite generalized when r=1, at which point we get the I-function. Second, we showed that an integral is formed by multiplying the I-function of a single variable by the Bessel function and the General polynomial class. A two-variable I-function may be obtained by evaluating the Sine transform of the product of the Hermite polynomial and Fox's H-function, and its special case, the Cosine transform of the product of the Legendre polynomial and Fox's H-function. We have also provided exceptional examples, basic features, and circumstances under which convergence occurs. Results for H-functions in "r" variables are generalized in this study.

Keywords - I-Function, Variables, H-Function, Polynomials and Equation

- X -

INTRODUCTION

Functions defined on the complex coordinate space are the subject of the mathematical field known as the theory of functions of many complex variables. $\mathbb{C}^n$, meaning complex numbers in n-tuple form. The Mathematics Subject Classification includes a top-level heading titled "several complex variables (and analytic space)" for the discipline that studies the characteristics of these functions.

The functions under study are holomorphic or complex analytic, such that, locally, they are power series in the variables zi, just as they are in the complex analysis of functions of one variable, which is the case n = 1. They may also be thought of as locally square-integrable solutions to the n-dimensional Cauchy-Riemann equations or as locally uniform limits of polynomials. Any given domain for a complex variable $(D \subset \mathbb{C})$, is holomorphic to some function, or, put another way, every domain is holomorphic to some function. However, this is not the case when there are several complicated factors involved. $(D \subset \mathbb{C}^n,\ n \geq 2)$ It is not necessarily the realm of holomorphy, hence the concept of holomorphy's domain is one of the major topics in this area of study. The Cousin issue is the challenge of constructing a global meromorphic function from zeros and poles, or "patching" the local data of meromorphic functions. Furthermore, the study of compact complex manifolds and complex projective varieties relies heavily on the intriguing phenomena that occur in many complex variables. $(\mathbb{CP}^n)$ has a unique flavor compared to other forms of complex analytic geometry, $\mathbb{C}^n$, or on Stein manifolds, are more akin to complex analytic geometry than the study of algebraic varieties.

LITERATURE REVIEW

Rezepin, Alexandr et.al (2017). This article discusses the production function model as a tool for describing the production process and finding solutions to real-world issues like determining the most appropriate production technology or allocating capital efficiently. The manufacturing process is analyzed using Urals Stampings Plant as a case study. There are two elements to the analysis. In the first section, we utilize regression analysis to assess the production function and determine the revenue elasticity of the primary resources. Using artificial neural networks, the authors of the second section of the investigation look into how the dynamics of the number of production variables and productivity affect the issue's physical volume. In conclusion, the authors provide suggestions for improving the efficiency of the Urals Stampings Plant's investment plan.

Gerck, Edgardo et.al (2019). This paper introduces new avenues for the use of complex analysis in the physical and engineering sciences. Despite the fact that there is no such thing as a "complex number," it does provide a summary of the themes explored throughout four separate studies. The origin and destination of results are in real number theory; however, the complex plane plays a role and is not explicitly revealed. Also covered are the Cauchy principal value, conformal mapping, and visual representation; the Laplace equation; harmonic functions; subharmonic analysis; and the residue theorem.

Shadab, Mohd et.al (2018). In this paper, we introduce an extended beta function via the Mittag-Leffler function and explore various properties and formulas of the Mohd Shadab, Saima Jabee, and Junesang Choi 236 extended beta function, including integral repr, as a result of the widespread use of Euler's beta, hypergeometric, and confluent hypergeometric functions and their extensions in a wide variety of research fields, including engineering, chemical, and physical problems. We also give a graphical representation of the expanded beta function and a tabular summary of its specialized values, both of which may be calculated with relative ease by means of object-oriented programming languages.

Rachel Webb, (2023) For I-functions of quasimaps, we establish their abelian-nonabelian correspondence. If Z is an affine l.c.i. variety under the action of a complex reductive group G, then we provide a rigorous proof of a formula that relates the I-functions of the quasimaps of the geometric invariant theory quotients Z//G and Z//T, where T is a maximal torus of G. We use the formula to determine the J-functions of certain Grassmannian bundles on Calabi-Yau hypersurfaces in Grassmannian varieties.

Guala, Francesco et.al (2019). Define an institution. And what makes one institution different from another? Token institutions are concrete solutions to these difficulties, or strategic game equilibria, and are the focus of our answers to these concerns from a functionalist perspective. The functionalist perspective sheds light on how far institutions like marriage, property, and democracy may be altered before they cease to be the same thing altogether.

