A Study of Some Important Theorem on Fuzzy Linear Space

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Abstract:- ***In this paper we introduced the proof of some new theorem on fuzzy linear space.***

Key Words:- ***Fuzzy set, fuzzy linear space.***

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INTRODUCTION - 1.1

Let S be a nonempty set. Then a characteristic function μ :S $\rightarrow$[0,1] is called a fuzzy set of S Let X be a field and be a fuzzy field in X with characteristic function μ_F . Let Y be a linear space over the fuzzy field F and V is a fuzzy subset of Y with characteristic function μ_v. Then V is called a fuzzy linear space in Y if and only if, it satisfied the following condition.

(i) μ_v (x+y) $\geq$ min{ μ_v (x), μ_v (y)} for all x,y $\in$ Y

(ii) μ_v (λ x) $\geq$ min{ μ_v (λ), μ_v (x)} for all $\lambda \in$ F and x $\in$ Y

(iii) μ_v (o)=1

THEOREM – 1.2

Let us suppose that Y and Z be vector space (linear space) over a field F in a field X and f be liner transformation from Y in to Z and W be a fuzzy linear space in Z, then the inverse image of W that is f^{-1}(w) be a fuzzy linear space in Y.

PROOF:-

Let us assume that Y and Z be two vector space over a fuzzy field F in a field X. Again we assume that f be a linear transformation of Y into Z. Let us suppose λ, μ□X and a,b□Y. Then we take from the Properties of linear transformation that

$$f(x+y) = f(x)+(y)$$
and
$$f(\lambda x) = \lambda f(x)$$

Again assume that w, be a fuzzy linear space in Z and f^{-1}[w] be the inverse image of W in Y

Then we prove that f^{-1}(w) be a fuzzy linear space in Y for all x,y□Y and λ, μ □X

Then

$$\mu_{f^{-1}(W)}(\lambda x+\mu y) = \mu_W [f\{\lambda(x) + \mu y\}]$$

= μ_W {λf(x) + μ f(y)}

≥ min[min{ μ_F (λ), μ_W f(x)},min{ μ_F (μ), μ_W f(y)}]

=min[min{ μ_F □□□, $\mu_{f^{-1}(W)}$ (x)}min{ μ_F (μ)□ $\mu_{f^{-1}(W)}$ (y)}]

Hence

$\mu_{f^{-1}(W)}$ (λx+ μ y)□□min[min{ μ_F □□□, $\mu_{f^{-1}(W)}$ (x)}]min{ μ_F (μ), $\mu_{f^{-1}(W)}$ (y)}]

Thus it is clear that $\mu_{f^{-1}(W)}$ the inverse image of W in a fuzzy linear spaces in Y.

THEOREM - 1.3

If V be a fuzzy linear space in a linear spaces Y over an ordinary filed F in X, then μ_V (□x) = μ_V (x), for every x□Y and for all o□□□□X.

PROOF:-

Let us suppose that V be a fuzzy linear space in a linear space Y over an ordinary field F in X.

Then for all x□Y, we take

μ_V (□x)□□ μ_V (x)□
□□
□i)

Now for all □□□□□F and all x□Y

Then we have

μ_V (X) = μ_V (EX)= μ_V {(λ^{-1} λ)X}

= μ_V {(λ^{-1} (λ X)}□□ μ_V (□X)

i.e. μ_V (X)□□ μ_V (□X) ………………..(II)

Thus From (I) And (Ii), We Get

μ_V (□X)= μ_V (X) FOR EVERY O□□□□X AND X□Y.

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