THE I-FUNCTION OF SEVERAL VARIABLES

The following defines and illustrates the generalized Fox H-function, often known as the I-function of "r" variables:

$$\mathrm{I}[z_1,\ldots,z_r] = \mathrm{I}^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c} z_1 \\ \vdots \\ z_r \end{array}\middle| \begin{array}{l} \left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)};A_j\right)_{1,p} : \left(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)}\right)_{1,p_r} \\ \left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)};B_j\right)_{1,q} : \left(d_j^{(1)},\delta_j^{(1)};D_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)};D_j^{(r)}\right)_{1,q_r} \end{array}\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{\mathscr{L}_1}\cdots\int_{\mathscr{L}_r}\phi(s_1,\ldots,s_r)\,\theta_1(s_1)\cdots\theta_r(s_r)\,z_1^{s_1}\cdots z_r^{s_r}\,ds_1\cdots ds_r,$$

where $\phi(s_1,\ldots,s_r)$, $\theta_i(s_i)$, $i = 1,\ldots,r$ are given by

$$\phi(s_1,\ldots,s_r) = \prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right) \times\left(\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_i\right) \times\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_i\right)\right)^{-1},$$

$$\theta_i(s_i) = \left(\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right) \times\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)\right) \times\left(\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right) \times\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)\right)^{-1},$$

where $i = 1, \ldots, r$.

Also,

(i) $zi = 0$/, for $i = 1, \ldots, r$;

(ii) $i = \sqrt{-1}$;

(iii) An empty product is equivalent to oneness;

(iv) the parameters mj, nj, pj, qj ($j = 1, \ldots, r$), n, p, there exists a pair of positive integers and q such that $0 \le n \le p$, $q \ge 0$, $0 \le nj \le pj$, and $0 \le mj \le qj$ ($j = 1, \ldots, r$) (not all zero simultaneously);

(v) $\alpha(i)\ j$ ($j = 1, \ldots, p$, $i = 1, \ldots, r$), $\beta(i)\ j$ ($j = 1, \ldots, q$, $i = 1, \ldots, r$), $\gamma(i)\ j$ ($j = 1, \ldots, pi$, $i = 1, \ldots, r$), and $\delta(i)\ j$ ($j = 1, \ldots, qi$, $i = 1, \ldots, r$) are standardized to be positive numbers for convenience. Even if some of the values are 0 or negative, the definition of the I-function of "r" variables will still make sense. In a subsequent section, we will provide possible transformation formulae for these;

(vi) aj ($j = 1, \ldots, p$), bj ($j = 1, \ldots, q$), $c\ (i)\ j$ ($j = 1, \ldots, pi$, $i = 1, \ldots, r$), and $d(i)\ j$ ($j = 1, \ldots, qi$, $i = 1, \ldots, r$) are complex numbers;

(vii) the exponents Aj ($j = 1, \ldots, p$), Bj ($j = 1, \ldots, q$), $C(i)\ j$ ($j = 1, \ldots, pi$, $i = 1, \ldots, r$), and $D(i)\ j$ ($j = 1, \ldots, qi$, $i = 1, \ldots, r$) values that are not integers for the different gamma functions used in

(viii) The Mellin Barnes type Li contour in the complicated si-plane extends from $c - i\infty$ to $c + i\infty$, (c

real) using appropriate indentation to ensure that all singularities of $\Gamma D(i)\ j\ (d(i)\ j - \delta(i)\ j\ si)$, $j = 1, \ldots, mi$ lie to the right and $\Gamma C(i)\ j\ (1 - c(i)\ j + \gamma(i)\ j\ si)$, $j = 1, \ldots, ni$ and Li's left side. Braaksma stablished that the I-function of "r" variables is analytic if and only if.

$$\mu_i = \sum_{j=1}^{p} A_j \alpha_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)}$$

$$- \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \le 0, \quad i = 1, \ldots, r.$$

I-FUNCTION OF ONE VARIABLE, BESSEL'S FUNCTION AND GENERAL CLASS OF POLYNOMIALS

Integrals involving the product of the I-function, the Bessel function, and the General class of polynomials have been obtained in this subsection.

$$\int_0^\infty x^{\lambda-1} J_\nu\left[x + a + \sqrt{x^2 + 2ax}\right] S_h^w\left\{\left[x + a + \sqrt{x^2 + 2ax}\right]^{-\gamma}\right\}$$

$$\times I_{p_i,q_i:r}^{m,n}\left[y\left[x + a + \sqrt{(x^2 + 2ax)}\right]^{-\mu} \Big| \begin{matrix} T \\ T' \end{matrix}\right] dx$$

$$= 2\left(\frac{a}{2}\right)^\lambda \Gamma(2\lambda) \sum_{l=0}^{\infty} f(l) a^{\nu+2l} \sum_{k=0}^{[h/w]} F(k)\, a^{-\gamma k}$$

$$\times I_{p_i+2,q_i+3:r}^{m,n+2}\left[y a^{-\mu} \Big| \begin{matrix} (\nu + 2l - \gamma k, \mu), (1 + \nu + 2l + \lambda - \gamma k, \mu), T \\ T', (-\nu - l, 0), (1 + \nu + 2l - \gamma k, \mu), (\nu + 2l - \lambda - \gamma k, \mu) \end{matrix}\right]$$

where
$\nu > 0, \gamma > 0, \mu > 0, 0 < Re(\lambda) < (\mu\xi + \gamma k - \nu - 2l)$.

Proof: Let

$$I = \int_0^\infty x^{\lambda-1} J_\nu\left[x + a + \sqrt{x^2 + 2ax}\right] S_h^w\left\{\left[x + a + \sqrt{x^2 + 2ax}\right]^{-\gamma}\right\}$$
$$\times I_{p_i,q_i:r}^{m,n}\left[y\left[x + a + \sqrt{(x^2 + 2ax)}\right]^{-\mu} \Big| \begin{matrix} T \\ T' \end{matrix}\right] dx$$

The I-function of one variable is expressed in terms of a Mellin-Barne type contour integral, which is obtained by generalizing the class of polynomials and Bessel's function in series form.

$$I = \int_0^\infty x^{\lambda-1} \left\{ \sum_{l=0}^{\infty} \frac{1}{\Gamma(1+\nu+l)} f(l) \left(x + a + \sqrt{x^2 + 2ax}\right)^{\nu+2l} \sum_{k=0}^{[h/w]} F(k) \right.$$

$$\left. \times \left(x + a + \sqrt{x^2 + 2ax}\right)^{-\gamma k} \frac{1}{2\pi\omega} \int_L \theta(\xi) y^\xi \left(x + a + \sqrt{x^2 + 2ax}\right)^{-\mu\xi} d\xi \right\} dx$$

In cases where it is allowed, we have modified the order of integration.

$$I = \frac{1}{2\pi\omega} \int_L \theta(\xi) y^\xi \sum_{l=0}^{\infty} f(l) \frac{1}{\Gamma(1+\nu+l)} \sum_{k=0}^{[h/w]} F(k)$$
$$\times \left\{ \int_0^\infty x^{\lambda-1} \left(x + a + \sqrt{x^2 + 2ax}\right)^{-(\mu\xi+\gamma k-\nu-2l)} dx \right\} d\xi$$

Using x-integral evaluation with equation, we get

$$I = \frac{1}{2\pi\omega} \int_L \theta(\xi) y^\xi \sum_{l=0}^{\infty} f(l) \frac{1}{\Gamma(1+\nu+l)} \sum_{k=0}^{[h/w]} F(k)$$

$$\times \left\{ 2a^{-(\mu\xi+\gamma k-\nu-2l)} (\mu\xi + \gamma k - \nu - 2l) \left(\frac{a}{2}\right)^\lambda \Gamma(2\lambda) \frac{\Gamma(\mu\xi + \gamma k - \nu - 2l - \lambda)}{\Gamma(1 + \lambda + \mu\xi + \gamma k - \nu - 2l)} \right\} d\xi$$

by shuffling the words around, we obtain

$$I = 2\left(\frac{a}{2}\right)^\lambda \Gamma(2\lambda) \sum_{l=0}^{\infty} f(l) \sum_{k=0}^{[h/w]} F(k)\, a^{-(\mu\xi+\gamma k-\nu-2l)} \times$$

$$\left\{ \frac{1}{2\pi\omega} \int_L \theta(\xi) y^\xi \frac{\Gamma(1 - (\nu + 2l - \gamma k) + \mu\xi)\Gamma(1 - (1 + \nu + 2l + \lambda - \gamma k) + \mu\xi)}{\Gamma(1 + \nu + l)\Gamma(1 - (1 + \nu + 2l - \gamma k) + \mu\xi)\Gamma(1 - (\nu + 2l - \lambda - \gamma k) + \mu\xi)} \right\} d\xi$$

The right-hand side of equation is found by expressing the contour integral of Mellin-Barne's type in terms of an I-function of one variable.

I-FUNCTION OF TWO VARIABLES AND GENERAL CLASS OF POLYNOMIALS

Here, we obtain three integrals for the combination of the I-function in two variables provided by Goyal and Agrawal and the second class of multivariable polynomials given by Srivastava.

$$\int_0^\infty \left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-\rho-1} S_{V_1 \ldots V_t}^{U_1 \ldots U_t}\left[d_1\left(\left(ax + \frac{b}{x}\right)^2 + c\right)^{-\lambda_1} \ldots d_t\left(\left(ax + \frac{b}{x}\right)^2 + c\right)^{-\lambda_t}\right]$$
$$\times I_{p,q:R'}^{m_1,n_1:R}\left[\begin{matrix} z_1\left(\left(ax + \frac{b}{x}\right)^2 + c\right)^{-\sigma_1} \\ z_2\left(\left(ax + \frac{b}{x}\right)^2 + c\right)^{-\sigma_2} \end{matrix}\right] dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab + c)^{\rho+\frac{1}{2}}} \sum_{k_1=0}^{[V_1/U_1]} \ldots \sum_{k_t=0}^{[V_t/U_t]} f(k_1 \ldots k_t)\, (4ab + c)^{-(\sum_{i=1}^{t} \lambda_i k_i)} \times$$

$$I_{p+1,q+1:R'}^{m_1+1,n_1:R}\left[\begin{matrix} z_1(4ab + c)^{-\sigma_1} \\ z_2(4ab + c)^{-\sigma_2} \end{matrix} \Big| \begin{matrix} (1/2 - \rho - \sum_{i=1}^{t} \lambda_i k_i, \sigma_1, \sigma_2), (e_p: E_p, E_p'): W \\ (f_q: F_q, F_q'), (-\rho - \sum_{i=1}^{t} \lambda_i k_i, \sigma_1, \sigma_2): W' \end{matrix}\right]$$

The necessary conditions for equation to hold are

(i)
$a > 0; b \ge 0; c + 4ab > 0, \rho > 0, \lambda_i > 0, \sigma_1 > 0, \sigma_2 > 0$

(ii) $\Re(\rho) + \sigma_1 \min_{1\le j\le m_2} \Re\left(\frac{b_j}{\beta_j}\right) + \sigma_2 \min_{1\le j\le m_3} \Re\left(\frac{d_j}{\delta_j}\right) + \frac{1}{2} > 0$

(iii) $|\arg z_1| < \frac{A_i\pi}{2}, |\arg z_2| < \frac{B_i\pi}{2}$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\rho-1} S_{V_1\dots V_t}^{U_1\dots U_t}\left[d_1\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\lambda_1}\dots d_t\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\lambda_t}\right]$$

$$\times I_{p,q:R'}^{m_1,n_1:R}\begin{bmatrix} z_1\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\sigma_1} \\ z_2\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\sigma_2}\end{bmatrix}dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\rho+\frac{1}{2}}} \sum_{k_1=0}^{[V_1/U_1]} \dots \sum_{k_t=0}^{[V_t/U_t]} f(k_1\dots k_t)\,(4ab+c)^{-(\sum_{i=1}^t \lambda_i k_i)}$$

$$\times I_{p+1,q+1:R'}^{m_1,n_1+1:R}\left[\begin{matrix} z_1(4ab+c)^{-\sigma_1} \\ z_2(4ab+c)^{-\sigma_2}\end{matrix}\middle| \begin{matrix}(1/2-\rho-\sum_{i=1}^t \lambda_i k_i,\sigma_1,\sigma_2),(e_p:E_p,E_p'):W \\ (f_q:F_q,F_q'),(-\rho-\sum_{i=1}^t \lambda_i k_i,\sigma_1,\sigma_2):W'\end{matrix}\right]$$

The necessary conditions for equation to hold are

(i) $a \ge 0; b > 0; c + 4ab > 0, \rho > 0, \lambda_i > 0, \sigma_1 > 0, \sigma_2 > 0$

(ii) $\Re(\rho) + \sigma_1 \min_{1\le j\le m_2} \Re\left(\frac{b_j}{\beta_j}\right) + \sigma_2 \min_{1\le j\le m_3} \Re\left(\frac{d_j}{\delta_j}\right) + \frac{1}{2} > 0$

(iii) $|\arg z_1| < \frac{A_i\pi}{2}, |\arg z_2| < \frac{B_i\pi}{2}$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\rho-1} S_{V_1\dots V_t}^{U_1\dots U_t}\left[d_1\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\lambda_1}\dots d_t\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\lambda_t}\right]$$

$$\times I_{p,q:R'}^{m_1,n_1:R}\begin{bmatrix} z_1\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\sigma_1} \\ z_2\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\sigma_2}\end{bmatrix}dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\rho+\frac{1}{2}}} \sum_{k_1=0}^{[V_1/U_1]} \dots \sum_{k_t=0}^{[V_t/U_t]} f(k_1\dots k_t)\,(4ab+c)^{-(\sum_{i=1}^t \lambda_i k_i)}$$

$$\times I_{p+1,q+1:R'}^{m_1,n_1+1:R}\left[\begin{matrix} z_1(4ab+c)^{-\sigma_1} \\ z_2(4ab+c)^{-\sigma_2}\end{matrix}\middle| \begin{matrix}(1/2-\rho-\sum_{i=1}^t \lambda_i k_i,\sigma_1,\sigma_2),(e_p:E_p,E_p'):W \\ (f_q:F_q,F_q'),(-\rho-\sum_{i=1}^t \lambda_i k_i,\sigma_1,\sigma_2):W'\end{matrix}\right]$$

The necessary conditions for equation to hold are

(i) $a > 0; b > 0; c + 4ab > 0, \rho > 0, \lambda_i > 0, \sigma_1 > 0, \sigma_2 > 0$

(ii) $\Re(\rho) + \sigma_1 \min_{1\le j\le m_2} \Re\left(\frac{b_j}{\beta_j}\right) + \sigma_2 \min_{1\le j\le m_3} \Re\left(\frac{d_j}{\delta_j}\right) + \frac{1}{2} > 0$

(iii) $|\arg z_1| < \frac{A_i\pi}{2}, |\arg z_2| < \frac{B_i\pi}{2}$

Explanation of Proof: Let

$$I = \int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-\rho-1} S_{V_1\dots V_t}^{U_1\dots U_t}\left[d_1\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\lambda_1}\dots d_t\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\lambda_t}\right]$$

$$\times I_{p,q:R'}^{m_1,n_1:R}\begin{bmatrix} z_1\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\sigma_1} \\ z_2\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-\sigma_2}\end{bmatrix}dx$$

By writing the series expansion of the polynomials and the I-function in terms of a double Mellin-Barne's type contour integral, we get

$$I = \int_0^\infty x^{-\rho-1} \sum_{k_1=0}^{[V_1/U_1]} \dots \sum_{k_t=0}^{[V_t/U_t]} f(k_1\dots k_t)\left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-(\sum_{i=1}^t \lambda_i k_1)}$$

$$\times \left\{\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \varphi_1(\xi)\varphi_2(\eta)\psi(\xi,\eta) z_1^\xi z_2^\eta \left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-(\sigma_1\xi+\sigma_2\eta)} d\xi d\eta\right\}dx$$

If we are allowed to alter the order of integration, we get

$$I = \sum_{k_1=0}^{[V_1/U_1]} \dots \sum_{k_t=0}^{[V_t/U_t]} f(k_1\dots k_t)\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \varphi_1(\xi)\varphi_2(\eta)\psi(\xi,\eta) z_1^\xi z_2^\eta$$

$$\times \left\{\int_0^\infty \left(\left(ax+\frac{b}{x}\right)^2+c\right)^{-(\rho+\sum_{i=1}^t \lambda_i k_1+\sigma_1\xi+\sigma_2\eta)-1} dx\right\} d\xi d\eta$$

By plugging into the previous equation to get its inner integral, we get

$$I = \sum_{k_1=0}^{[V_1/U_1]} \dots \sum_{k_t=0}^{[V_t/U_t]} f(k_1\dots k_t)\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \varphi_1(\xi)\varphi_2(\eta)\psi(\xi,\eta) z_1^\xi z_2^\eta \times$$

$$\left[\frac{\sqrt{\pi}}{2a(4ab+c)^{(\rho+\sum_{i=1}^t \lambda_i k_1+\sigma_1\xi+\sigma_2\eta+\frac{1}{2})}} \frac{\Gamma\left(\rho+\sum_{i=1}^t \lambda_i k_i+\sigma_1\xi+\sigma_2\eta+\frac{1}{2}\right)}{\Gamma(\rho+\sum_{i=1}^t \lambda_i k_i+\sigma_1\xi+\sigma_2\eta+1)}\right] d\xi\, d\eta$$

The right-hand side of equation is found by reinterpreting the resultant contour integral in terms of a two-variable I-function. Integrals are derived in a similar fashion using the solutions to equations.

Special Cases

All exponents cancel at r=2, and Aj $(j = 1, . . ., p)$, Bj $(j = 1, . . ., q)$, $C(i)\ j$ $(j = 1, . . ., pi,\ i = 1 . . ., r)$, and $D(i)\ j$ $(1, . . ., qi,\ i = 1 . . ., r)$ Similar findings in H-function of two variables are obtained as the I-function of "r" variables reduces to H-function of two variables.

CONCLUSION

The theory of functions of several complex variables is the branch of mathematics dealing with functions The results presented in this paper generalize the results of H-function of "r" variables. The generality of the integrals is quite broad. It is possible to get many different specialized functions by appropriately specializing the I-function of two variables. Again, they may be reduced to other multivariate hypergeometric polynomials and classical orthogonal polynomials of one or more variables by appropriately specializing the coefficients. Thus, the equivalent conclusions involving a large number of simpler functions and polynomials would follow from our findings.

REFERENCES

1. Rezepin, alexandr & amirova, taya & mishina, vera. (2017). The research of the production function of an industrial enterprise. Shs web of conferences. 35. 01038. 10.1051/shsconf/20173501038.
2. Gerck, edgardo & gerck, ed. (2019). Overview of complex analysis and applications.
3. Shadab, mohd & jabee, saima & choi, junesang. (2018). An extended beta function and its applications. Far east journal of mathematical sciences. 103. 235-251. 10.17654/ms103010235.
4. Rachel webb, the abelian–nonabelian correspondence for i-functions, international mathematics research notices, volume 2023, issue 3, february 2023, pages 2592–2648,
5. Guala, francesco & hindriks, frank. (2019). Institutions and functions. 10.1007/978-3-030-32618-0_2.
6. Rachel webb, the abelian–nonabelian correspondence for i-functions, international mathematics research notices, volume 2023, issue 3, february 2023, pages 2592–2648,
7. Abayatilake, pujani & blessing, lucienne. (2021). The application of function models in software design: a survey within the software community. International journal of software engineering. 9. 27-62.
8. Balwani, mukesh & hiwase, prashant & agrawal, n. (2015). Application of function analysis for project elements. 1.
9. Shadab, mohd & jabee, saima & choi, junesang. (2018). An extended beta function and its applications. Far east journal of mathematical sciences. 103. 235-251. 10.17654/ms103010235.
10. Cusick, thomas & stănică, pante. (2017). Cryptographic boolean functions and applications: second edition.
11. Tuong, hoa & moslehpour, massoud & hau, nguyen & huy, lam minh & wong, wing-keung. (2021). Applications of the theory of derivative in decision science and education. 24. 1-20. 10.6084/m9.figshare. 14931546.v1.
12. Hofmann, peter & jöhnk, jan & protschky, dominik & urbach, nils. (2020). Developing purposeful ai use cases - a structured method and its application in project management.
13. Gyöngyi, szanyi. (2019). "On the way" to the function concept - experiences of a teaching experiment. Teaching mathematics and computer science. 17. 17-39. 10.5485/tmcs.2019.0455.
14. Leonhardt, daniel & haffke, ingmar & kranz, johann & benlian, alexander. (2017). Reinventing the it functions: the role of it agility and it ambidexterity in supporting digital business transformation.
15. Rawash, hassan. (2014). Electronic management's contribution to the development of managerial functions. Academic research international. 5. 213-225.

Corresponding Author

Priti Pandey*

Research Scholar Mathematics, Awadhesh Pratap Singh University, Rewa, Madhya Pradesh.